Investigation of State Transition Phenomena in Cross-Coupled Chaotic Circuits

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***Abstract*— Studies on chaos synchronization in coupled chaotic circuits are extensively carried out in various fields. In this study, two simple chaotic circuits cross-coupled by inductors are investigated. Interesting state transition phenomenon around chaos synchronization is observed by computer simulations and circuit experiments.**

I. Introduction

Synchronization phenomena in complex systems are very good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. Studies on synchronization phenomena of coupled chaotic circuits are extensively carried out in various fields [1]-[9]. We consider that it is very important to investigate the phenomena related with chaos synchronization to realize future engineering application utilizing chaos. We have reported an interesting state transition phenomenon observed in simple coupled chaotic circuits [10].

In this study, we investigate the state transition phenomena in detail by computer simulations and circuit experiments.

II. Circuit Model

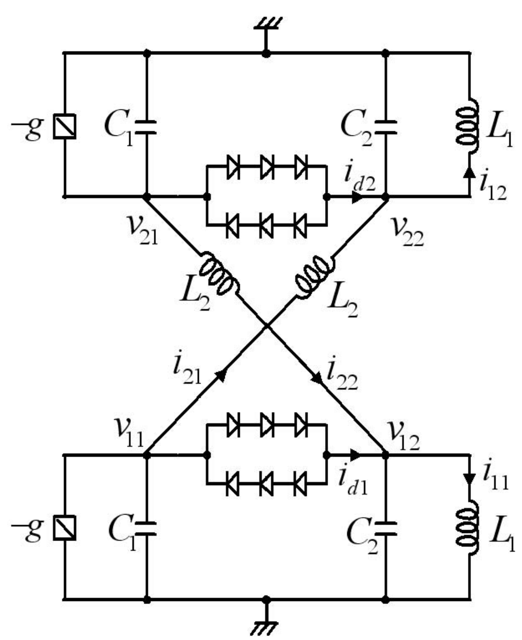

Fig. 1. Circuit model.

Figure 1 shows the circuit model [10]. In the circuit, two Shinriki-Mori chaotic circuits [11][12] are cross-coupled via inductors L_2.

By using the following variables and the parameters,

$$\begin{cases} x_1 = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{11}}{V}, \quad y_1 = \dfrac{v_{11}}{V}, \quad z_1 = \dfrac{v_{12}}{V}, \\ x_2 = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{21}}{V}, \quad y_2 = \dfrac{v_{21}}{V}, \quad z_2 = \dfrac{v_{22}}{V}, \\ w_1 = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{12}}{V}, \quad w_2 = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{22}}{V}, \\ \alpha = \dfrac{C_2}{C_1}, \quad \beta = \sqrt{\dfrac{L_1}{C_2}}\,G, \quad \gamma = \sqrt{\dfrac{L_1}{C_2}}\,g, \\ \delta = \dfrac{L_1}{L_2}, \quad t = \sqrt{L_1 C_2}\,\tau, \end{cases} \tag{1}$$

the normalized circuit equations are given as follows.

$$\begin{cases} \dot{x}_1 = z_1 \\ \dot{x}_2 = z_2 \\ \dot{y}_1 = \alpha\{\gamma y_1 - w_1 - \beta f(y_1 - z_1)\} \\ \dot{y}_2 = \alpha\{\gamma y_2 - w_2 - \beta f(y_2 - z_2)\} \\ \dot{z}_1 = \beta f(y_1 - z_1) + w_2 - x_1 \\ \dot{z}_2 = \beta f(y_2 - z_2) + w_1 - x_2 \\ \dot{w}_1 = \delta(y_1 - z_2) \\ \dot{w}_2 = \delta(y_2 - z_1) \end{cases} \tag{2}$$

where f are the nonlinear functions corresponding to the $v - i$ characteristics of the nonlinear resistors consisting of the diodes and are assumed to be described by the following 3-segment piecewise-linear functions:

$$f(y_1 - z_1) = \begin{cases} y_1 - z_1 - 1 & (y_1 - z_1 > 1) \\ 0 & (|y_1 - z_1| \le 1) \\ y_1 - z_1 + 1 & (y_1 - z_1 < -1) \end{cases} \tag{3}$$

$$f(y_2 - z_2) = \begin{cases} y_2 - z_2 - 1 & (y_2 - z_2 > 1) \\ 0 & (|y_2 - z_2| \le 1) \\ y_2 - z_2 + 1 & (y_2 - z_2 < -1). \end{cases} \tag{4}$$

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III. State Transition Phenomenon

From the circuit in Fig. 1, we could observe interesting state transition phenomenon [10]. An example of the phenomenon is shown in Figs. 2 and 3. Figure 2 shows computer calculated results of Eq. (2) with the Runge-Kutta method. While, Fig. 3 shows the corresponding circuit experimental result. The two circuits exhibited chaos but almost synchronized in in-phase in the sense that the attractor was almost in the quadrant I or III on the $y_1 - y_2$ plane. The behaviors of the circuits are very interesting because the solutions on the $y_i - z_i$ planes seem to be attracted to the fixed points located at around $(y_i,\ z_i)$=($\pm$1.2, 0). However, after converging to the fixed points the solution abruptly moves toward the other fixed point. When one circuit switches to/from the positive region from/to the negative region in this way, the other follows the transition after a few instants.

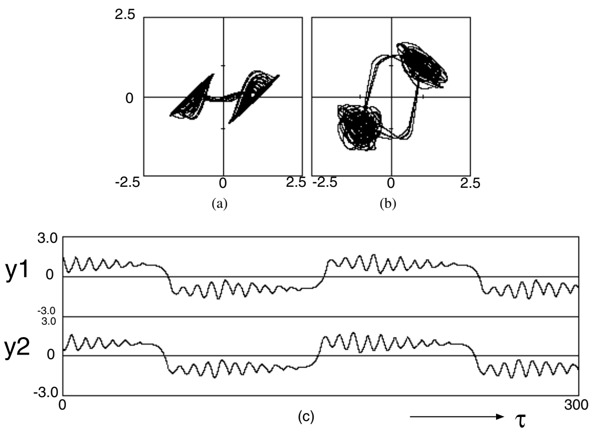

Fig. 2. State transition phenomenon around in-phase synchronization (computer calculated result). $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and $\delta = 0.005$. (a) Attractor on $y_1 - z_1$ plane. (b) Attractor on $y_1 - y_2$ plane. (c) Time waveform.

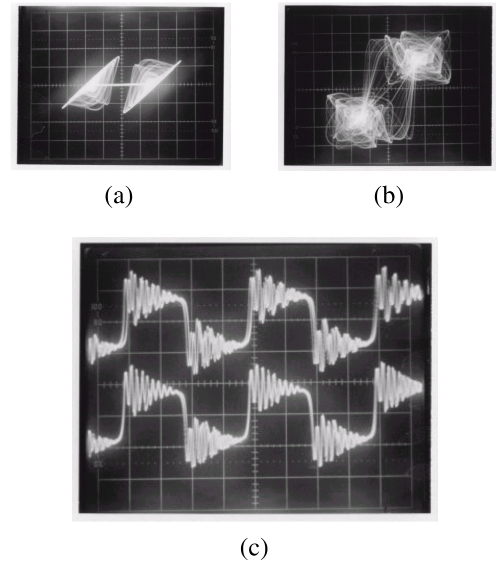

Fig. 3. State transition phenomenon around in-phase synchronization (circuit experimental result). L_1 = 9.93mH, L_2 = 800mH, C_1=32.8nF, and C_2=49.5nF, and g=683mS. (a) Attractor on $v_{11} - v_{12}$ plane. Horizontal and vertical: 1 V/div. (b) Attractor on $v_{11} - v_{21}$ plane. Horizontal and vertical: 1 V/div. (c) Time waveform v_{11} and v_{21}. Horizontal 1.0 ms/div and vertical: 2 V/div.

IV. Detailed investigation

In this study, we investigate the above-mentioned state transition phenomena in detail. First, we vary the coupling parameter δ and observe the phenomena. The results are shown in Fig. 4. We can see that the sojourn time between the transitions becomes shorter as increasing the coupling parameter δ. The corresponding circuit experimental results are shown in Fig. 5. We can say that this interesting phenomenon can be observed from both computer calculations and circuit experiments.

Next, Fig. 6 shows how the variables change around the transitions. The variables x and z correspond to the current and the voltage of the LC resonator in the original chaotic circuit. Hence, they oscillate around zero and its amplitude decays as time goes. On the other hand, the variable y corresponds to the voltage across the capacitor in parallel with the negative resistor and the value tends to go toward the equilibrium points located around $\pm$1.2.

When the amplitude of the oscillation of the LC resonator approaches to zero, the state transition occurs; namely y abruptly moves toward to the other fixed point quickly. Though the mechanism of this transition cannot be explained very well, we consider that this is because the coupling inductor can be regarded as a short circuit when the oscillation stops. In other words, the coupling inductor plays a role of a kind of switch depending on the oscillation amplitude of the LC resonator.

Moreover, Fig. 7 shows the magnification of the time waveform of y. We can see that the switching timing of y_1 and y_2 are synchronized in in-phase, however, small oscillations between the transitions are synchronized in anti-phase.

V. Anti-phase state transition

Next, similar state transition can be observed around anti-phase synchronization as shown in Figs. 8 and 9. Similar to the in-phase, the sojourn time between the transitions decreases as the coupling δ increases.

VI. Quadrature-phase state transition

We also observed an interesting state transition around quadrature-phase, namely 90 degrees. The computer calculated results are shown in Fig. 10 and the corresponding circuit experimental results are shown in Fig. 11. In this state, the solution moves to the quadrant I, II, III, and IV on the $y_1 - y_2$ plane in order as shown in the figures.

VII. Conclusions

In this study, we have investigated interesting state transition phenomenon observed from two Shinriki-Mori chaotic circuits cross-coupled by inductors.

Investigating the coexistence of the states and statistical analysis of the observed phenomena are our important future work as well as more detailed explanation of the mechanism of their generations.

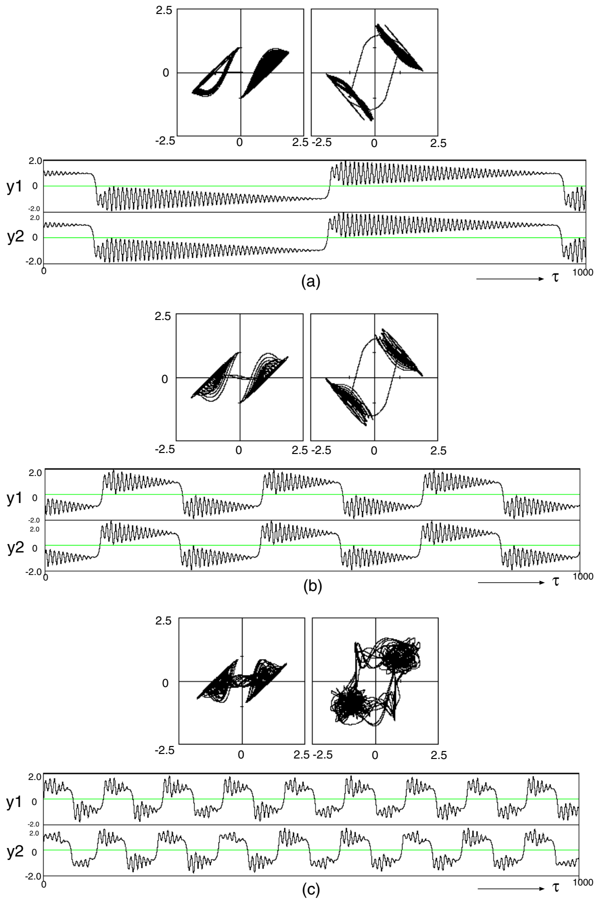

Fig. 4. State transition phenomenon for different coupling parameters (computer calculated result). $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and δ= (a) 0.001, (b) 0.003, (c) 0.008.

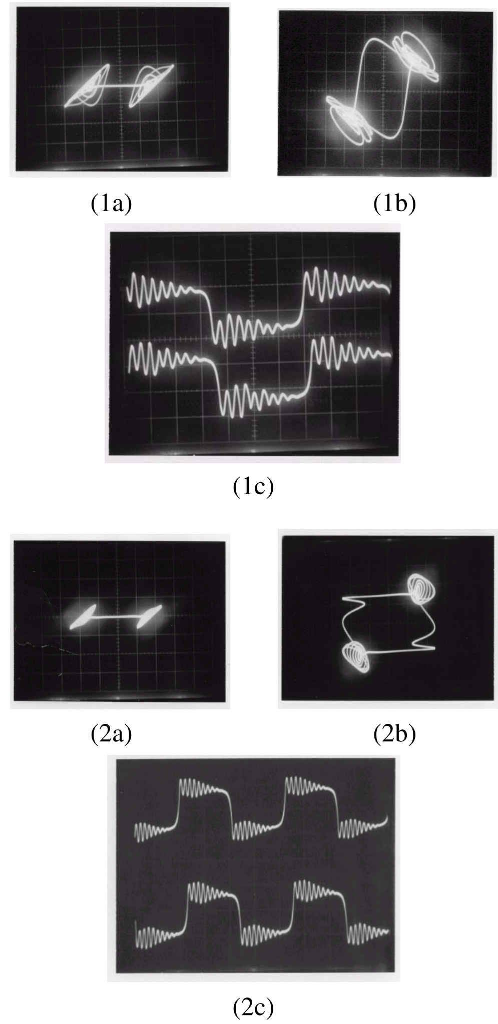

Fig. 5. State transition phenomenon for different coupling parameters (circuit experimental result). L_1 = 9.93mH, C_1=32.8nF, and C_2=49.5nF. (1) L_2 = 800mH and g=626mS. (2) L_2 = 1.2H and g=495mS. (a) Attractor on $v_{11} - v_{12}$ plane. Horizontal and vertical: 1 V/div. (b) Attractor on $v_{11} - v_{21}$ plane. Horizontal and vertical: 1 V/div. (c) Time waveform v_{11} and v_{21}. Horizontal 0.5 ms/div for (1) and 1.0 ms/div for (2), and vertical: 2 V/div.

Acknowledgments

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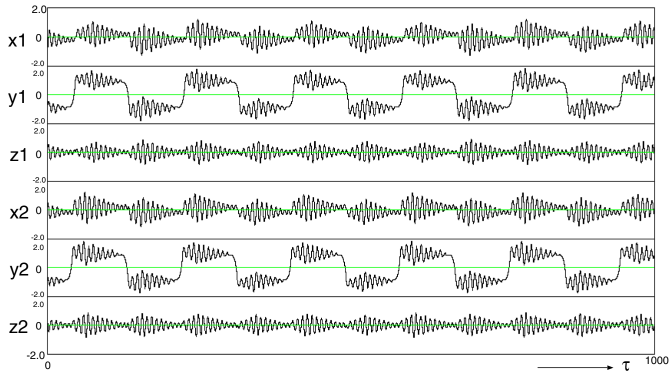

Fig. 6. Time waveforms around transition. $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and $\delta = 0.005$.

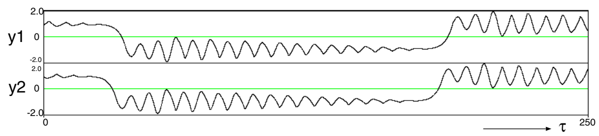

Fig. 7. Magnification of the time waveform around transition. $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and $\delta = 0.003$.

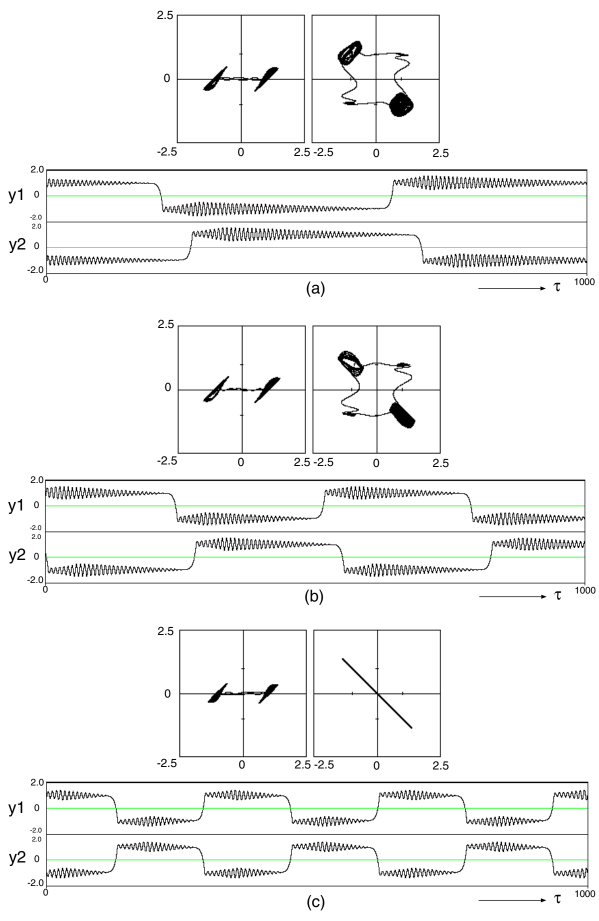

Fig. 8. Anti-phase state transition (computer calculated result). $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.1$,and (a) $\delta = 0.0005$, (b) $\delta = 0.0008$, (c) $\delta = 0.0014$.

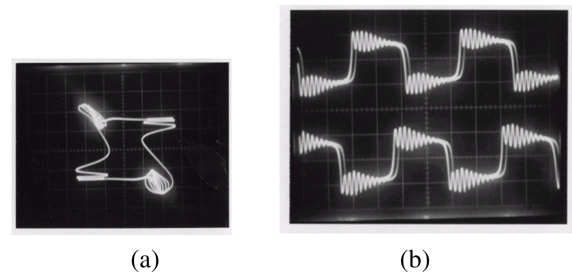

Fig. 9. Anti-phase state transition (circuit experimental result). L_1 = 9.93mH, L_2 = 1.2H, C_1=32.8nF, C_2=49.5nF, and g=495mS. (a) Attractor on $v_{11} - v_{21}$ plane. Horizontal and vertical: 1 V/div. (b) Time waveform v_{11} and v_{21}. Horizontal 1.0 ms/div and vertical: 2 V/div.

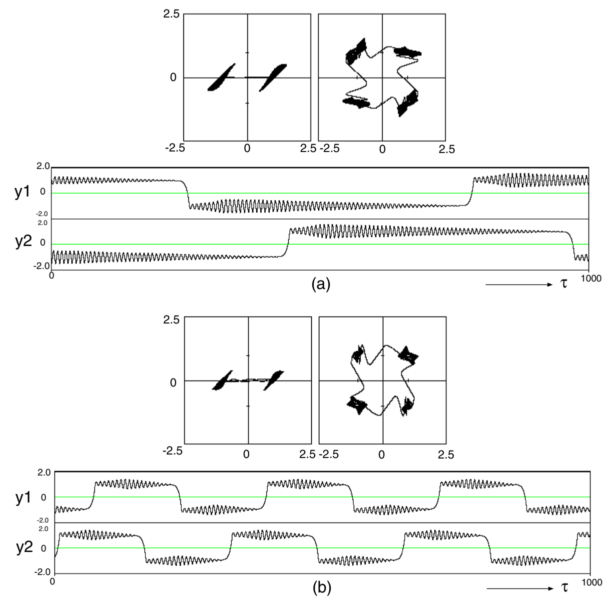

Fig. 10. Quadrature-phase state transition (computer calculated result). $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.1$,and (a) $\delta = 0.0004$, (b) $\delta = 0.0014$.

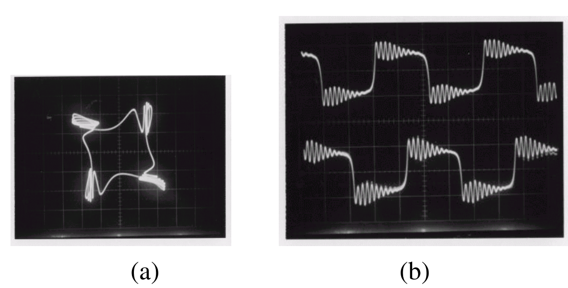

Fig. 11. Quadrature-phase state transition (circuit experimental result). L_1 = 9.93mH, L_2 = 1.2H, C_1=32.8nF, C_2=49.5nF, and g=495mS. (a) Attractor on $v_{11} - v_{21}$ plane. Horizontal and vertical: 1 V/div. (b) Time waveform v_{11} and v_{21}. Horizontal 1.0 ms/div and vertical: 2 V/div.