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Spice-Oriented Harmonic Balance and Volterra Series Methods

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Abstract

There are typical two types of frequency-domain analysis of nonlinear circuits. One is Volterra series method which is widely used in European countries, because the algorithm is based on the bilinear theorem, and that the higher order solutions can be obtained in the analytical forms. Unfortunately, it can be only applied to the weakly nonlinear systems. The other is the harmonic balance (HB) method. In this paper, we have greatly improved the HB by combining with MATLAB and Spice, such that it can be efficiently applied to any kind of nonlinear electronic circuits. Namely, the Fourier coefficients can be obtained in the symbolic forms with MATLAB. The determining equation of harmonic balance method is given by the net-lists, which can be solved by the DC analysis of Spice. We will compare the Volterra series with HB methods in the point of their accuracies.

1. Introduction

For designing integrated circuits and communication systems, it is very important to analyze the frequency-domain characteristics of nonlinear circuits. There are two kinds of the time-domain and frequency-domain methods. Although, the first one can get the accurate steady-state waveform, it is inefficient to calculate the frequency response curve[1,2]. The latter is useful to obtain the frequency response curve of nonlinear circuits. There are two methods of Volterra series and harmonic balance. The Volterra series method is widely used for the purposes [3-5] which gives the solution in analytical form, and the algorithm can be easily used combining with MATLAB [6]. The algorithms are based on bilinear theorem, and can be also effectively applied to the intermodulation's analysis driven by multiple inputs. However, they can be only applied to the weakly nonlinear circuits, and the solutions for strong nonlinear circuits become erroneous.

The harmonic balance (HB) is also a well-known method for the frequency domain analysis, which gives good results even for relatively strong nonlinear circuits. There are two types of classical HB [7-9] and computer-aided HB methods [10,11]. The classical HB can be applied to relatively small scale circuits, but they have found many interesting nonlinear phenomena such as bifurcations and chaos. The latter HBs are rather numerical methods, and find the solutions with the Newton-Raphson and/or relaxation methods. Therefore, they are inefficient to calculate the frequency response curves. We propose here another type of Spice-oriented HB method which is based on the classical HB. It is largely improved such that that it can be easily applied to large scale nonlinear circuits combining with MATLAB and Spice, where the determining equation called *Sine-Cosine circuit* [11,12] can be obtained in the form of schematic and/or net-list of Spice. The circuit can be solved by DC analysis of Spice simulator and easily get the frequency characteristic.

We briefly show the Volttera series method in Section 2, and our HB method in Section 3. Comparisons between our HB and the Volterra methods are given in Section 4.

2. Volterra series method

To understand the Volterra series method [3-5], we consider a simple circuit shown in Fig. 1, where nonlinear resistor is described by the following polynomial function;

$$i_G = k_1 v_G + k_2 v_G^2 + k_3 v_G^3. \tag{1}$$

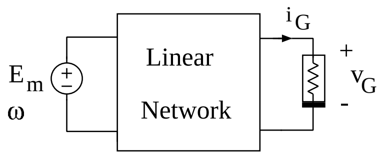

Fig. 1 Nonlinear resistive circuit.

Now, let us caluculate the kernels in complex frequency-domain "s". The schematic diagram is shown in Fig. 2. The first order kernel corresponds to the output voltage for the normalized unit inputs $U(s)$.

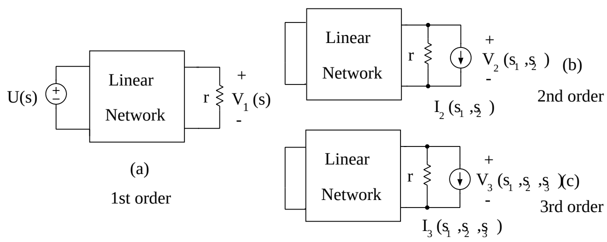

Fig. 2 Volterra kernels.

and the 1st order kernel is given by

$$H_1(s) = V_1(s) = K(s)U(s) \tag{2.1}$$

where $K(s)$ is the transfer function.

Next, we consider the 2nd order kernel. The current is obtained by the 1st order kernel.

$$I_2(s_1, s_2) = k_2 H_1(s_1) H_1(s_2) \tag{2.2}$$

and the 2nd order kernel is obtained as follows.

$$H_2(s_1, s_2) = Y(s_1 + s_2) I_2(s_1, s_2) \tag{2.3}$$

where $Y(s_1 + s_2)$ is the admittance at the output port.

Finally, the 3rd order current is obtained as follows;

$$\begin{aligned} I_3(s_1, s_2, s_3) &= k_3(H_1(s_1)jH_1(s_2)H_1(s_2) \\ &+ \tfrac{2}{3}k_2(H_1(s_1)H_2(s_2, s_3) + H_1(s_2)H_2(s_1, s_3) + H_1(s_3)H_2(s_1, s_2)) \end{aligned} \tag{2.4}$$

and the 3rd order voltage is obtained as follows;

$$\begin{aligned} V_3(s_1, s_2, s_3) &= H_3(s_1, s_2, s_3), \\ &= Y(s_1 + s_2 + s_3) I_3(s_1, s_2, s_3). \end{aligned} \tag{2.5}$$

Thus, the output is given by

$$H_{out}(s_1, s_2, s_3) = H_1(s_1) + H_2(s_1, s_2) + H_3(s_1, s_2, s_3). \tag{2.6}$$

For the input

$$e(t) = E_m \sin(\omega t), \tag{3}$$

by setting $s_i = \pm j\omega$, (i=1,2,3), the output amplitudes can be estimated by the following Table 1.

Table 1 Frequency response by Volterra series method.

Frequency	Amplitude of response
DC	$\lvert \frac{1}{2}E_m^2 H_2(j\omega, -j\omega)\rvert$
ω	$\lvert E_m H_1(j\omega) + \frac{3}{4}E_m^3 H_3(\omega, j\omega, -j\omega)\rvert$
2ω	$\lvert \frac{1}{2}E_m^2 H_2(j\omega, j\omega)\rvert$
3ω	$\lvert \frac{1}{4}E_m^3 H_3(\omega, j\omega, j\omega)\rvert$

Note that the solution of Volterra series method can be easily solved by the use of MATLAB.

3. Spice-oriented HB method

In this Section, we propose our Spice-oriented HB method. Analog integrated circuits are usually composed of many kinds of nonlinear devices such as diodes, bipolar transistors and MOSFETs, whose Spice models are described by the several special functions such as exponential, square-root, piecewise continuous functions and so on [13]. For the special functions, the Fourier expansions can not be described by analytical forms. Therefore, we need to approximate them with the Taylor expansions as follows;

$$\begin{aligned} i &= f(v) \\ &= k_0 + k_1 v + k_2 v^2 + \cdots + k_K v^K. \end{aligned} \tag{4}$$

Suppose the circuit is driven by the independent "p" frequency components $\omega_1, \omega_2, \ldots, \omega_p$. The input and output waveforms of (4) is assumed as follows;

$$\left.\begin{aligned} v(t) &= V_0 + \sum_{k=1}^{M}(V_{2k-1}\cos\nu_k t + V_{2k}\sin\nu_k t) \\ i(t) &= I_0 + \sum_{k=1}^{M}(I_{2k-1}\cos\nu_k t + I_{2k}\sin\nu_k t) \end{aligned}\right\}, \tag{5.1}$$

where

$$\nu_k \equiv m_{1k}\omega_1 + m_{2k}\omega_2 + \cdots + m_{pk}\omega_p \tag{5.2}$$

for integers $m_{1k}, m_{2k}, \ldots, m_{pk}$. The Fourier expansion $i(t)$ for the input $v(t)$ can be carried out symbolically with MATLAB.

Now, we apply Fourier expansion to the capacitive element described by

$$q_C = \hat{q}_C(v_C), \tag{6.1}$$

and get the current as follow:

$$i_C = \sum_{k=1}^{M}\nu_k(-Q_{C,2k-1}\sin\nu_k t + Q_{Q,2k}\cos\nu_k t) \tag{6.2}$$

where $Q_1, Q_2, \ldots, Q_{2M-1}, Q_{2M}$ are Fourier coefficient from (6.1). Thus, we have

$$I_{C,2k-1} = \nu_k Q_{C,2k}, \quad I_{C,2k} = -\nu_k Q_{C,2k-1}, \quad k = 1, 2, \ldots, M. \tag{6.3}$$

Thus, the HB models of capacitors are replaced by the voltage-controlled current sources which are function of the capacitor voltages.

Next, we apply Fourier expansion to the inductive element described by

$$\phi_L = \hat{\Phi}_L(i_L), \tag{7.1}$$

and get the voltage as follow:

$$v_L = \sum_{k=1}^{M}\nu_k(-\Phi_{L,2k-1}\sin\nu_k t + \Phi_{L,2k}\cos\nu_k t) \tag{7.2}$$

where $\Phi_1, \Phi_2, \ldots, \Phi_{2M-1}, \Phi_{2M}$ are Fourier coefficient from (7.1). Thus, we have

$$V_{C,2k-1} = \nu_k \Phi_{L,2k}, \quad V_{C,2k} = -\nu_k \Phi_{L,2k-1}, \quad k = 1, 2, \ldots, M. \tag{7.3}$$

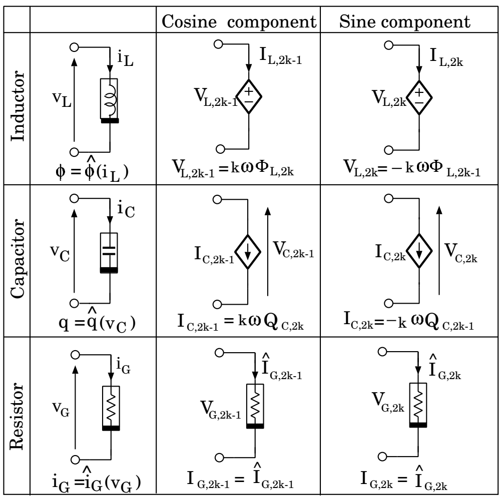

Fig. 3 Sine-Cosine equivalent circuit models.

Thus, the HB models of inductors are replaced by the current-controlled voltage sources, which are function of the inductor currents. All the nonlinear devices such as bipolar transistors and MOSFETs can be replaced by the equivalent HB modules with MATLAB.

Replacing each element by the corresponding HB model, we can formulate the *Sine-Cosine circuit* that is equal to the *HB determining equation*. The circuit is efficiently solved by the DC analysis of Spice.

4. Comparisons between Volterra and HB

Consider a simple diode circuit driven by a single input as shown in Fig. 4(a). In this Section, we compare the Volterra series method with our HB method in the point of accuracy.

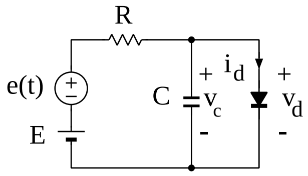

Fig. 4 Diode circuit. R=1[kΩ], C=1[μF], E=1[V], $e(t)$=$E_m \cos \omega t$.

Assume the diode characteristic as follow;

$$i_d = 10^{-8} \exp(40 v_d). \tag{8.1}$$

It is approximated by the 3rd order Taylor expansion at v_{d0} as follow;

$$i_d = k_0(V_{d0}) + k_1(V_{d0}) v_d + k_2(V_{d0}) v_d^2 + k_3(V_{d0}) v_d^3 \tag{8.2}$$

for an unknown value V_{d0}. We assume the waveform up to 3rd higher harmonic as follows;

$$v_d = V_{d,0} + \sum_{k=1}^{3} (V_{d,2k-1} \cos k\omega t + V_{d,2k} \sin k\omega t). \tag{9}$$

Substituting it into (8.2), the Fourier series is described as follow;

$$i_d = I_{d,0} + \sum_{k=1}^{3} (I_{d,2k-1} \cos k\omega t + I_{d,2k} \sin k\omega t). \tag{10}$$

In the arithmetic computaion, we have

$$1 + 7 + 7^2 + 7^3 = 400$$

terms. It is not easy to estimate all the Fourier coefficients by hand calculations. Hence, we apply MATLAB and get them in the formula symbolic forms. Thus, we have the equivalent HB circuit as shown in Fig. 5.

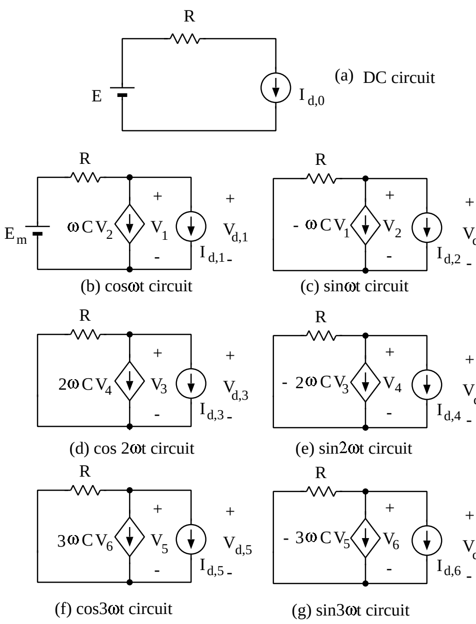

Fig. 5 Equivalent HB circuit of Fig. 4.

Note that the equivalent HB circuit of Fig. 5 corresponds to the *HB determining equation*, and can be solved by DC analysis of Spice. We have calculated the 2nd-3rd order harmonic distortions (HD2, HD3) to the fundamental harmonic. Thus, we obtained the results shown by Table 2. We have compare the results with those of Volterra method and the results from the transient analysis[1]. When the input is small, the values of both methods are similar. For the large inputs, the results from Volterra method may be erroneous. They are coming from the philosophy of Volterra series method.

Table 2 Comparisons ($\omega = 20,000$).

	Volterra		HB		Transient	
E_m	HD2	HD3	HD2	HD3	HD2	HD3
0.1	0.0161	0.0003	0.0160	0.0003	0.0128	0.0018
0.5	0.0804	0.0077	0.0753	0.0068	0.0711	0.0049
0.7	0.1126	0.0151	0.0990	0.0118	0.0980	0.0100
1.0	0.1609	0.0309	0.1244	0.0189	0.1298	0.0187
2.0	0.3218	0.1237	0.1404	0.0344	0.1698	0.0355
5.0	0.8043	0.7729	0.0872	0.0709	0.1280	0.0371

We have also solved the circuit in Fig. 4 with Volterra series and our HB methods in the frequency-domain. The 2nd and 3rd order harmonic distortion curves are shown in Figs. 6(a) and (b)[2].

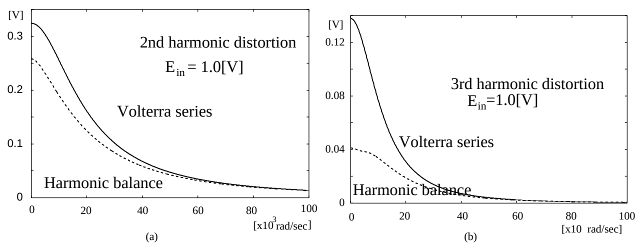

Fig. 6 (a) 2nd order harmonic distortion. (b) 3rd order harmonic distortion.

5. An illustrative example

We consider a base module circuit driven by two inputs as shown in Fig. 7(a) [14]. Firstly, let us solve it with Volterra series method.

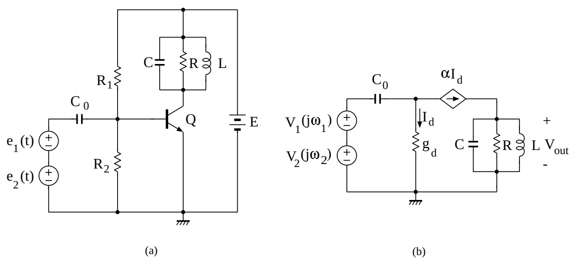

Fig. 7 (a) Base modulation circuit. (b) 1st order Volterra equivalent circuit.
R_1=50[kΩ], R_2=10[kΩ], R=1[kΩ], C=1[nF], C_0=1[nF], C_0=1[nF], L=10[μF], E=10[V], v_1=0.01$\sin \omega_1 t$, v_2=0.01$\sin \omega_2 t$, for ω_1=8.2×10^6[rad/sec].

[1] We used Ebers-Moll model for the bipolar transistor in the transient analysis. Note that the approximation error of HB method may be arisen from the Taylor expansion and neglecting the higher harmonic component than 3rd order.

[2] At the lower frequency, the results from Volterra are erroneous because the diode voltage is large at the frequencies.

We consider here 8 intermodulation frequencies given in Table 3.

Table 3 The frequency components to take into the consideration.

$\nu_1 = \omega_1$	$\nu_5 = \omega_1 - \omega_2$
$\nu_2 = \omega_2$	$\nu_6 = \omega_1 + \omega_2$
$\nu_3 = 2\omega_1$	$\nu_7 = 2\omega_1 - \omega_2$
$\nu_4 = 2\omega_2$	$\nu_8 = 2\omega_1 + \omega_2$

We have modeled the transistor by the Ebers-Moll model where

$$i_d = I_s e^{\lambda v_d}, \quad \text{for} \quad I_s = 10^{-12}[A], \quad \lambda = 40, \quad \text{and } \alpha = 0.99. \quad (11)$$

Firstly, we calculate the operating point of diode voltage, and have $v_{d0} = 0.5822[V]$. It is approximated by the Taylor expansion at the point v_{d0} as follow;

$$i_d = 0.0130 + 0.52 v_d + 10.40 v_d^2 + 138.60 v_d^3. \quad (12)$$

Thus, we have

$$g_d = 0.52, \quad k_2 = 10.40, \quad k_3 = 138.60. \quad (13)$$

We have changed the frequency ω_2 in the region $[0, 3\times 10^7]$, and get the frequency characteristics as shown Figs. 8(a) and (b), where we consider up to the 3rd order Volterra kernels, and solved with MATLAB.

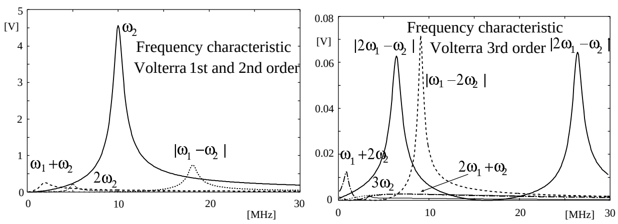

Fig. 8 (a) 1st and 2nd order solutions. (b) 3rd order solutions.

Next, we approximate the nonlinear diode by Taylor expansion in our HB method as follow;

$$i_d = k_0(v_{d0}) + k_1(v_{d0})v + k_2(v_{d0})v^2 + k_3(v_{d0})v^3. \quad (14)$$

Remark that v_{d0} is unknown variable in HB. The waveform is approximated by the frequency components in Table 3;

$$v(t) = V_0 + \sum_{k=1}^{8} (V_{2k-1} \cos \nu_k t + V_{2k} \sin \nu_k t). \quad (15)$$

The Sine-Cosine circuit can be formulated by MATLAB, and is solved by the DC analysis of Spice.

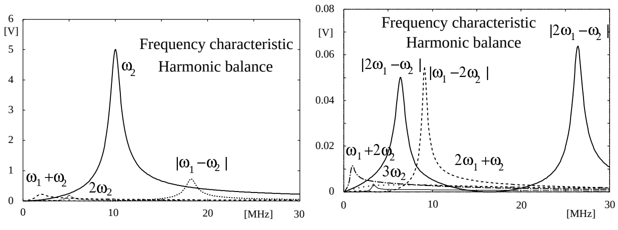

Fig. 9 (a) Frequency characteristic by HB. (b) Other frequency components.

Since the input voltages in this example are small, the both results of Volterra series method and HB method are almost the same.

6. Conclusions and remarks

In this paper, we proposed Spice-oriented HB method which can efficiently trace the characteristic curves such as frequency responses. For the nonlinear elements such as bipolar transistor, the Fourier coefficients are calculated by MATLAB in the symbolic forms. Thus, we can formulate the HB determining equation in the form of net-list and solve it with DC analysis of Spice. We have compared the method with Volterra series method, and found that our HB method can get much accurate solutions compared to Volterra series method.

As the future work, in order to apply our method to large scale networks, we need to develop device HB modules for bipolar transistors and MOSFETs.

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