Solving Ability of Hopfield Neural Network with Scale-Rule Noise for QAP

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***Abstract*— One of the applications of neural network is solving combinatorial optimization problems. In our past study, the solving ability of the Hopfield Neural Network with noise for quadratic assignment problem is investigated. However, even if we injected the noise to the network, the optimal solution cannot occasionally be found. In this study, we propose the method adding scale-rule noise to the Hopfield Neural Network to achieve better performance. By computer simulations solving quadratic assignment problem, we evaluate the performance of the method.**

I. Introduction

Hopfield Neural Network (abbr. NN) [1] is one of the important tools of solving combinatorial optimization problems. The global minimum can be searched by energy decent principle of the Hopfield NN. However, in a lot of cases, the network finds a local minimum, and can not escape from there. In order to avoid this critical problem, many researchers proposed the method adding some kinds of noises to the Hopfield NN. Hayakawa and Sawada pointed out the chaos near the three-periodic window of the logistic map gains the best performance as noise [2]. They concluded that the good result might be obtained by a property of the chaos noise; short time correlations of the time-sequence. Hasegawa et al. investigated solving abilities of the Hopfield NN with various surrogate noise, and they concluded that the effects of the chaotic sequence for solving optimization problems can be replaced by stochastic noise with similar autocorrelation [3]. In our past study, we proposed the method to change the amplitude of the chaos noise according to the state; Hopping chaos [4]. By using this method, the network could search many good solutions. However, this method was not still effective to find near optimal solutions for difficult problems.

Since the scale-free network were discovered by Barabasi et al. [5], studies assessing the influence of this property on the efficiency of networks have been carried out in various fields. Figure 1 shows the scale-free network model. In the scale-free network, a lot of nodes have only few connections, some nodes (marked in black) act as highly connected hubs. So, the small number of important nodes have a lot of connections. This distinction is captured in a more quantitative way by the distribution of the number of links vs. the number of nodes, as shown in Fig. 2. Mathematically, scale-free network are

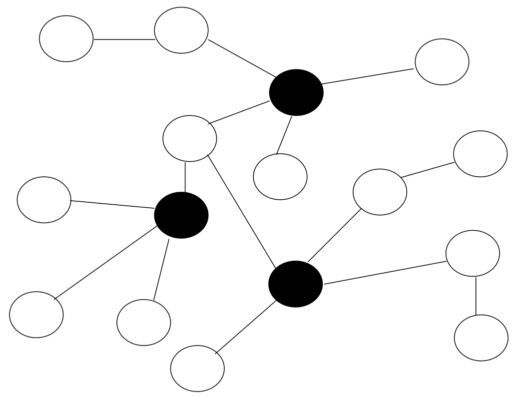

Fig. 1. Scale-free network model.

characterized by power law distributions (Fig. 2). Because scale-rules emerge in many areas and disciplines of science (e.g. engineering, economics, social sciences and so on), we expect that also in the development of neural networks, they will play an important role.

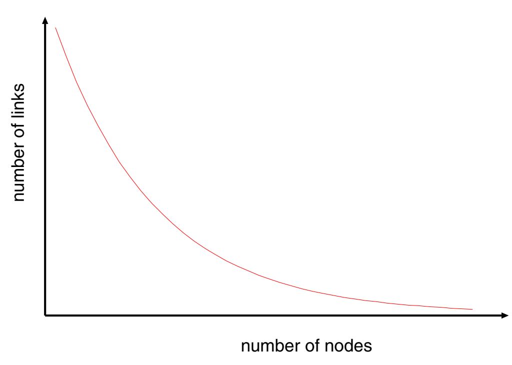

Fig. 2. Scale-free distribution.

In this study, we propose a method to add noise of different amplitudes determined by a scale-rule. Adding the noise is effective for the solution to escape from the local minimum, but at the same time it may obstruct the energy decent principle. If the noise with a small amplitude is injected to important neurons and the noise with a large amplitude

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is injected to unimportant neurons, the solution may search only around the global optimum without wandering away. We investigate solving ability of the Hopfield NN with the scale-rule noise for QAP. We confirm that the method is effective to solve QAP by computer simulations.

II. Solving QAP with Hopfield NN

Various methods are proposed for solving QAP which is one of the NP-hard combinatorial optimization problems. We explain QAP with a factory arrangement problem. The problem is given by two matrices, distance matrix C denoting the distances between the factories and flow matrix D denoting the flow of the products between the factories, and is to find the permutation P which corresponds to the minimum value of the objective function $f(P)$ in Eq. (1).

$$f(P) = \sum_{\mathrm{i}=1}^{N} \sum_{\mathrm{j}=1}^{N} C_{\mathrm{ij}} D_{\mathrm{p(i)p(j)}}, \tag{1}$$

where C_{ij} and D_{ij} are the (i,j)-th elements of C and D, respectively, p(i) is the i-th element of vector P, and N is the size of the problem. There are many real applications which are formulated by Eq. (1). Other examples are the placement of logical modules in an IC chip and the distribution of medical services in large hospital.

Because the QAP is very difficult, it is almost impossible to solve the optimum solution in large problems. The largest problem which is solved by deterministic methods may be only 24 in recent study. Further, computation times is very long to obtain the exact optimum solution. Therefore, it is usual to develop heuristic methods which search nearly optimal solutions in reasonable time.

For solving N-element QAP by the Hopfield NN, $N \times N$ neurons are required and the following energy function is defined to fire (i,j)-th neuron at the optimal position:

$$E = \sum_{\mathrm{i,m}=1}^{N} \sum_{\mathrm{j,n}=1}^{N} w_{\mathrm{im;jn}} x_{\mathrm{im}} x_{\mathrm{jn}} + \sum_{\mathrm{i,m}=1}^{N} \theta_{\mathrm{im}} x_{\mathrm{im}}. \tag{2}$$

The neurons are coupled each other with weight between (i,m)-th neuron and (j,n)-th neuron and the threshold of the (i,m)-th neuron are described by:

$$w_{\mathrm{im;jn}} = -2\left\{ A(1-\delta_{\mathrm{mn}})\delta_{\mathrm{ij}} + B\delta_{\mathrm{mn}}(1-\delta_{\mathrm{ij}}) + \frac{C_{\mathrm{ij}} D_{\mathrm{mn}}}{q} \right\} \tag{3}$$

$$\theta_{\mathrm{im}} = A + B \tag{4}$$

where A and B are positive constant, and δ_{ij} is Kroneker's delta. The state of $N \times N$ neurons are asynchronously updated due to the following difference equation:

$$x_{\mathrm{im}}(t+1) = f\left(\sum_{\mathrm{j,n}=1}^{N} w_{\mathrm{im;jn}} x_{\mathrm{im}}(t) x_{\mathrm{jn}}(t) - \theta_{\mathrm{im}} + \beta z_{\mathrm{im}}(t) \right) \tag{5}$$

where f is sigmoidal function defined as follows:

$$f(x) = \frac{1}{1 + \exp\left(-\frac{x}{\epsilon}\right)} \tag{6}$$

z_{im} is additional noise injected to the network, and β limits amplitude of the noise.

III. Chaos noise

In this section, we describe chaos noise injected to the Hopfield NN. The logistic map is used to generate the chaos noise:

$$\hat{l}_{\mathrm{im}}(t+1) = \alpha_l(t)(1 - \hat{l}_{\mathrm{im}}(t)). \tag{7}$$

Varying parameter α_l, Eq. (7) behaves chaotically via a periodic-doubling cascade. Further, it is well known that the map produces intermittent bursts just before periodic-windows appear. Figure 3 shows an example of the intermittency chaos near the three-periodic window obtained from Eq. (7) for $\alpha_l = 3.8274$. As we can see from the figure, the chaotic time series could be divided into two phases; laminar parts of periodic behaviour with period three and burst parts of spread points over the invariant interval. As increasing α_l, the ratio of the laminar parts becomes larger and finally the three-periodic window appears.

We use the intermittency chaos after the following normalization.

$$l_{\mathrm{im}}(t+1) = \frac{\hat{l}_{\mathrm{im}}(t) - \bar{y}}{\sigma_l} \tag{8}$$

where $\bar{l}$ and σ_l are the average and the standard deviation of $\hat{l}(t)$, respectively.

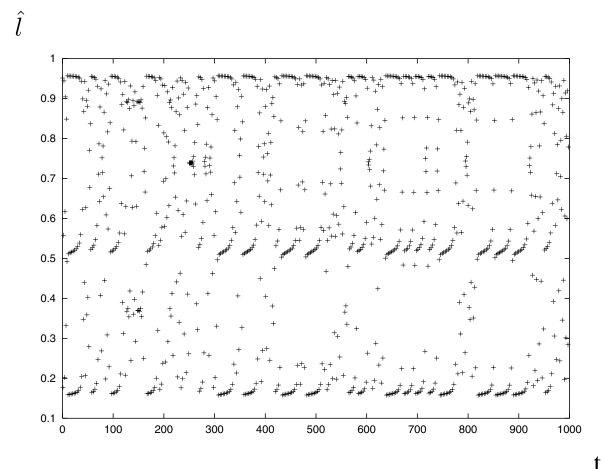

Fig. 3. Time series obtained from logistic map for $\alpha_l = 3.8274$.

IV. Scale-rule noise

We consider that there are some important cities and unimportant cities in a given problem. In other words, there exist some key cities in many problems, whose arrangements should be decided prior to the others to find better solutions. In order to avoid to obstruct the steepest decent principle of the neurons corresponding to such important cities, it may be better not to inject noise with a large amplitude to such neurons. In this study, we propose the method to add scale-rule noise based on the concept of the important city and the scale-rule.

Figure 4 shows the Hopfield NN model for N-element QAP. The neuron of each column expresses the city. The scale-rule noise is injected to all the neurons. However, their amplitudes are determined by the column number n_C of the neurons. Namely,

$$\beta = U^{n_C} \tag{9}$$

where U is the constant between 0 and 1, and n_C is the column number.

For the purpose of the comparison, we consider the case that the amplitude is determined by a linear function with the same maximum value.

Further, in order to compare them with the conventional method, the average value of the functions is set up to β_0 of the conventional method.

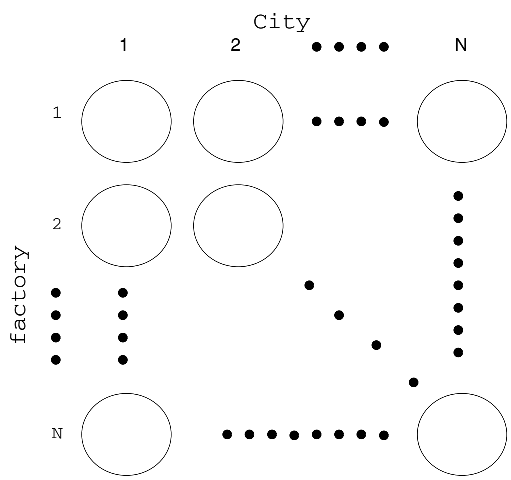

Fig. 4. Neurons of Hopfield NN model for N-elemnt QAP.

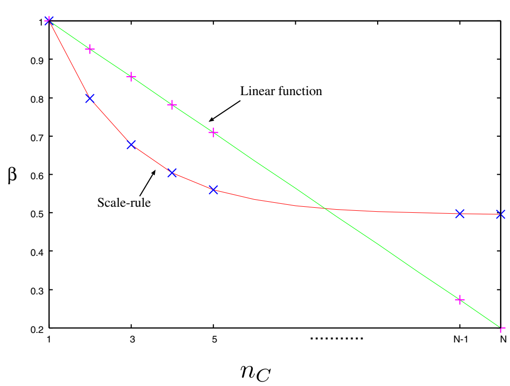

Fig. 5. Amplitude of scale-rule noise.

However, which is important city? Usually, we cannot know how important is each city. Hence, we combine a rearranging the column with the method. In this algorithm, we memorize the column with the first fired neuron. The first fired neuron means the largest output value among the all neurons. So, the column is rearranged to the last (right in Fig. 4). Next, the column with the second fired neuron is the rearranged to the next to the last. In this way, we rearrange the columns in the order of frequency of the firing.

V. Simulation Results

In this section, the simulated results of the Hopfield NN with the scale-rule noise for 12-elements of QAP are shown. The problem was chosen from the site QAPLIB [6] named "Nug12". The global minimum is known as 578. The parameters of the Hopfield NN are fixed as A = 0.9, B = 0.9, q = 140, and ϵ = 0.2. We carried out 100 trails of 10000 iterations. The parameter corresponding to the noise amplitude of the conventional method is fixed as β_0 = 0.6. The control parameter of the logistic map is fixed as α_l = 3.8274. The maximum value of the scale-rule and the linear functions are set to β_{max}=1.0. The parameter of the scale-rule function is fixed as U = 0.6.

TABLE I

Solving abilities for Nug12.

Iteration	Conventional	Scale-rule	Linear
1000	632.96	617.96	623.86
2000	623.30	613.32	616.12
3000	619.82	608.50	612.54
4000	616.18	605.78	609.68
5000	613.56	603.02	607.54
6000	612.68	600.94	606.20
7000	611.74	598.84	604.94
8000	610.48	597.82	604.12
9000	610.30	596.74	603.24
10000	609.96	**595.84**	602.36

Next, we tried another problem, problem name is "Tai12a". The global minimum is 224416. The parameters of the Hopfield NN are fixed as A = 0.9, B = 0.9, q = 16000, and ϵ = 0.02. We carried out 100 trails of 40000 iterations. Other parameters are β_0 = 0.5, α_l = 3.82676, β_{max}=1.3, and U = 0.5.

TABLE II

Solving abilities for Tai12a.

Iteration	Conventional	Scale-rule	Linear
4000	252624.44	242148.18	244849.92
8000	251337.90	239571.98	242123.36
12000	250291.38	238373.66	240738.44
16000	249638.16	237750.34	239504.80
20000	249520.72	237236.76	238803.02
24000	249495.62	236733.68	238198.66
28000	249458.04	236347.96	237838.76
32000	249335.58	236021.02	237469.98
36000	249228.40	235834.22	237209.98
40000	249225.84	**235549.42**	237121.14

Tables 1 and 2 show the average values of the best solutions obtained during a given iteration numbers for Nug12 and Tai12a, respectively. From these results, the proposed method gains better performance than the conventional method, and the scale-rule noise is better than the linear function noise. For Nug12, all the methods could find the global minimum. While, for Tai12a, the conventional method could not find the global minimum, but the proposed methods (both scale-rule and linear) could find it.

Finally, we investigate the performance against changing the distribution of the scale-rule function (Fig. 6). As the value of U increases, the scale-rule function becomes similar to the linear function.

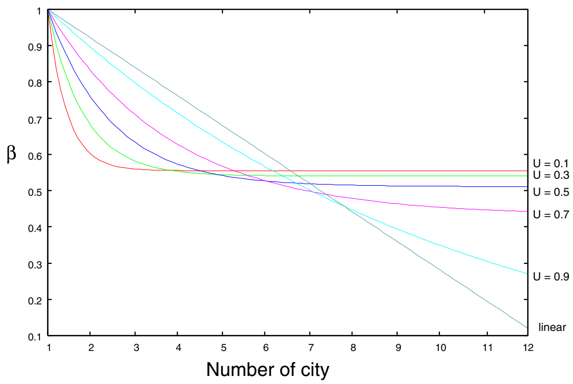

Fig. 6. Value of changing U.

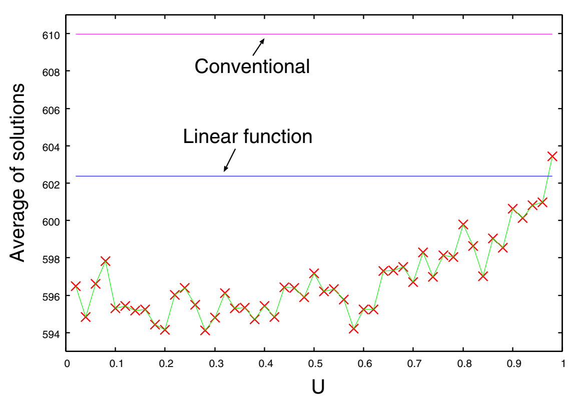

Fig. 7. Simulation result of Nug12.

From the results of changing U (Figs. 7 and 8), we can say that the scale-rule noise gains better performance than the linear function for almost U. When U approaches to 1.0, the performance becomes similar to the linear function.

VI. Conclusions

In this study, we proposed the method to add scale-rule noise based on the concept of the important city and the scale-rule. By combining the method with the rearrangement of the columns, we could inject the noise with a large amplitude to unimportant neurons and the noise with a small amplitude to important neurons. By computer simulations, we investigated the solving ability of the Hopfield NN with the scale-rule for QAP. We confirmed that the proposed method is effective to solve QAP.

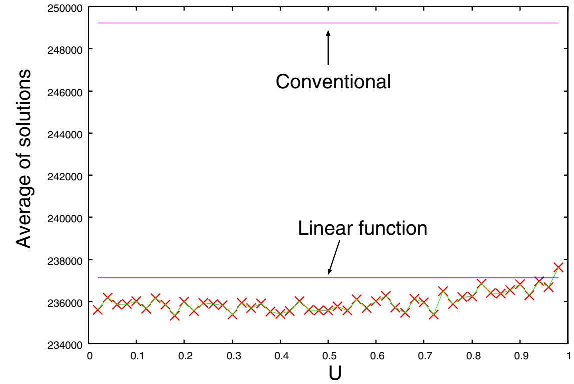

Fig. 8. Simulation result of Tai12a.

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