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Clustering Phenomena on Cellular Neural Networks Using Two kinds of Templates

Junji Fujii†, Yasuteru Hosokawa† and Yoshifumi Nishio‡

†Shikoku University
Furukawa, Ojin-cho, Tokushima, Japan
Email: {i1739082,hosokawa}@keiei.shikoku-u.ac.jp

‡Tokushima University
2-1 Minami-Josanjima, Tokushima, Japan
Email: nishio@ee.tokushima-u.ac.jp

Abstract

In this study, we investigate clustering phenomena on cellular neural networks using two kinds of templates. It was confirmed that these clusters are classified into four kinds of patterns.

1. Introduction

Cellular Neural Network (CNN) [1]-[3] is one kind of neural networks. The main characteristic is the local connection. There have been many studies on CNN and many kinds of CNN have been proposed. One of them is two-layer CNN. Two-layer CNN can generate many interesting phenomena. For instance, self-organizing pattern [4], active wave propagation [5] and so on. Investigating these phenomena contributes to understand complex systems and to apply them to engineering systems.

In our earlier study[6], we proposed CNN using two kinds of templates. In this system, clustering phenomena, pattern formation, active wave propagation and so on are observed. This system has a peculiar characteristic in order to investigate a new class of coupled oscillatory systems. Namely, a pair of cells which have two kinds of templates are needed for a simple oscillation. The cell connects with four the other cells and the other cell also connects with four neighbor cells. Like this, these cells are sharing a factor of the oscillation. This type of connection may be difficult to realize by coupling normal oscillators.

In this study, we investigate clustering phenomena observed in this system. In computer simulations, it is confirmed that these clusters are classified into four kinds of patterns. The relationship of patterns is also investigated.

2. CNN Using Two Kinds of Templates

Figure 1 shows a system model of our proposed system. We assume that the system has a two-dimensional M by N array structure. Each cell in the array is denoted as $c(i, j)$, where (i, j) is the position of the cell, $1 \leq i \leq M$ and $1 \leq j \leq N$. The coupling radius is assumed to be one. In this proposed CNN, two kinds of templates are used. Cells having

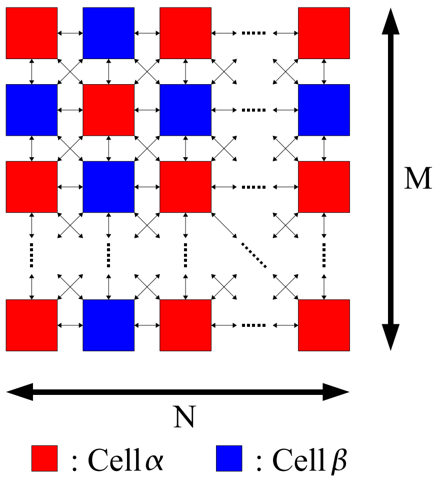


Figure 1: Structure of CNN using two kinds of templates.

one template are called as Cell α and the other are called as Cell β. These two types of the cells are placed as checkered. The state equations of the cells are given as follows:

1: The case that $i + j$ is an even number.

$$\frac{dx_{ij}}{dt} = -x_{ij} + I_\alpha + \sum_{c(k,l)} A_\alpha(i, j; k, l) y_{kl} + \sum_{c(k,l)} B_\alpha(i, j; k, l) u_{kl} \quad (1)$$

2: The case that $i + j$ is an odd number.

$$\frac{dx_{ij}}{dt} = -x_{ij} + I_\beta + \sum_{c(k,l)} A_\beta(i, j; k, l) y_{kl} + \sum_{c(k,l)} B_\beta(i, j; k, l) u_{kl} \quad (2)$$

$A_{\{\alpha\beta\}}(i, j; k, l) y_{kl}$, $B_{\{\alpha\beta\}}(i, j; k, l) u_{kl}$ and $I_{\{\alpha\beta\}}$ are called as the feedback coefficient, the control coefficient and the bias current, respectively.

The output equation of the cell is given as follows:

$$y_{ij} = f(x_{ij}). \tag{3}$$

where,

$$f(x) = 0.5(|x+1| - |x-1|). \tag{4}$$

The variables u and y are the input and output variables of the cell, respectively. $\boldsymbol{A}_\alpha$, $\boldsymbol{B}_\alpha$, $\boldsymbol{A}_\beta$ and $\boldsymbol{B}_\beta$ are 3 times 3 matrices, which can be described to have a similar form to Eq. (5).

$$\begin{pmatrix} A_\alpha(i,j;i-1,j-1) & A_\alpha(i,j;i-1,j) & A_\alpha(i,j;i-1,j+1) \\ A_\alpha(i,j;i,j-1) & A_\alpha(i,j;i,j) & A_\alpha(i,j;i,j+1) \\ A_\alpha(i,j;i+1,j-1) & A_\alpha(i,j;i+1,j) & A_\alpha(i,j;i+1,j+1) \end{pmatrix} \tag{5}$$

This proposed system is more complex than the normal CNN. This system has a peculiar characteristic in order to investigate a new class of coupled oscillatory systems. Namely, a pair of cell α and cell β are needed for a simple oscillation. Additionally, one cell α connects with four neighbor cells β and one cell β also connects with four neighbor cells α. Like this, these cells are sharing a factor of oscillation. This type of connection may be difficult to realize by coupling normal oscillators.

3. Computer Simulations

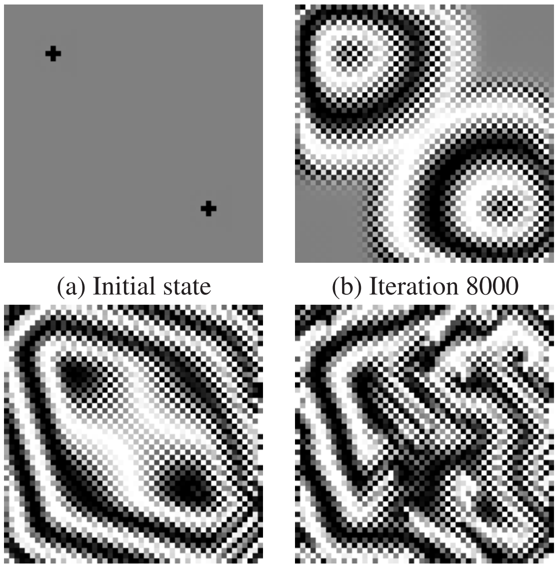
In all simulation results of this study, we define -1 and 1 as white and black, respectively. Initial state values of dummy cells which is placed at outside of border cells are set as 0. Figures 2 show one of the computer simulation results using following templates.

$$\boldsymbol{A}_\alpha = \begin{pmatrix} 0.1 & 0 & 0.1 \\ 0 & 1.1 & -1 \\ 0.1 & 0 & 0.1 \end{pmatrix}, \quad \boldsymbol{A}_\beta = \begin{pmatrix} -0.01 & 0 & -0.01 \\ 1 & 1.04 & 0 \\ -0.01 & 0 & -0.01 \end{pmatrix},$$
$$\boldsymbol{B}_\alpha = 0, \quad \boldsymbol{B}_\beta = 0, \quad I_\alpha = 0, \quad I_\beta = 0, \tag{6}$$

Two concentric circular waves are generated. Their wave fronts propagate in all directions through the network from their initialized positions shown in Fig. 2 (a) (b). When the waves hit the end of CNN, the waves reflect shown in Fig. 2 (c). After that the pattern becomes complex pattern shown in Fig. 2 (d). Like this, the proposed system generates some interesting phenomena.

In this study, the following template case is investigated.

$$\boldsymbol{A}_\alpha = \begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{pmatrix}, \quad \boldsymbol{A}_\beta = \begin{pmatrix} 1 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & 1 \end{pmatrix}, \tag{7}$$
$$\boldsymbol{B}_\alpha = 0, \quad \boldsymbol{B}_\beta = 0, \quad I_\alpha = 0, \quad I_\beta = 0,$$



(a) Initial state (b) Iteration 8000

(c) Iteration 20000 (d) Iteration 100000

Figure 2: Active Propagation Phenomena.

This template is used only three values which are 1, 0 and -1. Investigating simple templates like this, peculiar characteristics of this system can be revealed. Figure 3 shows one of the computer simulation results. All initial state values are set as random numbers. In this case, clustering phenomena are observed. Border lines keep oscillating and these positions don't move. These clusters are classified into four kinds of patterns as shown in Fig. 4. Pattern A as shown in Fig. 4 shows the case that cells α which are placed at rows and columns of odd numbers become black and the others become white. In the same way, Pattern B shows the case that cells α which are placed at rows and columns of even numbers become black. Pattern C shows the case that cells α which are placed at rows and columns of odd numbers become white. Pattern D shows the case that cells α which are placed at rows and columns of even numbers become white.

This result shows that this system has four stable states. Therefore, it is supposed that patterns are placed at arbitrary positions by these patterns are set as initial values.

In order to confirm this supposition, some investigations are carried out by computer simulations. At first, the case that one of four patterns are set as initial values are investigated. In this case, we can confirm that one pattern fills the system. Figure 5 shows the computer simulation result in the case of pattern A. In all cases, border cells don't become white or black. However, border cells don't influence inner cells. Figures 6 show left upper sides of the computer simulation results in cases that one of four patterns are set as initial values. Next, we investigate cases that one pattern is placed at the center of the other pattern. The results of this

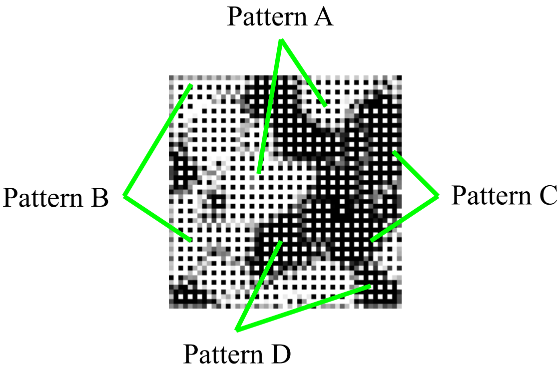


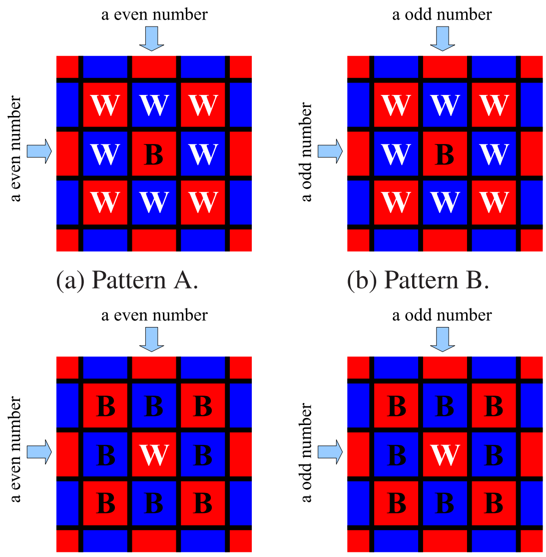
Figure 3: Clustering phenomena.

investigation are shown in Table 1. Two kinds of results are obtained. One is that the center pattern doesn't move and the border line keeps oscillating. The other is that the center pattern disappears. Figure 7 shows one of the results. In this case, two patterns don't move and the border line keeps oscillating. These results show that the center pattern disappears in the case of combinations of "A and D" or "B and C." Next, we investigate cases that two patterns are placed at right and left sides. The results of this investigation are shown in Table 2. Figure 8 shows one of the results. In this case, two patterns don't move and the border line keeps oscillating. These results show that patterns A and C disappear in the case of combinations of "A and D" or "B and C."

Finally, we try to set four patterns as the initial state. Figures 9 show the one of the computer simulation results in the case of setting four patterns. Pattern A, B, C and D are set at left upper side, right upper side, left lower side and right lower side, respectively. In this case, these four patterns exist stable states and border lines keep oscillating. However, four patterns don't exist stable states in some cases. By placements of patterns, the result changes. Namely, it depends on relationship between two patterns.

4. Conclusions

In this study, we investigated clustering phenomena on cellular neural networks using two kinds of templates. It was confirmed that these clusters are classified into four kinds of patterns. Relationships among four patterns also investigated. As a result of these investigations, we could confirm that patterns are placed at arbitrary positions with conditions. In future, we will analyze this phenomena theoretically.



(a) Pattern A. (b) Pattern B.

(c) Pattern C. (d) Pattern D.

Figure 4: Four kinds of patterns observed in clustering phenomena.

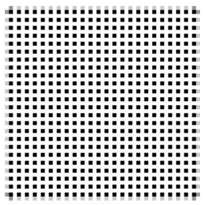
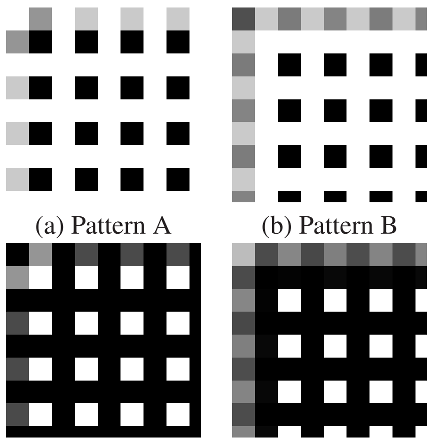


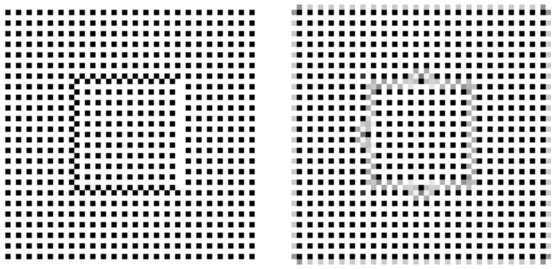
Figure 5: Computer simulation result in the case that initial values are set as Pattern A.



(a) Pattern A (b) Pattern B

(c) Pattern C (d) Pattern D

Figure 6: left upper sides of the computer simulation results in cases that one of four patters are set as initial values.



(a) Initial state. (b) Steady state.

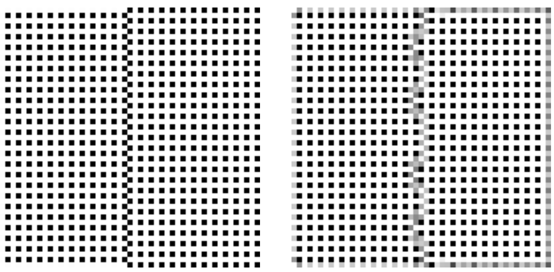
Figure 7: Computer simulation result in the case that pattern B is placed at center of pattern A.

Table 1: Relationship between two patterns 1

	Center pattern			
Base pattern	A	B	C	D
A	-	O	O	D
B	O	-	D	O
C	O	D	-	O
D	D	O	O	-

O: The center pattern doesn't move and the border line keeps oscillating.

D: The center pattern disappears.



(a) Initial state. (b) Steady state.

Figure 8: Computer simulation result in the case that pattern A and B are placed at left and right side respectively.

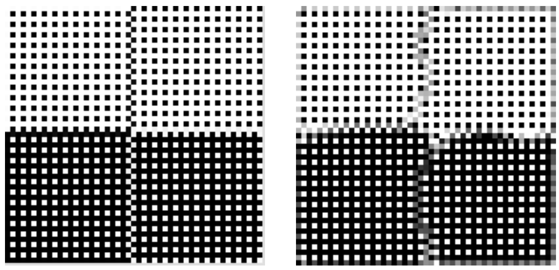
Table 2: Relationship between two patterns 2

	Right pattern			
Left pattern	A	B	C	D
A	-	O	O	D
B	O	-	B	O
C	O	B	-	O
D	D	O	O	-

O: Two patterns don't move and the border line keeps oscillating.

B: The system is filled by pattern B.

D: The system is filled by pattern D.



(a) Initial state. (b) Steady state.

Figure 9: Computer simulation result in the case of Pattern B is placed at center of Pattern A.

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