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Phase Synchronization Phenomena on Coupled Multi-State Oscillators

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Abstract

This paper presents several phase synchronization phenomena on multi–state oscillators coupled by inductors as a ring. Each oscillator circuit can individually behave some periodic oscillations (limit cycles) in the same parameters asynchronously. In this study, a coupled oscillator circuit which is composed by some oscillators is proposed and classification of phase synchronization modes is investigated. In numerical simulation, many types of phase synchronization modes are confirmed in the proposed systems.

1. Introduction

Oscillator is an important device and one of essential component in the natural world. Nonlinear dynamics on coupled oscillators is considerable interesting for a wide variety of systems in several scientific fields and some engineering applications. Many types of coupled circuit systems have been widely studied in order to clarify inherent features and many researchers have already proposed and investigated mechanism of them. The dynamics of multimode oscillations or phase synchronization on several coupled systems is still considerable interest from the viewpoint of both natural scientific fields and several applications.

They have been confirmed in several systems; e.g., coupled van der Pol oscillators [1], laser systems [2], and so on. Phase synchronization and pattern dynamics are also interesting for several engineering applications. On the other hand, many types of chaotic systems and circuits have already been proposed and investigated in detail. As interesting phenomena, there are famous chaotic attractors such a double-scroll family [3], n-double scroll [4]–[6] and scroll grid attractors [7]. If the active elements including in the systems have complexity constructed by compound some nonlinear elements, it can be easily considered that they yield several interesting features.

In our previous studies, the circuit which can individually behave both chaotic or periodic oscillations in the same parameters had been investigated [9]-[12]. This type of circuit was called a multi–state chaotic circuit (abbr. MSCC). Multimode oscillations in coupled two or more multi–state chaotic circuits had been shown [10] on real circuit. A complicated and interesting phenomena of phase synchronization had also been investigated [11][12]. It is known that complex behavior can be confirmed such chaotic itinerancy and spatio–temporal chaos on the large scale coupled networks. Some kinds of oscillation modes had been reported on large scale coupled chaotic circuits such phase synchronization, phase propagation and frustration of oscillation and so on [13]-[15]. On the other hand, the coupled van del Pol oscillators with hard nonlinearity had been investigated [16] and also stability analysis of them [17]. There are many oscillators from very low to very high frequency, which can be easily constructed on the real electrical circuits. We consider that it is important to investigate phase synchronization and pattern dynamics in such coupled oscillator systems.

This paper presents several phase synchronization modes of multi–state oscillator circuits (abbr. MSOC) coupled by inductors as a ring. This is a kind of val der Pol type oscillator and a typical two dimensional autonomous system, which consists of two memory elements and a designed piecewise linear resistor. We substitute a symmetrical continuous segments piecewise linear resistor for the negative active resistor including in the original chaotic circuit. Each circuit can individually behave some periodic oscillations (limit cycles) in the same parameters asynchronously. This proposed circuit can behave three limit cycles when we supply with different initial conditions. In this study, a coupled system which is connected to some oscillators is proposed and classification of phase synchronization modes is investigated. In numerical simulation, many types of phase synchronization modes are asynchronously confirmed in the proposed systems, further all parameters are the same. This means several phase synchronization modes are coexisting in the same parameters.

2. Model Description

The circuit shown in Fig. 1 is an oscillator circuit of a well–known two dimensional van der Pol type oscillator. The original circuit consists of two memory elements and a designed negative resistor.

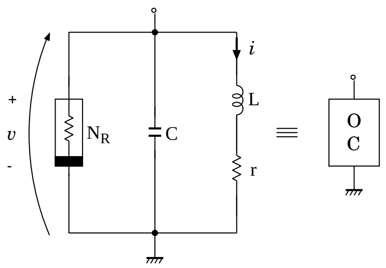

Figure 1: Based oscillator circuit with piecewise linear resistors N_R.

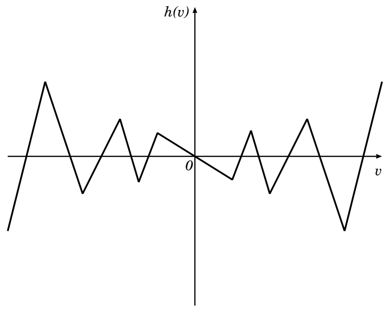

Figure 2: Design of N_R in the oscillator for sawtooth piecewise linear resistor with respect to the origin.

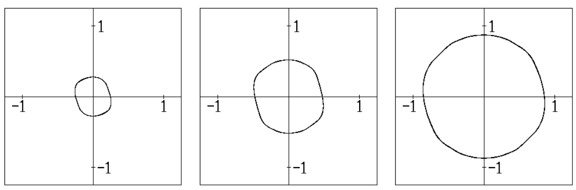

Figure 3: Three steady state of the oscillator in the same parameters.

In this study, we substitute a symmetrical continuous piecewise linear resistor for the negative active resistor including in the circuit. It can be realized the segment-wise nonlinear resistor which can be easily constructed by some subcircuits. The designed piecewise linear resistor is shown in Fig. 2. The piecewise linear resistor can be easily constructed by combining some components in parallel [9][10]. Let us consider the circuit equations by changing the following variables and parameters.

$$i = \sqrt{\frac{C}{L}}x, \;\; v = y, \;\; t = \sqrt{LC}\tau, \;\; \text{“}\cdot\text{”} = d/d\tau,$$
$$\gamma = g\sqrt{\frac{L}{C}}, \text{ and } r \simeq 0 \tag{1}$$

where g is a linear negative conductance value of N_R if we assume the negative resistor is as an ideal linear. Replace the part of negative value N_R in Fig. 1 to the function $h(y)$ represented by a voltage source y as a canonical form. Then the normalized circuit equations can be rewritten as follows.

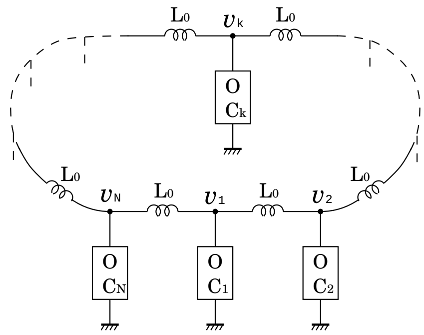

Figure 4: Concept of a model of oscillators coupled by inductors as a ring.

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= -x - \gamma h(y) \end{cases} \tag{2}$$

$$h(y) = m_N y + \frac{1}{2}\sum_{k=1}^{N}(m_k - m_{k-1})\{|y - b_k| - |y + b_k|\} \tag{3}$$

where $h(y)$ is the piecewise nonlinear resistor with respect to the origin. The parameter γ is used for a basic common value, hence the values m_k (k = 0, 1, 2, $\cdots$, N) mean magnitude of the slope to the ratio for γ, the values b_k means break points. Therefore, the piecewise linear function has $2N + 1$ slopes and N breakpoints.

For example, the circuit parameters in the case of $N = 5$ are as follows.

$$\begin{aligned} &\gamma = 0.78 \\ &\{m_0, m_1, m_2, m_3, m_4, m_5\} \\ &\quad = \{-0.5, 4.0, -2.0, 4.0, -2.0, 2.0\} \\ &\{b_1, b_2, b_3, b_4, b_5\} \\ &\quad = \{0.2, 0.25, 0.4, 0.5, 0.7\} \end{aligned} \tag{4}$$

This case corresponds to 11-segments piecewise linear resistor as shown in Fig. 2.

Figure 3 shows some computer simulation results in the case of (4) when the parameters are all the same and the initial conditions of each circuit are only different. In these parameters, we can confirm that three limit cycles coexist in the same parameters as a steady state. Hereafter we use these parameters (4) in numerical simulation.

3. Simulation for coupled MSOCs

In this section, the model of coupled MSOCs by inductors are investigated. We now consider the coupled model which oscillator circuits to the number of N are connected by inductors L_0 as a ring structure as shown in Fig. 4. All circuit

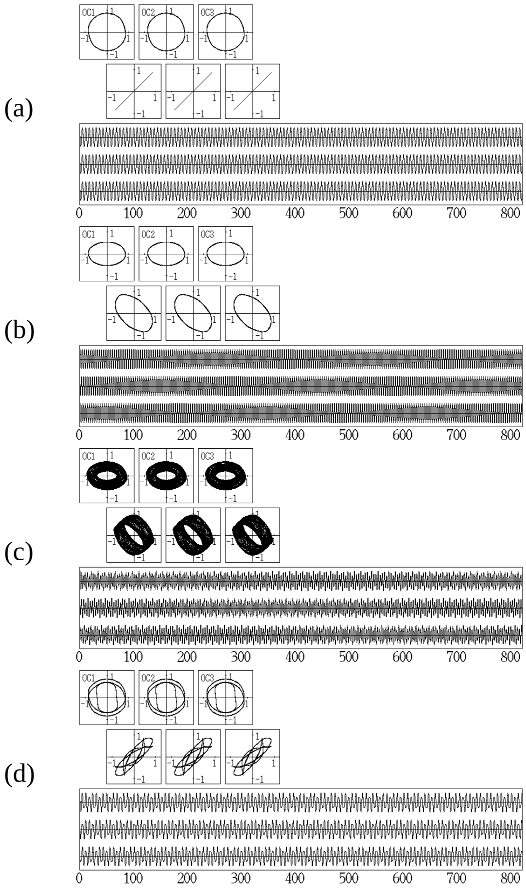

Figure 5: Some simulation results in the same parameters for the case $N = 3$.

parameters are set in the same values. The circuit equation of coupled MSOCs can be normalized by changing the variables (1) and a new parameter $\alpha = L/L_0$ as coupling strength, then the circuit equations can be rewritten as follows.

$$\begin{cases} \dot{x_k} &= y_k \\ \dot{y_k} &= \alpha(x_{k-1} - 2x_k + x_{k+1}) \\ & \qquad -x_k - \gamma h(y_k) \end{cases} \qquad (5)$$

Here, we show some computer simulation results by using 4-th order Runge–Kutta method with time step size $\Delta t = 0.001$ for the circuit equation (5) and (3) in some cases of $N = 2 \sim 6$ as follows.

First, we consider that the number of the coupled MSOCs is two. In this case, some asynchronous oscillation modes could be confirmed consequently by numerical simulations when the initial conditions are changed. We can confirm several types of phase synchronization phenomena in this model for their size of the limit cycles; in-phase synchronous limit cycles, anti-phase synchronous limit cycles, and multimode oscillations, in the same parameters, respectively.

Second, we consider the case of $N = 3$ or more. Compare with the case $N = 2$, several different synchronization phenomena can be found. Because all types of the results can not be represented, some simulation results are only shown here. Figures 5 and 6 show some computer simulation results obtained by the circuit parameters (4) and $\alpha = 0.50$. From top of the figure, attractors drawing onto y–x plane, synchronization state for both of phase $y_k - y_{k+1}$ and $x_k - x_{k+1}$, and waveform of each oscillator, which k obeys in the order of cyclic rule for any number of coupled oscillators. Figure 5 shows some simulation results in the case of $N = 3$; (a) in-phase synchronization of three limit cycles, (b) three phase synchronization, (c) double-mode oscillation, and (d) phase locking phenomena. We could confirm to coexist a lot of synchronization modes in this situation.

In a large number of coupled systems for $N \geq 4$, it is easily expected to be confirmed more complex behavior. We show some results in Fig. 6 for the case of N = 4 and 6; (a) in-phase synchronization, (b) two-pair of in-phase and anti-phase synchronization of limit cycles, (c) two-pair of synchronization with phase difference, (d) phase locking phenomena, respectively for N = 4. On the other hand, in the case of N =6, several phase synchronization modes had been confirmed, for examples as shown in Fig. 6(e)–(g). Thus, several complex behavior and phase synchronization phenomena could be also confirmed in the coupled multi–state oscillators in the same parameters.

4. Conclusions

In this study, we have investigated several synchronization modes in coupled multi–state oscillator circuits. Coexistence of several types oscillation modes have been confirmed in coupled MSOCs by inductors as a ring for several cases. On large scale coupled oscillator including chaotic circuits such a scale-free network, we consider that several types of complex behavior are expected to yield novel complicated phenomena e.g., spatio–temporal behavior, multi–agent systems, soliton like wave propagation phenomena, and inherent emergent property, in which concerned with other current topics. Furthermore, it is also an important work to analyze stability of the multi–state oscillator.

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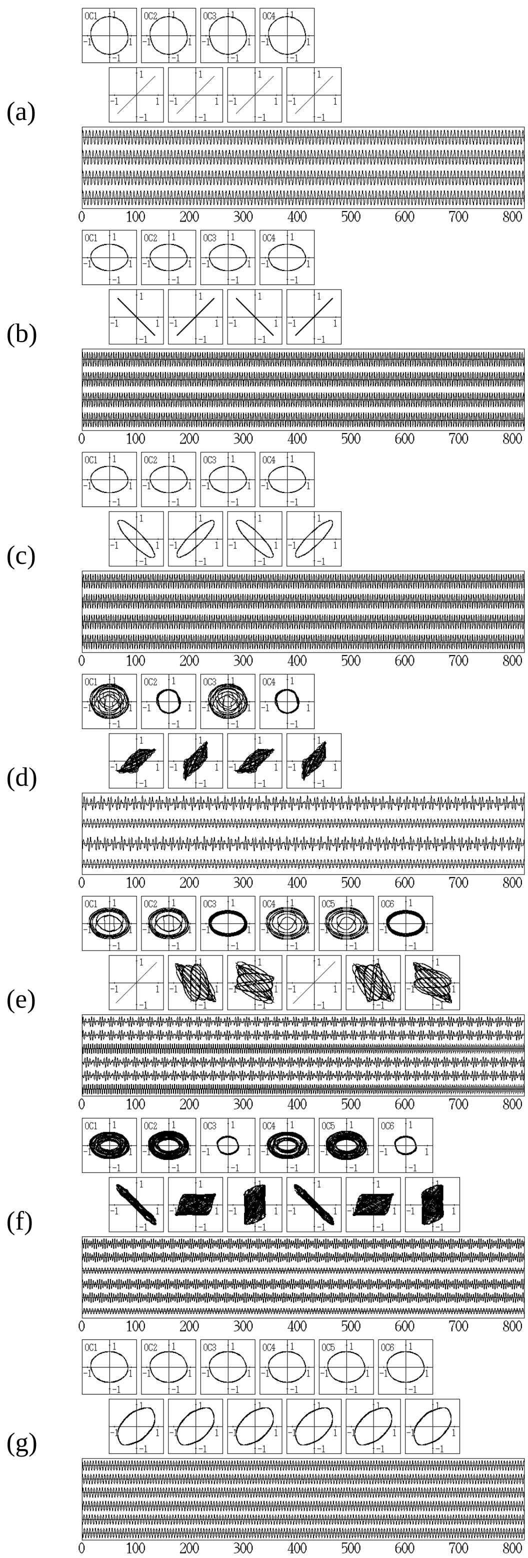

Figure 6: Some simulation results in the same parameters for the cases $N = 4$ and $N = 6$.