2008 International Workshop on Nonlinear Circuits and Signal Processing
NCSP'08, Gold Coast, Australia, March 6-8, 2008

Three-Neuron Nonlinear Spring Model of Self-Organizing Map

Haruna Matsushita and Yoshifumi Nishio

Tokushima University, Japan
Phone:+81-88-656-7470, Fax:+81-88-656-7471
Email: {haruna, nishio}@ee.tokushima-u.ac.jp

Abstract

In our previous research, as the first step to realize a new Self-Organizing Map model, we have proposed a simple one dimensional 2-neuron model connected by a nonlinear spring. This study proposes one dimensional 3-neuron model connected by a nonlinear spring in order to represent a relationship between the winner and its neighboring neurons in SOM algorithm. Furthermore, we consider two kinds of forces by external input vectors and investigate their behaviors.

1. Introduction

The Self-Organizing Map (SOM) is a subtype of artificial neural networks. It is trained using unsupervised learning to produce low dimensional representation of the training samples while preserving the topological properties of the input space. SOM is introduced by Kohonen in 1982 [1] and is a model simplifying self-organization process of the brain.

However, SOM is still far away from the realization of the brain mechanism. In order to realize more powerful and more flexible mechanism, it is important to propose new models of the brain mechanism and to investigate their behaviors.

In our previous research, as the first step to realize a new nonlinear spring model of SOM, we have proposed a simple one dimensional 2-neuron model connected by a nonlinear spring [2]. We have investigated its behavior under a simple assumption where input vectors are given to the model periodically.

In this study, we propose one dimensional 3-neuron model connected by a nonlinear spring. In the SOM algorithm, the neuron nearer to the winner can be updated more significantly. By increasing the number of neurons from two to three, we represent a relationship between the winner and its neighboring neurons. Furthermore, we consider two kinds of forces by external input vectors; a rectangular wave and a sine curve.

In Section 2, we explain the 3-neuron nonlinear spring model of SOM in detail. The behaviors of the proposed model are investigated by calculating the projection of attractors and Poincaré map in Section 3. Furthermore, in order to investigate chaotic behavior in detail, one-parameter bifurcation diagram and the largest Lyapunov exponent are calculated. We investigate different behaviors between two kinds of the external force. Computer simulated results show that the neurons oscillate chaotically.

2. 3-Neuron Nonlinear Spring Model of SOM

This study proposes one dimensional 3-neuron SOM model connected by a nonlinear spring. The model is shown in Fig. 1. The three neurons are assumed to have the same mass m and to be connected by the nonlinear spring with the natural length l, whose restoring force F against the variation x is represented by

$$F = -bx^3 \tag{1}$$

where b denotes the stiffness of each spring.

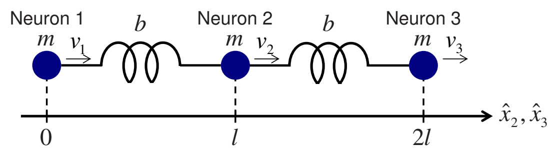

Figure 1: Three-neuron nonlinear spring model of SOM.

Without loss of generality, we fix the position of the Neuron 1 as the origin of the x-coordinate. The position of the Neuron 2 and the Neuron 3, denoted by $\hat{x}_2$ and $\hat{x}_3$, and the velocities of the neurons (v_1, v_2 and v_3) are chosen as the state variables. The motion equation of the model can be described as

$$\begin{cases} \dfrac{d\hat{x}_2}{dt} = v_2 \\ \dfrac{d\hat{x}_3}{dt} = v_3 \\ m\dfrac{dv_1}{dt} = -av_1 + b(\hat{x}_2 - l)^3 \\ m\dfrac{dv_2}{dt} = -av_2 - b(\hat{x}_2 - l)^3 + b(\hat{x}_3 - \hat{x}_2 - l)^3 \\ m\dfrac{dv_3}{dt} = -av_3 - b(\hat{x}_3 - \hat{x}_2 - l)^3, \end{cases} \tag{2}$$

where a is the friction parameter. By changing the variables and parameters;

$$\hat{x}_2 - l = x_2, \qquad \hat{x}_3 - 2l = x_3,$$
$$v_1 = \sqrt{\frac{b}{m}} y_1, \quad v_2 = \sqrt{\frac{b}{m}} y_2, \quad v_3 = \sqrt{\frac{b}{m}} y_3, \tag{3}$$
$$t = \sqrt{\frac{m}{b}} \tau, \qquad k = \frac{a}{\sqrt{bm}},$$

the normalized equations are given as

$$\begin{cases} \dfrac{dx_2}{d\tau} &= y_2 \\ \dfrac{dx_3}{d\tau} &= y_3 \\ \dfrac{dy_1}{d\tau} &= -ky_1 + x_2^3 \\ \dfrac{dy_2}{d\tau} &= -ky_2 - x_2^3 + (x_3 - x_2)^3 \\ \dfrac{dy_3}{d\tau} &= -ky_3 - (x_3 - x_2)^3, \end{cases} \tag{4}$$

Next, we model the learning process of the SOM by the external force by input vectors. When an input vector is given to the 3-neuron model as Fig. 2, the winner, which is the neuron nearest to the input vector, is attracted to the input. In this study, we consider two kinds of forces by the external input vector; a rectangular wave;

$$\hat{f}(t) = \hat{B}\mathrm{sign}\left(\sin\sqrt{\frac{b}{m}}t\right), \qquad \left(0 \le t \le \sqrt{\frac{m}{b}}\pi\right), \tag{5}$$

and a sine curve;

$$\hat{f}(t) = \hat{B}\sin\sqrt{\frac{m}{b}}t, \qquad \left(0 \le t \le \sqrt{\frac{m}{b}}\pi\right), \tag{6}$$

where $\mathrm{sign}(\cdot)$ is the signum function and $t = 0$ is the time when the input vector is given. The shapes of these functions are shown in Fig. 3. Note that the other neurons do not receive a direct effect from the input vector.

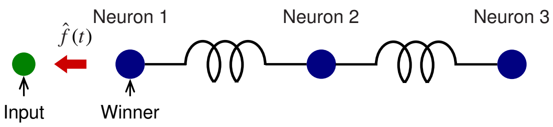

Figure 2: Input vector and winner

In this study, in order to investigate the simplest learning process of 3-neuron model, we concentrate on the case that the input vectors are given to the right-hand side and left-hand side of the model alternately with the fixed frequency $\sqrt{b/m}/(2\pi)$. Therefore, only the Neuron 1 or the Neuron 3 becomes the winner and is attracted to the input with the force

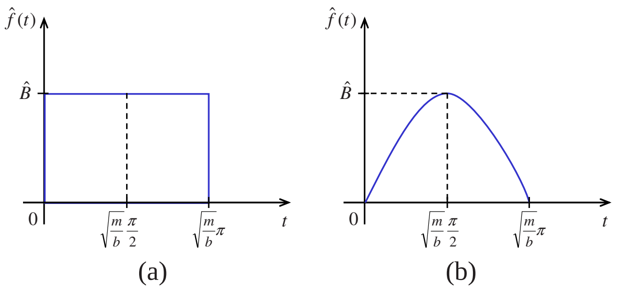

Figure 3: Force by external input vector. (a) Rectangular wave described by Eq. (5). (b) Sine wave described by Eq. (6).

as Eq. (5) or Eq. (6). The Neuron 2 always does not receive the direct effect from the input vector and is influenced only by the restoring force of the nonlinear spring. In this case, the motion equation is modified as

$$\begin{cases} \dfrac{dx_2}{d\tau} &= y_2 \\ \dfrac{dx_3}{d\tau} &= y_3 \\ \dfrac{dy_1}{d\tau} &= -ky_1 + x_2^3 - f(\tau) \\ \dfrac{dy_2}{d\tau} &= -ky_2 - x_2^3 + (x_3 - x_2)^3 \\ \dfrac{dy_3}{d\tau} &= -ky_3 - (x_3 - x_2)^3 + f(\tau - \pi), \end{cases} \tag{7}$$

where $f(\tau)$ is

$$f(\tau) = \frac{B}{2}(\mathrm{sign}(\sin\tau) + 1) \tag{8}$$

when the external force is Eq. (5), and is

$$f(\tau) = \frac{B}{2}(\sin\tau + |\sin\tau|), \tag{9}$$

when the external force is Eq. (6), and

$$B = \frac{\hat{B}}{b}. \tag{10}$$

The shape of each $f(\tau)$ is shown in Fig. 4.

3. Computer Simulation Results

In this section, we show some computer calculation results obtained by using Runge-Kutta method with time step $\delta t = 2\pi/500$ for Eq. (7).

3.1. Attractors and Poincaré maps

The projection of attractors onto x_2–y_2 and x_3–y_3 plane, when the external force is the rectangular wave as Eq. (8), are

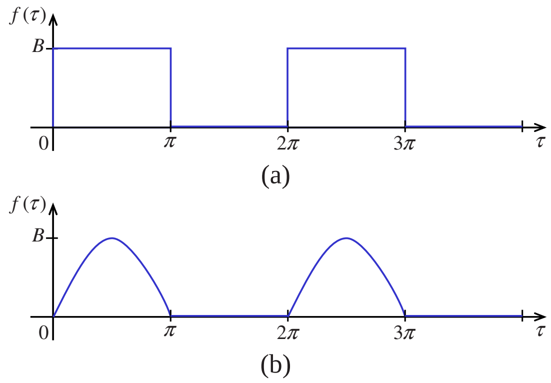

Figure 4: Force by periodic external input vector. (a) Force described by Eq. (8). (b) Force described by Eq. (9).

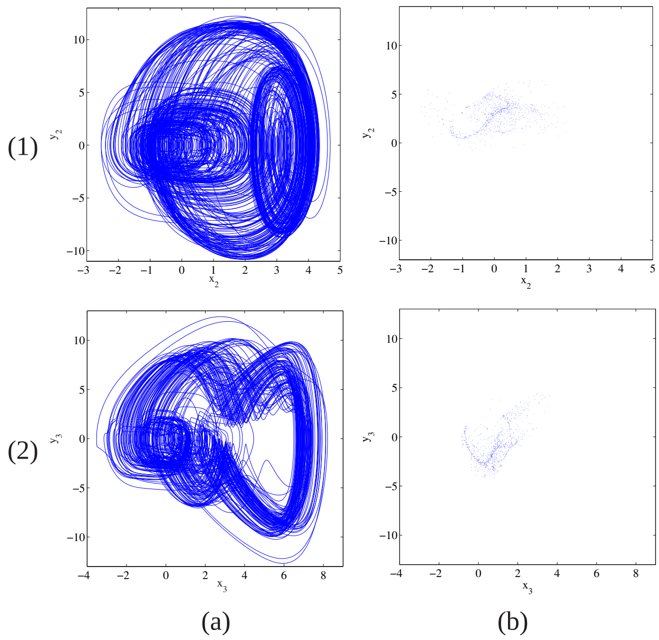

Figure 5: Projection of attractors and their Poincaré maps when force by external input vector is rectangular wave (8). Fixed parameter $k = 0.15$ and $B = 15$. (a) Attractors. (b) Poincaré maps. (1) x_2–y_2 plane. (2) x_3–y_3 plane.

shown in Fig. 5(a). We can confirm that the orbits of both attractors look chaos.

In order to investigate the chaotic behavior of the model in detail, we define the Poincaré section as $\tau = 2n\pi$ and plot the discrete data on the Poincaré section onto x_2–y_2 and x_3–y_3. The Poincaré maps are shown in Fig 5(b). We can see that the Poincaré maps are folded have the shape like strange attractors [3]. We can estimate the attractors are chaos from the shape of Poincaré maps.

Similarly, the projection of attractors in case of using the sine curve as Eq. (9) and their Poincaré map are shown in Fig. 6. The attractors look more complex chaos than Fig. 5(a). The Poincaré maps are also folded and have the shape like strange attractors.

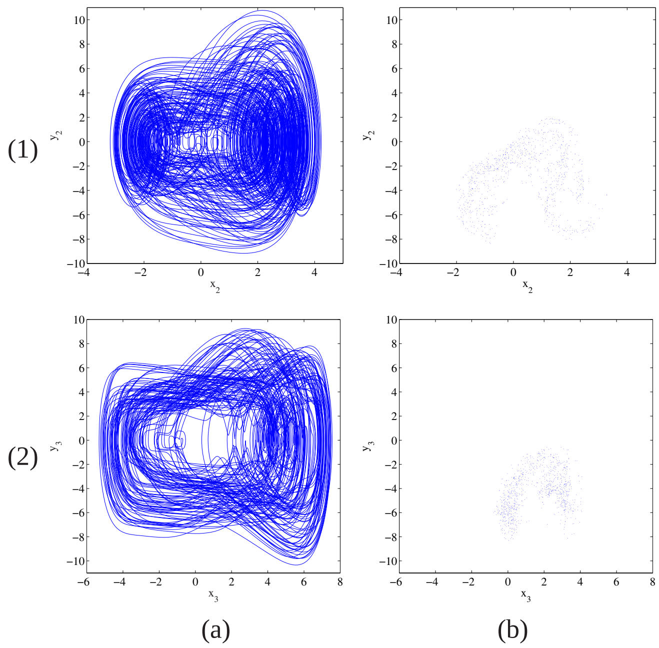

Figure 6: Projection of attractors and their Poincaré maps when force by external input vector is sine curve (9). Fixed parameter $k = 0.15$ and $B = 15$. (a) Attractors. (b) Poincaré maps. (1) x_2–y_2 plane. (2) x_3–y_3 plane.

3.2. Bifurcation Diagram and Lyapunov Exponent

In order to evaluate the chaotic behavior of the model in detail, we calculate the largest Lyapunov exponent of the attractors. By using the Jacobian Matrix **DT** obtained by integrating the variational equations of Eq. (7), we can calculate the largest Lyapunov exponent [4];

$$\lambda_1 = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \log |\mathbf{DT}(j) \cdot \boldsymbol{e}(j)|, \tag{11}$$

where $\boldsymbol{e}(j)$ is a normalized basis.

Figures 7 and 8 show One-parameter bifurcation diagram of x_2 and the largest Lyapunov exponents of the attractors, which are calculated using Shimada-Nagashima algorithm [5]. The control parameter is B and another parameter k is fixed as 0.15. The Lyapunov exponent takes positive values for a wide range of B in both Fig. 7(b) and Fig. 8(b). Therefore, the nonlinear spring model can be said to generate chaos. We observe the bifurcation in detail. In case of using the rectangular wave, complicated bifurcation property can be confirmed from Fig. 7(a). For $B < 11.64$, chaotic attractors can be observed for almost parameter values, and we can confirm the wide chaos region. On the other hand, in case of using the sine wave, period doubling bifurcation can be confirmed from Fig. 8(a). We can confirm more and wider chaos region in smaller value of B than Fig. 7(a) For $B < 9.57$, chaotic attractors can be observed for almost parameter values.

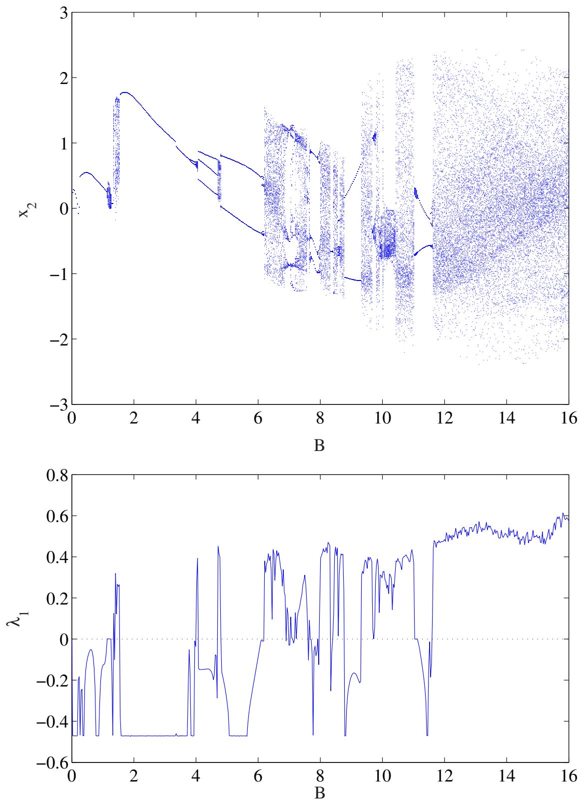

Figure 7: Bifurcation diagram and largest Lyapunov exponent changing B for $k = 0.15$ when external force is rectangular wave (8).

4. Conclusions

In this study, we have proposed one dimensional 3-neuron nonlinear spring model of SOM. By increasing the number of neurons from two to three, we have modeled the relationship between the winner and its neighboring neurons. Furthermore, we have considered two kinds of forces by external input vectors; a rectangular wave and a sine curve. In order to investigate chaotic behavior in detail, one-parameter bifurcation diagram and the largest Lyapunov exponent have be calculated. We have investigated different behaviors between two kinds of the external force and have confirmed that the neurons oscillate chaotically.

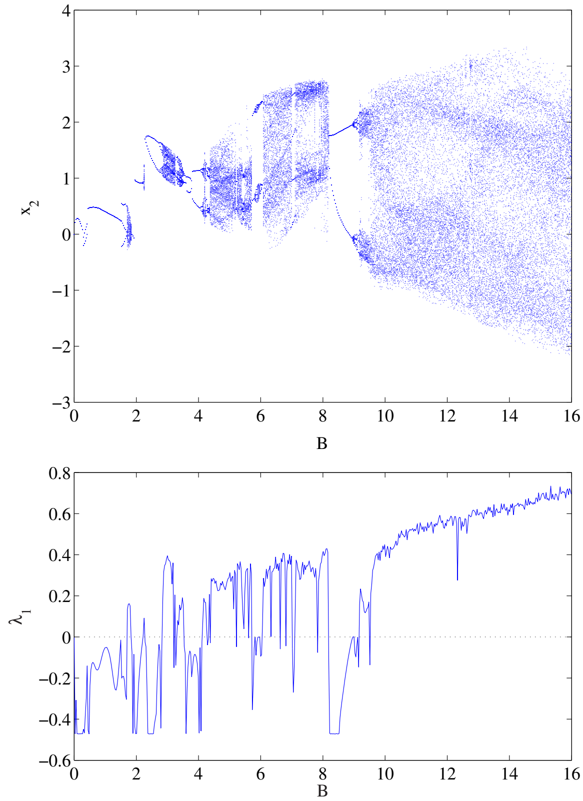

Figure 8: Bifurcation diagram and largest Lyapunov exponent changing B for $k = 0.15$ when external force is sine curve (9).

References

[1] T. Kohonen, *Self-organizing Maps*, Berlin, Springer, 1995.

[2] H. Matsushita and Y. Nishio, “Nonlinear Spring Model of Self-Organizing Map and its Chaotic Behavior,” *Proc. of ICCCAS*, vol. 2, pp. 1099–1102, 2007.

[3] Y. Ueda, *The Road to Chaos*, Santa Cruz, Aerial Press, 1992.

[4] H. Kawakami, “Bifurcation of Periodic Responses in Forced Dynamic Nonlinear Circuits: Computation of Bifurcation Values of the System Parameters,” IEEE Trans. Circuits and Systems, vol. CAS–31, No. 3, pp. 248–260, 1984.

[5] I. Shimada and T. Nagashima, “A Numerical Approach to Ergodic Problem of Dissipative Dynamical Systems,” Progress of Theoretical Physics, vol. 61, No. 6, pp. 1605–1616, 1979.