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Investigation of Asymmetrically Coupled Chaotic Circuits and Chaotic Maps

Yukari Kowatari,[†] Yasuteru Hosokawa,[†] and Yoshifumi Nishio[‡]

†Dept. of Information Science, Shikoku University
Furukawa, Ojin-cho, Tokushima, 771-1192 JAPAN
‡Dept. of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima, 770-8506 JAPAN
Email: s1839028@keiei.shikoku-u.ac.jp, hosokawa@keiei.shikoku-u.ac.jp, nishio@ee.tokushima-u.ac.jp

Abstract—In our previous study, we observed interesting phenomena in asymmetrically coupled chaotic circuits. Namely, the ratio of synchronization time of the one subsystem group increases in spite of increasing parameter mismatches in the other subsystem. In this study, we investigate the observed phenomena from asymmetrically coupled chaotic circuits and asymmetrically coupled chaotic maps.

1. Introduction

Coupled chaotic systems generate various kinds of complex higher-dimensional phenomena such as spatio-temporal chaotic phenomena, clustering phenomena and so on. One of the most studied systems may be the coupled map lattice proposed by Kaneko [1]. The advantage of the coupled map lattice is its simplicity. However, many of nonlinear phenomena generated in nature would be not so simple. Therefore, it is also important to compare with the complex phenomena observed in natural physical systems such as electrical circuits systems [2]-[6]. One of useful tools for investigating these phenomena is electrical circuits. There are many advantages of using electrical circuits. For instance, circuits are natural physical systems, circuit elements are low price and high quality, circuit experiments are easy to match theory, and so on.

In our previous study [7][8], some kinds of asymmetrically coupled chaotic systems are investigated. Especially, we paid our attentions to the relationships between synchronization phenomena and small parameter mismatches. In all the systems, the same interesting phenomenon is observed. The phenomenon is that the ratio of the synchronization time increases in spite of increasing parameter mismatches in the other system.

In this study, in order to investigate these phenomena, asymmetrical coupled chaotic maps are investigated. The logistic map is used as a chaotic map. Asymmetry of the system is realized by using two parameter values. We pay our attentions to the relationship between the ratio of synchronization time and parameter mismatches.

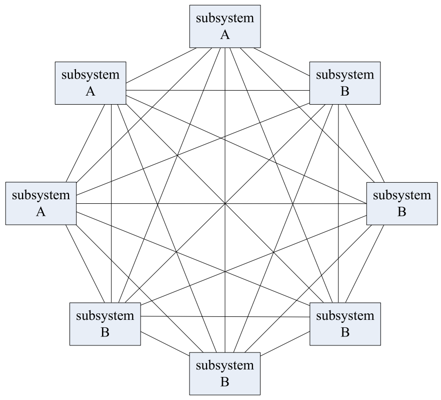


Figure 1: Asymmetrically coupled system.

2. Asymmetrically Coupled System

Asymmetrically coupled system is shown in Fig. 1. This system consists of two kinds of subsystems and coupling elements. Subsystems are coupled globally. An asymmetry of the system is realized by using two kinds of subsystems. Some kinds of chaotic circuits or autonomous oscillators were applied as subsystems in our previous studies. For instance, two different coupling points of one chaotic circuit, two different parameters of one chaotic circuit, chaotic circuits and van der Pol oscillators and so on. In this study, the logistic map is applied as subsystems. The asymmetry is realized by using two parameter values. This system is based on globally coupled map (GCM) proposed by Kaneko [9]. GCM is described by the following equation.

$$x_{n+1}(i) = (1-\varepsilon)f(x_n(i)) + \frac{\varepsilon}{N}\sum_{j=1}^{N} f(x_n(j)) \qquad (1)$$

where n is a discrete time step and i is the index of the elements. $f(x)$ is the function of the chaotic map.

In this study, the logistic map $f(x) = 1 - ax^2$ is chosen. The asymmetry of the system is realized by using two parameter values. Therefore, the function $f(x)$ is shown as follows.

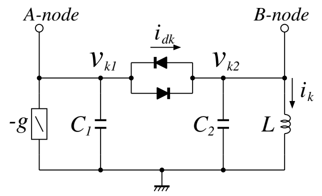


Figure 2: Chaotic Circuit.

Subsystem A ($1 \leq i \leq p$):

$$f_A(x_n(i)) = 1 - (1 + Q_a i)a_1 x_n(i)^2 \quad (2)$$

Subsystem B ($p + 1 \leq k \leq p + q$):

$$f_B(x_n(i)) = 1 - \{1 + Q_b(i - p)\}a_2 x_n(i)^2 \quad (3)$$

where i is the index number of the maps. n is the number of iterations. a_1 and a_2 are the parameter of the logistic map. p and q are the numbers of the elements in the subsystems A and B, respectively. Q_a and Q_b are parameter mismatch rates of the two subsystems.

3. Computer Simulations

3.1. Electrical Circuits

First, we introduce the results observed from the system using chaotic circuits. The applied chaotic circuit is shown in Fig. 2. This chaotic circuit is a simple three-dimensional autonomous circuit proposed by Shinriki et al. [10]. A-node is used as a coupling node. An asymmetry of the system is realized as a difference of parameters. Namely, parameters of subsystem A is different from subsystem B. Double scroll type attractors are observed from both of the subsystems. Normalized circuit equations are described as follows:
Subsystem A ($1 \leq k \leq m$):

$$\left\{\begin{array}{rcl} \dot{x}_k &=& \alpha\beta x_k - \alpha\gamma f(x_k - y_k) \\ & & +\alpha\delta\left\{\displaystyle\sum_{i=1}^{m+n} x_i - (m+n)x_k\right\}, \\ \dot{y}_k &=& -z_k + \gamma f(x_k - y_k), \\ \dot{z}_k &=& (1 + p_k)y_k, \end{array}\right. \quad (4)$$

Subsystem B ($m + 1 \leq k \leq m + n$):

$$\left\{\begin{array}{rcl} \dot{x}_k &=& \varepsilon\beta x_k - \varepsilon\gamma f(x_k - y_k) \\ & & +\varepsilon\delta\left\{\displaystyle\sum_{i=1}^{m+n} x_i - (m+n)x_k\right\}, \\ \dot{y}_k &=& \zeta\{-z_k + \gamma f(x_k - y_k)\}, \\ \dot{z}_k &=& \eta(1 + q_k)y_k, \end{array}\right. \quad (5)$$

where,

$$f(x) = x + \frac{(|x - 1| - |x + 1|)}{2}.$$

Figure 3 shows the voltage differences between the subsystems. Vertical axes show voltage differences and horizontal axes show time. Namely, in the case of synchronizing two subcircuits, the amplitude becomes zero. First graph shows the voltage difference between the two subsystem A. Synchronizations and un-synchronized burst appear alternately in a random way. The second graph shows the voltage difference between subsystem A and subsystem B. These are not synchronized at all. The third and fourth graphs show the voltage differences between two subsystem B. Here we define the synchronization as following equation and figure.

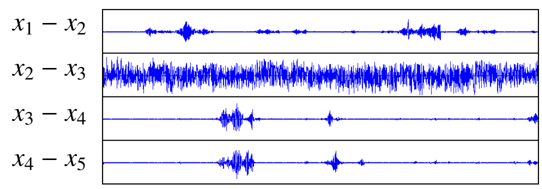


Figure 3: Voltage differences between two subsystems in the case of chaotic circuit.

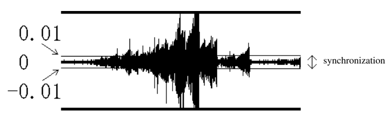


Figure 4: Definition of the synchronization.

$$|x_k - x_{k+1}| < 0.01 \quad (6)$$

Figure 5 shows ratios of the synchronization time and total time. Q is shown as following equation.

$$q_k = Q(k - 1) \quad (7)$$

Q is corresponding to small parameter mismatches q_k of subsystem B group. By increasing small parameter mismatch of subsystem B group, the synchronization time of

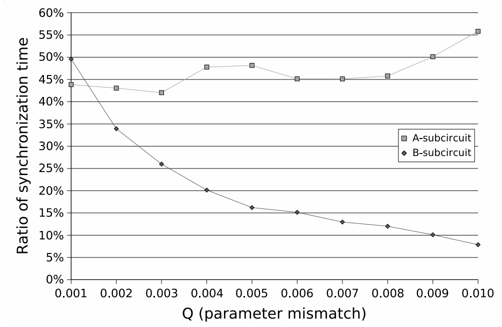
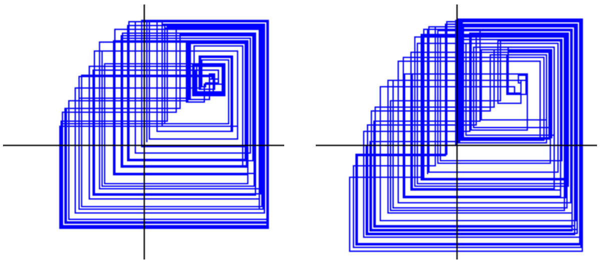


Figure 5: Relationship of the ratio of the synchronization time and small parameter mismatches in the case of System 2. $m = 2$, $n = 3$, $p_k = 0.001(k-1)$, $\alpha = 0.600$, $\beta = 0.500$, $\gamma = 20.0$, $\delta = 0.070$, $\varepsilon = 0.6$, $\zeta = 1.5$ and $\eta = 0.5$.



(a) $x_n(1)$ vs. $x_{n+1}(1)$ (b) $x_n(3)$ vs. $x_{n+1}(3)$

Figure 6: Maps of (a) subsystem A and (b) subsystem B. $p = 2$, $q = 3$, $Q_a = 0.050$, $a_1 = 1.70$, $a_2 = 1.98$ and $\varepsilon = 0.40$.

subsystem A group is increased. Namely, in spite of increasing small parameter mismatches of the system, the synchronization time of subsystem A group is increased.

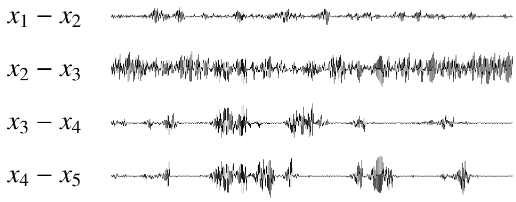
3.2. Chaotic Maps

We carry out computer simulations using Eq. (2) and Eq. (3). Here we define the synchronization as the following conditional equation:

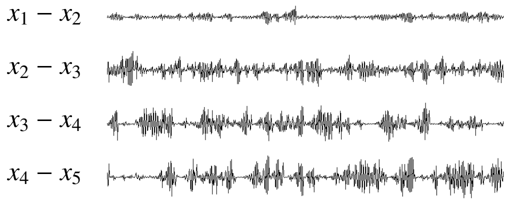
$$(|x_{n-1}(i)-x_{n-1}(i+1)| < 0.05) \cap (|x_n(i)-x_n(i+1)| < 0.05) \quad (8)$$

This equation shows that two consecutive closer values than a given threshold value (0.05) define synchronization.

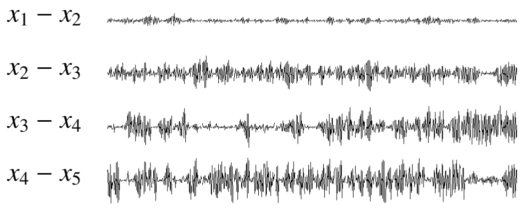
Figures 6 and 7 are examples of the computer simulation results. In Fig. 6, the horizontal axes show $x_n(i)$ and the vertical axes show $x_{n+1}(i)$. Figure 6(a) shows x_1 and Fig. 6(b) shows x_3. We can see the difference between the two maps. In Fig. 7, the horizontal axes show iterations and the vertical axes show the values of $x_k - x_{k+1}$. From these results, we can see that subsystem A and subsystem B are not synchronized at all and increasing Q_b causes increasing a ratio of synchronization time of subsystem A. Figure 8 is corresponding to Fig. 5. We can also see the similar result as Fig. 5. The chaotic circuit is continuous-



(a) $Q_b = 0.002$



(b) $Q_b = 0.020$



(c) $Q_b = 0.040$

Figure 7: Variable differences between two subsystems. $p = 2$, $q = 3$, $Q_a = 0.050$, $a_1 = 1.70$, $a_2 = 1.98$ and $\varepsilon = 0.40$.

time system. The logistic map is discrete-time system. In spite of this difference, we can observe the phenomena in both systems. Therefore, we consider that the phenomena are closely related to the structure of the system.

Next, we investigate the case of thirty maps. Figure 9 shows the relationship of the ratio of the synchronization time and small parameter mismatches. In this case, we can also the same phenomena. Figure 10 shows the relationship of the ratio of the synchronization time and the number of the maps in A group. This result shows that increasing rate of A group decrease the ratio of the synchronization time of A group.

4. Conclusions

In this study, asymmetrically coupled chaotic maps are proposed and investigated. Asymmetry of the system is realized by using two parameter values. In the case of five maps, it was confirmed that ratios of synchronization time of maps using one parameter set are increased by increasing a parameter mismatch rate of the other maps group. We consider that this result is corresponding to the result

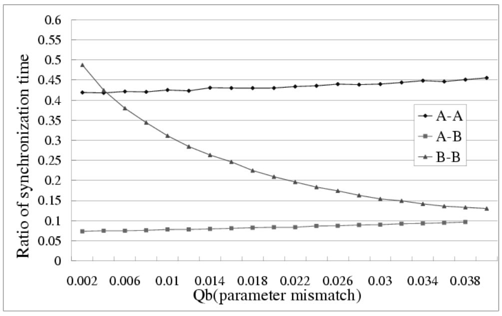


Figure 8: Relationship of the ratio of the synchronization time and small parameter mismatches. $p = 2$, $q = 3$, $Q_a = 0.050$, $a_1 = 1.70$, $a_2 = 1.98$ and $\varepsilon = 0.40$. The number of iteration is 1000000.

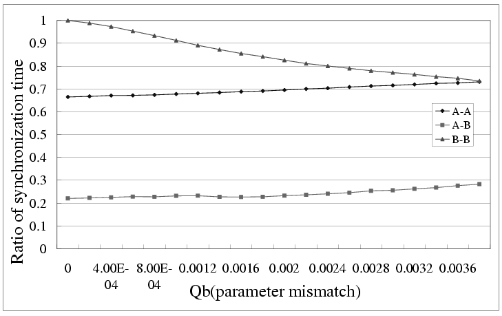


Figure 9: Relationship between the ratio of the synchronization time and small parameter mismatches. $p = 10$, $q = 20$, $Q_a = 0.002$, $a_1 = 1.70$, $a_2 = 1.95$ and $\varepsilon = 0.45$. The number of iteration is 1000000.

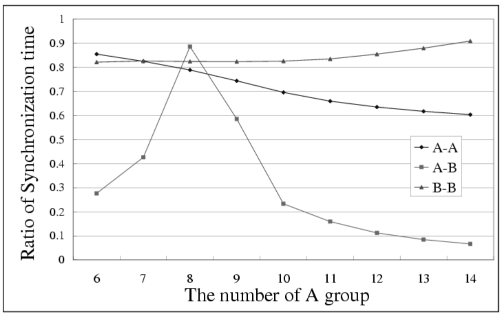


Figure 10: Relationship of the ratio of the synchronization time and small parameter mismatches. The number of maps is 30. $Q_a = 0.020$, $Q_b = 0.002$, $a_1 = 1.70$, $a_2 = 1.95$ and $\varepsilon = 0.45$. The number of iteration is 1000000.

of our previous study. We also investigated the case of thirty maps. In this case, we observed that increasing rate of A group decrease the ratio of the synchronization time of A group. Investigating this phenomenon is our future research.

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