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Coexistence of Several Phase Synchronization in a Ring of Coupled Multi-State Chaotic Oscillators

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Abstract—This paper presents several phase synchronization modes of multi–state chaotic oscillators coupled by inductors as a ring. Several phase patters occured in some types of coupled oscillators have been widely known. Each chaotic circuit which used in this paper can individually behave both chaotic and periodic oscillations in the same parameters asynchronously. In this study, such the coupled chaotic circuits are proposed and classifications of phase synchronization modes are investigated. In numerical simulation, many types of phase synchronization modes are confirmed.

1. Introduction

Nonlinear dynamics on coupled chaotic oscillators is considerable interesting for a wide variety of systems in several scientific fields and applications. Many types of coupled systems have been widely studied in order to clarify inherent features and many researchers have already proposed and investigated them. Coupled chaotic systems are as one of them which have several varieties of interesting behavior with emergent properties. The dynamics of chaotic multimode oscillations or chaotic itinerancy on several coupled systems is still considerable interest from the viewpoint of both natural scientific fields and several applications. They have been confirmed in several systems; e.g., coupled van der Pol oscillators [1], laser systems [2], and so on. Phase synchronization and pattern dynamics are also interesting for several engineering applications. On the other hand, many types of chaotic systems and circuits have already been proposed and investigated in detail. As interesting phenomena, there are famous chaotic attractors such a double-scroll family [3], n-double scroll [4]–[6] and scroll grid attractors [7]. If the active elements including in the systems have complexity constructed by compound some nonlinear elements, it can be easily considered that they yield several interesting features. The circuit which can individually behave both chaotic or periodic oscillations in the same parameters had been shown [8]. This type of circuit was called a multi–state chaotic circuit (abbr. MSCC). Multimode oscillations in coupled two or more multi–state chaotic circuits had also been investigated [10][11]. Such complex and strange nonlinear structures yield a wide variety of chaotic phenomena. It is an important idea that how does nonlinearity lead to various kinds of behaviors. They are also constructed by making several equilibrium points. A mechanism and search of chaotic regions in piecewise linear systems were investigated [9]. The purpose of our study is to clarify coexistence of both chaotic and non–chaotic behavior in the chaotic systems. Further complex behavior in the large scale network of the coupled chaotic circuits are also investigated. It is known that complex behavior can be confirmed such chaotic itinerancy and spatio-temporal chaos on the large scale coupled networks. Some kinds of oscillation modes had been reported on large scale coupled chaotic circuits such phase synchronization, phase propagation and frustration of oscillation and so on [12]-[14].

In this study, several phase patterns and multimode asynchronous oscillations on the coupled MSCCs are investigated. There is a typical three dimensional autonomous chaotic system, which consists of three memory elements, some diodes and designed negative resistors. It is well known that it can behave as Rössler type chaotic motion. We substitute a symmetrical continuous segments piecewise linear resistor for the negative active resistor including in the original chaotic circuit. This proposed circuit can behave both chaotic and periodic oscillations in the same parameters when we supply with different initial conditions. In this paper, phase synchronization and classification of several phase patterns in some MSCCs coupled by inductors as a ring are investigated. Several types of phase synchronization modes are confirmed asynchronously, but all circuit parameters are the same.

2. Model Description

The circuit shown in Fig. 1 is modified chaotic circuit from a well–known three dimensional chaotic circuit [15]. The original circuit consists of three memory elements, some diodes and designed negative resistors. It is well known that it can behave as Rössler type chaotic motion. We substitute a symmetrical piecewise linear resistor for the negative active resistor including in the original chaotic circuit. Further this circuit possesses another symmetrical piecewise nonlinear resistor with respect to the origin.

At first, we approximate the $i - v$ characteristics in the part of both diodes and E_1 by the following three-segment

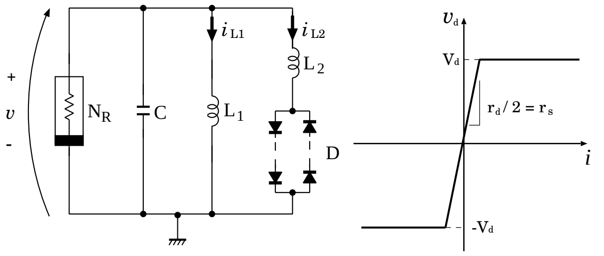

Figure 1: Based chaotic circuit with piecewise linear resistors.

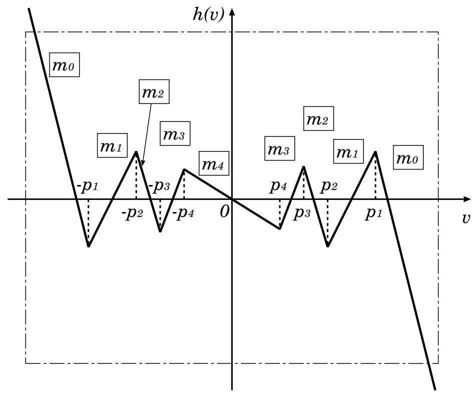

Figure 2: Design for sawtooth nonlinear resistor with respect to the origin.

piecewise linear functions $v_d(i_{L2k})$.

$$v_d(i_{L2k}) = \frac{1}{2}\left(r_s i_{L2k} + V_d - |r_s i_{L2k} - V_d|\right) \qquad (1)$$

where threshold voltage V_d is realized by the total of threshold of the diodes and r_s is a resistance value at the parallel diode in while off state. The variable $v_d(i_{L2k})$ determines their chaotic dynamics.

In this study, we substitute a symmetrical continuous piecewise linear resistor for the negative active resistor including in the chaotic circuit. The designed chaotic circuit is shown in Fig. 2. The piecewise linear resistor can be easily constructed by combining some components in parallel [8][10]. By changing the following variables and parameters as follows

$$\begin{aligned} & i_{L1} = \sqrt{\frac{C}{L_1}} V_d\, x\ ,\ \ i_{L2} = \sqrt{\frac{C}{L_1}} V_d\, y\ , \\ & v = V_d\, z\ ,\ \ t = \sqrt{L_1 C}\, d\tau\ ,\ \ \text{“}\cdot\text{”} = \frac{d}{d\tau}\ , \\ & \beta = \frac{L_1}{L_2}\ ,\ \ \gamma = g\sqrt{\frac{L_1}{C}}\ ,\ \ \delta = r_s\sqrt{\frac{C}{L_1}} \end{aligned} \qquad (2)$$

where g is a linear negative conductance value of N_R if we consider the negative resistor as an ideal. Consider that the part of negative resistance N_R in Fig. 1 replaces to the function $h(z)$ represented by a voltage source z as canonical form. When we chose V_d for a threshold voltage value of the diodes, then the circuit equations can be rewritten by

$$\left\{ \begin{array}{lcl} \dot{x} & = & z \\ \dot{y} & = & \beta\big(z - f(y)\big) \\ \dot{z} & = & -(x+y) - h(z) \end{array} \right. \qquad (3)$$

$$f(y) = \frac{1}{2}\big\{|\delta y + 1| - |\delta y - 1|\big\}, \qquad (4)$$

and

$$\begin{aligned} & h(z) = m_0\gamma^* z + \frac{\gamma^*}{2}\Big\{ \\ & \sum_{k=0}^{K}(m_k - m_{k+1})\big\{|z - p_{k+1}| - |z + p_{k+1}|\big\}\Big\} \end{aligned} \qquad (5)$$

where $f(y)$ is a function of the current y and $h(z)$ is a function of the voltage z, respectively. The function $h(z)$ which is designed for several segment piecewise linear as symmetric with respect to the origin. The parameter γ^* is used for a basic common value, hence the values $m_k(k =$ 0,1,2,$\cdots$,K) mean magnitude of the slope to the ratio for γ^*.

The detailed schematic design how to construct had been explained and the circuit settings are put in their figures' caption [8]-[11]. As a result, both chaotic and periodic attractors can be observed in the same circuit parameters. In the previous work, we could confirm that both chaotic and non-chaotic attractors coexist in the same parameters. If the sawtooth nonlinear resistor in the circuit was improved to have several segments, we can confirm coexistence of several attractors in the same parameters. Figure 3 also shows a typical chaotic attractor obtained for the parameters $\beta = 10.0$, $\gamma^* = 0.78$, $\delta = 100$, with piecewise linear characteristics realized by breakpoints p_1 = 0.65, p_2 = 0.55, p_3 = 0.40, p_4 = 0.30, slopes $m_0 = -1.0$, $m_1 = 2.0$, $m_2 = -1.0$, $m_3 = 1.0$ and $m_4 = -0.15$. We can confirm that both chaotic and two periodic attractors coexist in the same parameters. It is different from the previous work that we could confirm coexistence of chaotic attractor and two different size of limit cycles in this circuit.

3. Simulation for coupled MSCCs

We now consider the coupled model which combined number of N chaotic circuits are connected by inductors L_0 as a ring structure. The chaotic circuits are composed by all the same parameters. Therefore when we choose a threshold voltage value V_d as a criterion, the circuit equation of coupled MSCCs can be normalized by changing the variables (2) and a new parameter $\alpha = L_1/L_0$, then the entire circuit equations can be rewritten by

$$\left\{ \begin{array}{lcl} \dot{x_k} & = & z_k \\ \dot{y_k} & = & \beta\big(z_k - f(y_k)\big) \\ \dot{z_k} & = & \alpha(x_{k-1} - 2x_k + x_{k+1}) \\ & & \qquad -(x_k + y_k) - h(z_k) \end{array} \right. \qquad (6)$$

In this section, the model of coupled MSCCs by inductors are investigated. We show some computer calculation

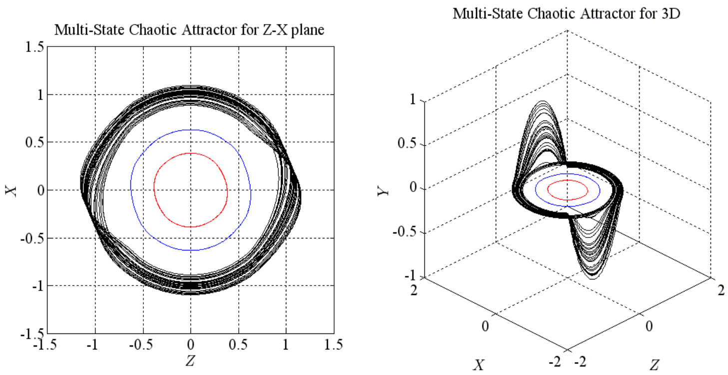

Figure 3: Drawing attractor onto the $z-x$ plane and 3-D trajectories for the parameters $\beta = 10.0$, $\gamma^* = 0.78$ and $\delta = 100$. $h(z)$: $\{p_1, p_2, p_3, p_4\}$={0.65, 0.55, 0.40, 0.30}, $\{m_0, m_1, m_2, m_3, m_4\}$ = {-1.0, 2.0, -1.0, 1.0, -0.15}.

results by using 4-th order Runge–Kutta method with time step size $\Delta t = 0.001$ for the circuit equation (6), (4) and (5) in some cases of $N = 2 \sim 7$ as follows.

3.1. Two subcircuits case $N = 2$

Now we consider that the number of the coupled MSCCs is two. This case is similar to the model in the sense of coexistence of chaotic and periodic oscillations for the previous work [10]. Although the detail results are omitted, we can confirm several types of phase synchronization modes in this model. In this case, some asynchronous oscillation modes could be confirmed consequently by numerical simulations when the initial conditions are varied. We could observe two different limit cycles by their oscillation size; in-phase synchronous limit cycles, anti-phase synchronous limit cycles, anti-phase chaotic synchronous state, and multimode oscillations in the same parameters.

3.2. Three or more subcircuits case $N \geq 3$

In this section, we consider the case of $N = 3$. The circuit parameters in each MSCC are set as all the same parameters in the section 2 with additional parameter $\alpha = 0.50$. Compare with the case $N = 2$, several different synchronization phenomena can be found. Because all types of the results can not be represented, some simulation results are only shown here. Figure 5(a) shows a case of in-phase synchronization of three limit cycles. From top of the figure, attractors drawing onto z–x plane, synchronization state of z_k–z_{k+1} plane, and waveform of difference between the two variables $z_k - z_{k+1}$. Figure 5(b) shows in-phase chaotic synchronization and pair of double-mode oscillation. They are corresponding normally to three phase synchronization in generic oscillators. We could confirm to coexist a lot of synchronization modes.

In large coupled systems for $N \geq 4$, it is easily expected to be confirmed more complex behavior. We now show only some results in Fig. 5 for the case of N = 4 and 7. In the case of $N = 4$, several types of synchronization modes are confirmed. Figure (c) shows two-pair of in-phase and anti-phase synchronization of limit cycles. Further, figure (d) shows a new type of synchronization mode applicable

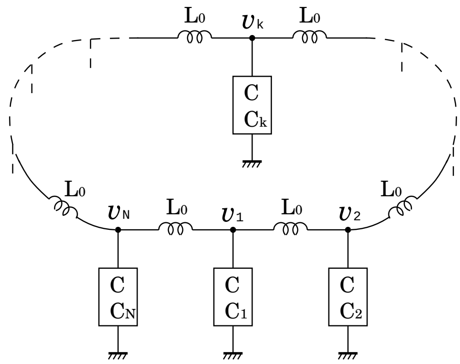

Figure 4: Coupled model of chaotic circuits by several inductors as a ring.

to no other one. In the case of $N = 7$, no much many synchronization modes have confirmed in such parameter settings. Figures (e) and (f) show typical simulation results for a large number of N. However when the number of N is large and the circuit parameters should be set appropriately, a certain kind of phase propagation phenomena may be confirmed. Thus, several complex behavior could be also confirmed in the coupled multi–state chaotic oscillators.

4. Conclusions

In this study, we have investigated several synchronization modes in coupled multi–state chaotic circuits. Coexistence of several types oscillation modes have been confirmed in coupled MSCCs by inductors as a ring for several cases. On large scale coupled chaotic circuits such a scale-free network, we consider that several types of complex behavior are expected to yield novel chaotic phenomena e.g., chaotic itinerancy, spatio–temporal chaos, multi–agent systems, soliton like wave propagation phenomena, and inherent emergent property, in which concerned with other current topics.

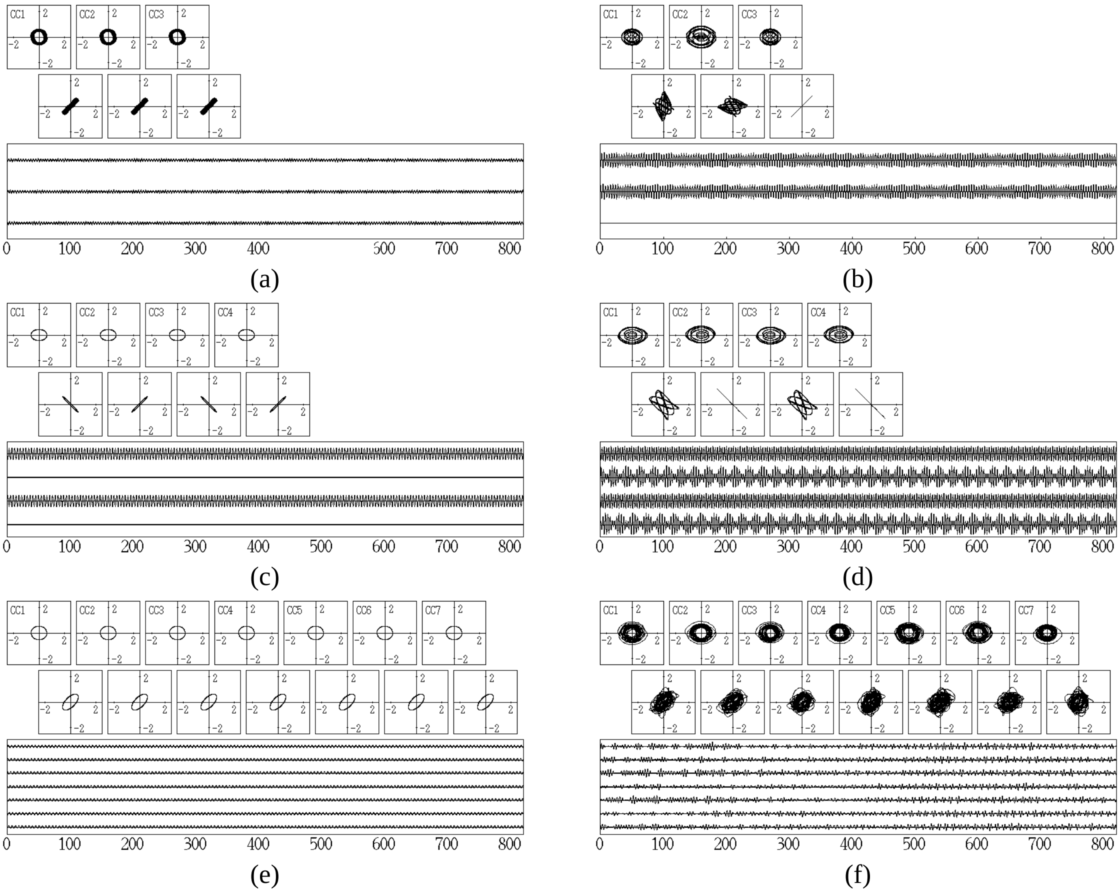

Figure 5: Some simulation results obtained from coupled three, four and seven MSCCs for α = 0.50, β = 10.0, γ^* = 0.78, δ = 100. $h(z)$: $\{p_1, p_2, p_3, p_4\}$={0.65, 0.55, 0.40, 0.30}, $\{m_0, m_1, m_2, m_3, m_4\}$ = {-1.0, 2.0, -1.0, 1.0, -0.15}. (a) in-phase synchronization, (b) double-mode and one-pair synchronization, (c) two-pari anti-phase synchronization, (d) two-pair double-mode and anti-phase synchronization, (e) seven-phase synchronization, and (f) multimode oscillation.

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