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State Transition Phenomenon in Cross-Coupled Chaotic Circuits

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Abstract—Studies on chaos synchronization in coupled chaotic circuits are extensively carried out in various fields. In this study, two simple chaotic circuits cross-coupled by inductors are investigated. Interesting state transition phenomenon around chaos synchronization is observed by computer simulations and circuit experiments.

1. Introduction

Synchronization phenomena in complex systems are very good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. Studies on synchronization phenomena of coupled chaotic circuits are extensively carried out in various fields [1]-[10]. We consider that it is very important to investigate the phenomena related with chaos synchronization to realize future engineering application utilizing chaos.

In this study, two Shinriki-Mori chaotic circuits [11][12] cross-coupled by inductors are investigated. We observe the generation of interesting state transition phenomenon around chaos synchronization. Computer simulations and circuit experiments are carried out to investigate the phenomenon in detail.

2. Circuit Model

Figure 1 shows the circuit model. In the circuit, two Shinriki-Mori chaotic circuits are cross-coupled via inductors L_2.

First, we approximate the $v-i$ characteristics of the nonlinear resistors consisting of the diodes by the following 3-segment piecewise-linear functions.

$$i_{d1} = \begin{cases} G(v_{11}-v_{12}-V) & (v_{11}-v_{12}>V) \\ 0 & (|v_{11}-v_{12}| \leq V) \\ G(v_{11}-v_{12}+V) & (v_{11}-v_{12}<-V) \end{cases} \quad (1)$$

$$i_{d2} = \begin{cases} G(v_{21}-v_{22}-V) & (v_{21}-v_{22}>V) \\ 0 & (|v_{21}-v_{22}| \leq V) \\ G(v_{21}-v_{22}+V) & (v_{21}-v_{22}<-V) \end{cases} \quad (2)$$

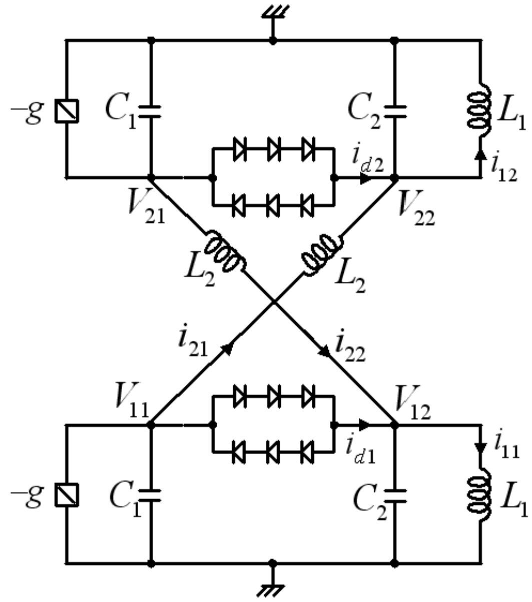

Figure 1: Circuit model.

The circuit equations are described as follows.

$$\begin{cases} L_1 \dfrac{di_{11}}{dt} = v_{12} \\ L_1 \dfrac{di_{12}}{dt} = v_{22} \\ C_1 \dfrac{dv_{11}}{dt} = -i_{d1} - i_{21} + gv_{11} \\ C_1 \dfrac{dv_{21}}{dt} = -i_{d2} - i_{22} + gv_{21} \\ C_2 \dfrac{dv_{12}}{dt} = i_{d1} + i_{22} - i_{11} \\ C_2 \dfrac{dv_{22}}{dt} = i_{d2} + i_{12} - i_{21} \\ L_2 \dfrac{di_{21}}{dt} = v_{11} - v_{22} \\ L_2 \dfrac{di_{22}}{dt} = v_{12} - v_{21} \end{cases} \quad (3)$$

By using the following variables and the parameters,

$$
\begin{cases}
i_{11} = \sqrt{\dfrac{C_2}{L_1}}\, V x_1, \quad v_{11} = V y_1, \quad v_{12} = V z_1, \\
i_{21} = \sqrt{\dfrac{C_2}{L_1}}\, V x_2, \quad v_{21} = V y_2, \quad v_{22} = V z_2, \\
i_{12} = \sqrt{\dfrac{C_2}{L_1}}\, V w_1, \quad i_{22} = \sqrt{\dfrac{C_2}{L_1}}\, V w_2, \\
\alpha = \dfrac{C_2}{C_1}, \quad \beta = \sqrt{\dfrac{L_1}{C_2}}\, G, \quad \gamma = \sqrt{\dfrac{L_1}{C_2}}\, g, \\
\delta = \dfrac{L_1}{L_2}, \quad t = \sqrt{L_1 C_2}\, \tau,
\end{cases}
\tag{4}
$$

the normalized circuit equations are given as follows.

$$
\begin{cases}
\dot{x}_1 = z_1 \\
\dot{x}_2 = z_2 \\
\dot{y}_1 = \alpha\{\gamma y_1 - w_1 - \beta f\,(y_1 - z_1)\} \\
\dot{y}_2 = \alpha\{\gamma y_2 - w_2 - \beta f\,(y_2 - z_2)\} \\
\dot{z}_1 = \beta\, f(y_1 - z_1) + w_2 - x_1 \\
\dot{z}_2 = \beta\, f(y_2 - z_2) + w_1 - x_2 \\
\dot{w}_1 = \delta(y_1 - z_2) \\
\dot{w}_2 = \delta(y_2 - z_1)
\end{cases}
\tag{5}
$$

where f are the nonlinear functions corresponding to the $v - i$ characteristics of the nonlinear resistors and are described as follows.

$$
f(y_1 - z_1) = \begin{cases}
y_1 - z_1 - 1 & (y_1 - z_1 > 1) \\
0 & (|y_1 - z_1| \le 1) \\
y_1 - z_1 + 1 & (y_1 - z_1 < -1)
\end{cases}
\tag{6}
$$

$$
f(y_2 - z_2) = \begin{cases}
y_2 - z_2 - 1 & (y_2 - z_2 > 1) \\
0 & (|y_2 - z_2| \le 1) \\
y_2 - z_2 + 1 & (y_2 - z_2 < -1).
\end{cases}
\tag{7}
$$

3. State Transition Phenomenon

From the circuit in Fig. 1, we can observe interesting state transition phenomenon around chaos synchronization.

Some examples of the phenomenon are shown in Figs. 2 and 3. These results are obtained by calculating Eq. (5) with the Runge-Kutta method. The two circuits exhibit chaos but almost synchronized in in-phase in the sense that the attractor is almost in the quadrant I or III on the $y_1 - y_2$ plane. When one circuit switches to/from the positive region from/to the negative region, the other follows the transition after a few instants. The sojourn time between the state transitions becomes longer as the coupling parameter δ decreases.

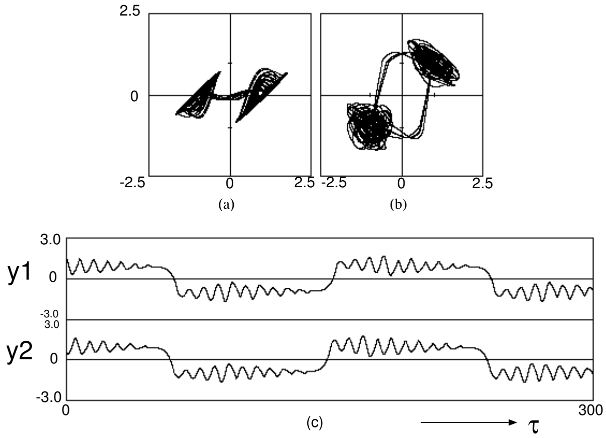

Figure 2: State transition phenomenon around in-phase synchronization (computer calculated result). $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and $\delta = 0.005$. (a) Attractor on $y_1 - z_1$ plane. (b) Attractor on $y_1 - y_2$ plane. (c) Time waveform.

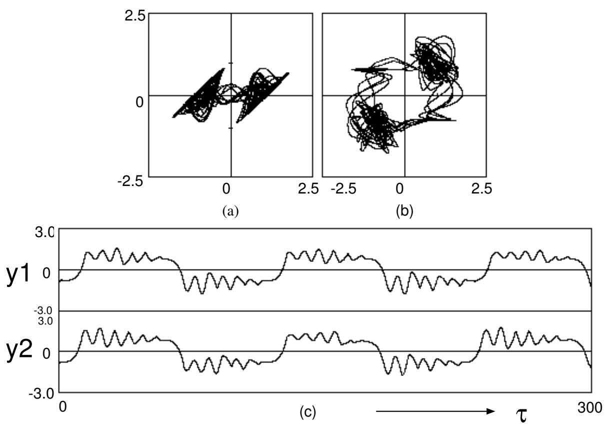

Figure 3: State transition phenomenon around in-phase synchronization (computer calculated result). $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and $\delta = 0.008$. (a) Attractor on $y_1 - z_1$ plane. (b) Attractor on $y_1 - y_2$ plane. (c) Time waveform.

Figure 4 shows how the sojourn times change as the coupling parameter changes. The curve of circles shows the average period of the state transitions of y_1. The curve of crosses shows the average time delay of the state transitions of y_2 when the state transitions of y_1 are considered to be the reference.

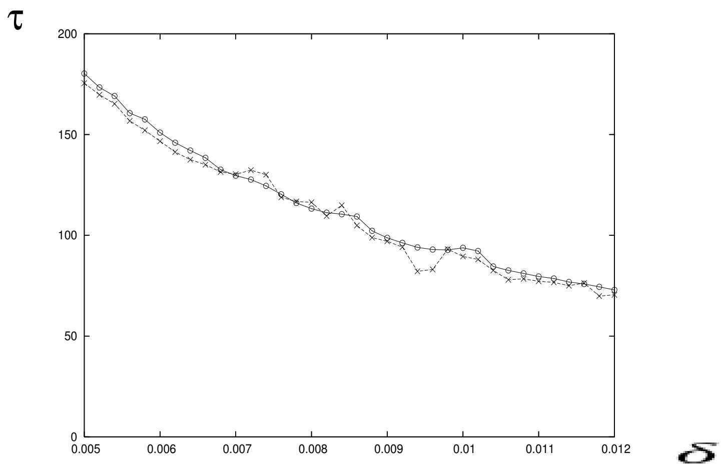

Figure 4: Sojourn time between state transitions versus coupling parameter (computer calculated result). $\alpha = 2.605$, $\beta = 4.0$, and $\gamma = 0.1$.

From this figure, we can confirm how the sojourn time between the state transitions changes according to the change of the coupling parameter.

It is very interesting that we can also confirm the generation of the state transition around the anti-phase synchronization for different set of parameter values. An example of such modes are shown in Fig. 5.

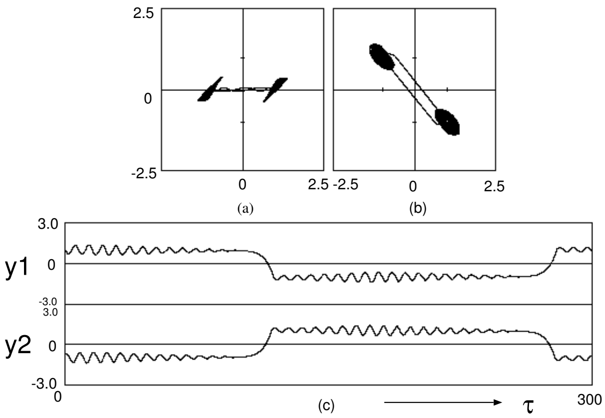

Figure 5: State transition phenomenon around anti-phase synchronization (computer calculated result). $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.1$, and $\delta = 0.0014$. (a) Attractor on $y_1 - z_1$ plane. (b) Attractor on $y_1 - y_2$ plane. (c) Time waveform.

4. Circuit Experimental Results

Because it is difficult to realize the very small coupling parameter like $\delta = 0.001$, the circuit experiments are carried out with relatively large δ.

The circuit experimental results are shown in Fig. 6. The corresponding computer calculated results are shown in Fig. 7.

We can say that the both results agree well.

5. Conclusions

In this study, we have investigated interesting state transition phenomenon observed from two Shinriki-Mori chaotic circuits cross-coupled by inductors.

Investigating the coexistence of the states and statistical analysis of the observed phenomena are our important future work.

Acknowledgments

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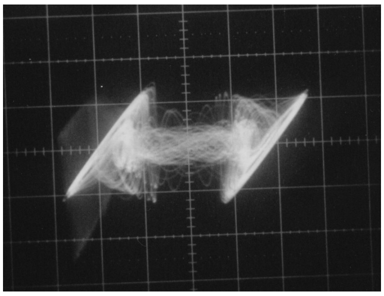
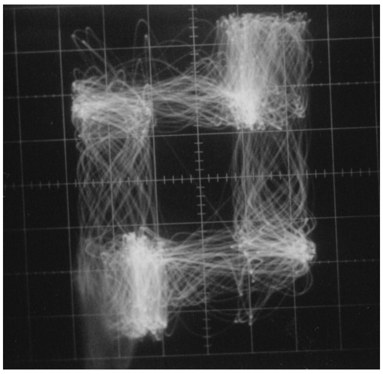
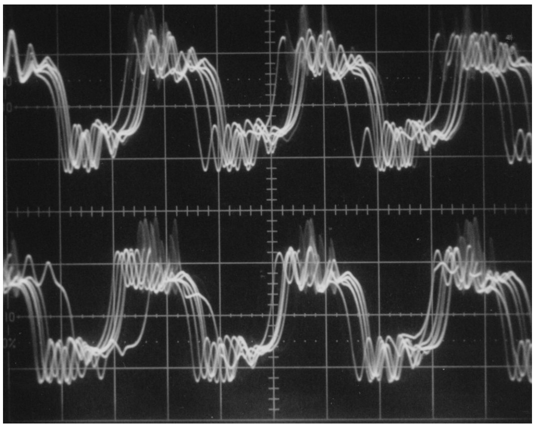


(a)



(b)



(c)

Figure 6: Circuit experimental result. $L_1 = 9.93$mH, $L_2 = 648$mH, C_1=32.8nF, C_2=49.5nF, and g=1.89mS. (a) Attractor on $v_{11} - v_{12}$ plane. Horizontal and vertical: 1 V/div. (b) Attractor on $v_{11} - v_{21}$ plane. Horizontal and vertical: 1 V/div. (c) Time waveform v_{11} and v_{21}. Horizontal 0.5 ms/div and vertical: 2 V/div.

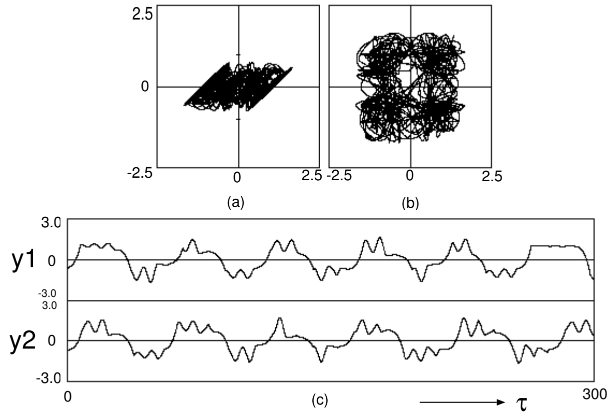


Figure 7: Computer calculated result corresponding to Fig. 6. $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and $\delta = 0.015$. (a) Attractor on $y_1 - z_1$ plane. (b) Attractor on $y_1 - y_2$ plane. (c) Time waveform.

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