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Effect of Parameter Mismatch in Chaotic Circuits Coupled by a Time-Varying Resistor

Yoko Uwate and Yoshifumi Nishio
Dept. of Electrical and Electronics Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima, Japan
Email: {uwate, nishio}@ee.tokushima-u.ac.jp

Abstract— **In this study, we investigate the effect of parameter mismatch in chaotic circuits coupled by a time-varying resistor. By computer simulations, we can observe very interesting synchronization phenomenon which means that the sojourn times of in-phase and anti-phase states become longer when the parameter mismatch increases.**

I. Introduction

Synchronization phenomena in complex systems are important to describe various higher-dimensional nonlinear phenomena in the field of natural science. Studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields; physics, biology, engineering and so on [1]-[4]. Because many researchers suggest that synchronization phenomena of coupled oscillators have some relations to information processing in the brain. We consider that it is very important to investigate the synchronization phenomena of coupled oscillators to realize a brain computer for the future engineering application.

On the other hand, there are some systems whose dissipation factors vary with time, for example, under the time-variation of the ambient temperature, an equation describing an object moving in a space with some friction and an equation governing a circuit with a resistor whose temperature coefficient is sensitive such as thermistor. However, there are few discussions about oscillators coupled by a time-varying resistor.

In our previous study, two chaotic circuits coupled by a time-varying resistor were investigated [5]. First, the coexistence of in-phase and anti-phase synchronization was observed. Next, we have confirmed that interesting synchronization phenomena with switching phase states between the two chaotic circuits can be observed when the strength of coupling parameter R is decreased. Furthermore, the sojourn times of the in-phase and the anti-phase have been investigated. We confirmed that the sojourn time depends on the frequency and the strength of the time-varying resistor.

In this study, we investigate the effect of parameter mismatch in chaotic circuits coupled by a time-varying resistor. Namely, the two chaotic circuits with different characteristics are coupled. We give the parameter mismatch of β to the two chaotic circuits. We consider that it is very important to observe the difference of synchronization phenomena between symmetric and asymmetric systems. By computer simulations, we can observe very interesting synchronization phenomenon which means that the sojourn times of in-phase and anti-phase states become longer when the parameter mismatch increases.

II. Circuit Model

Figure 1 shows the circuit model, which is the chaotic version of the circuit investigated in [6]. In the circuit, two identical chaotic circuits are coupled by a time-varying resistor whose characteristics are shown in Fig. 2 [7].

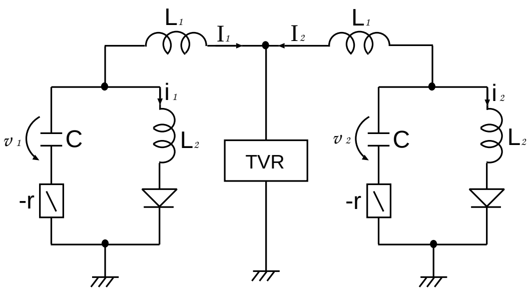


Fig. 1. Circuit model (TVR is a Time-Varying Resistor).

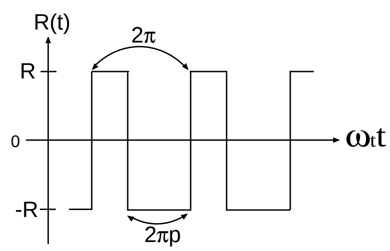


Fig. 2. Characteristics of the TVR.

First, the $i-v$ characteristics of the diodes are approximated by two-segment piecewise-linear functions as

$$v_d(i_k) = 0.5(r_d i_k + E - |r_d i_k - E|). \quad (1)$$

By changing the variables and parameters,

$$I_k = \sqrt{\frac{C}{L_1}} E x_k, \quad i_k = \sqrt{\frac{C}{L_1}} E y_k, \quad v_k = E z_k,$$
$$t = \sqrt{L_1 C}\tau, \quad \alpha = \frac{L_1}{L_2}, \quad \beta = r\sqrt{\frac{C}{L_1}}, \quad (2)$$
$$\gamma = R\sqrt{\frac{C}{L_1}}, \quad \delta = r_d\sqrt{\frac{C}{L_1}}, \quad \omega = \frac{1}{\sqrt{L_1 C}}\,\omega_t,$$

the normalized circuit equations are given as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \beta(x_k + y_k) - z_k \pm \gamma(x_1 + x_2) \\ \dfrac{dy_k}{d\tau} = \alpha\beta(x_k + y_k) - z_k - f(y_k) \\ \dfrac{dz_k}{d\tau} = x_k + y_k \qquad\qquad (k = 1, 2) \end{cases} \tag{3}$$

where the sign of the coupling term changes according to the value of the time-varying resistor. The normalized characteristics of the diodes are given as

$$f(y_k) = 0.5\ (\delta y_k + 1 - |\delta y_k - 1|). \tag{4}$$

III. Synchronization Phenomena

A. *In-Phase and Anti-Phase Synchronization*

We observed that the two coupled oscillators are synchronized in in-phase and anti-phase as shown in Figs. 3 and 4. These two synchronization states can be obtained by giving different initial conditions. The parameters of the chaotic circuits are fixed as $\alpha = 7.0$, $\beta = 0.084$, $\gamma = 0.1$, $\delta = 100$, and $\omega = 1.924$.

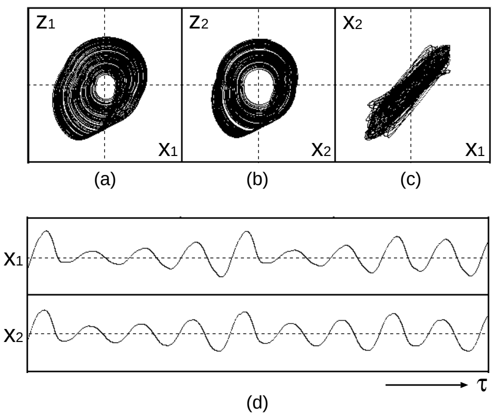


Fig. 3. In-phase synchronization (computer simulation results). (a) 1st circuit attractor (x_1 vs z_1). (b) 2nd circuit attractor (x_2 vs z_2). (c) Phase difference (x_1 vs x_2). (d) Time wave form (τ vs x_1 and x_2).

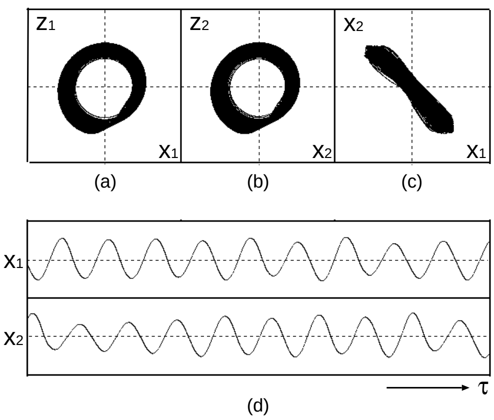
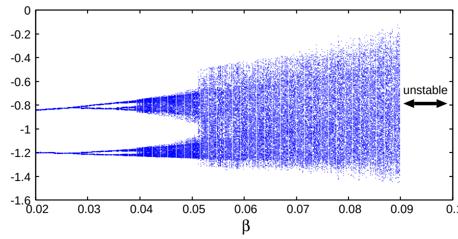
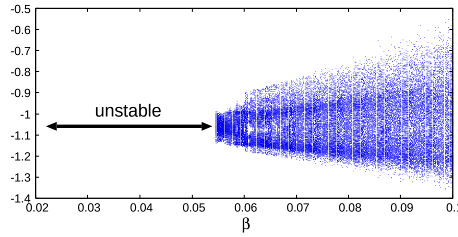


Fig. 4. Anti-phase synchronization (computer simulation results). (a) 1st circuit attractor (x_1 vs z_1). (b) 2nd circuit attractor (x_2 vs z_2). (c) Phase difference (x_1 vs x_2). (d) Time wave form (τ vs x_1 and x_2).

The one parameter bifurcation diagram of x_1 for in-phase and anti-phase synchronization modes are shown in Fig. 5. Figure 6 shows the one parameter diagram of the phase difference. We can confirm the coexistence of in-phase and anti-phase synchronizations for $0.055 < \beta < 0.090$.



(a) Bifurcation diagram of x_1 for in-phase synchronization mode.



(b) Bifurcation diagram of x_1 for anti-phase synchronization mode.

Fig. 5. One parameter bifurcation diagrams for $\alpha = 7.0$, $\gamma = 0.1$, $\omega = 1.924$. Horizontal axis: β.

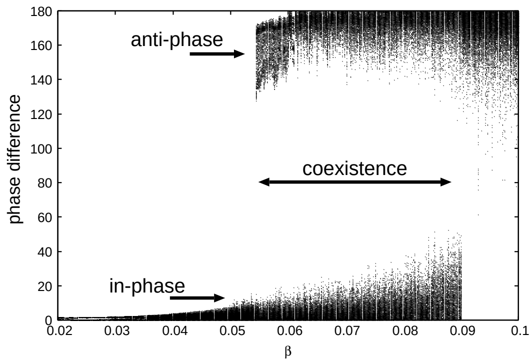


Fig. 6. Phase difference.

B. *Switching Phase States*

In this section, we investigate the synchronization phenomena when the strength of the coupling parameter γ is decreased. We can confirm that the switching of the phase states between the in-phase and the anti-phase is observed as shown in Fig. 7. These switching phenomena could not be confirmed in the two van der Pol oscillators coupled by time-varying resistor. The chaotic circuits coupled by time-varying resistor have possibility to generate complex phenomena.

Next, we pay our attention to the sojourn time of the in-phase state and the anti-phase state. We carry out the 30 moving average method of the phase difference between two coupled chaotic circuits to distinguish the in-phase state and the anti-phase state more correctly. The simulated result of the moving average of the phase difference is shown in Fig. 8.

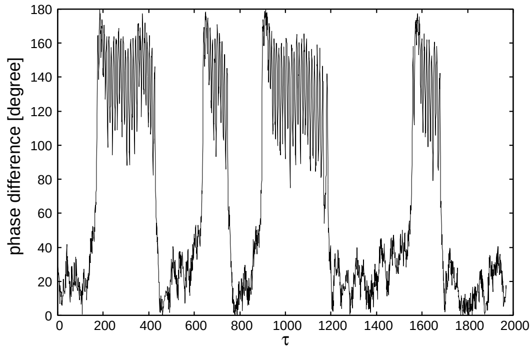


Fig. 7. Switching phase states ($\gamma = 0.095$).

We define in-phase or anti-phase synchronization, by using the 30 moving average of the phase difference. Namely, when the phase difference is smaller or larger than 90 degrees, the synchronization state of the two chaotic circuits is determined to in-phase or anti-phase state.

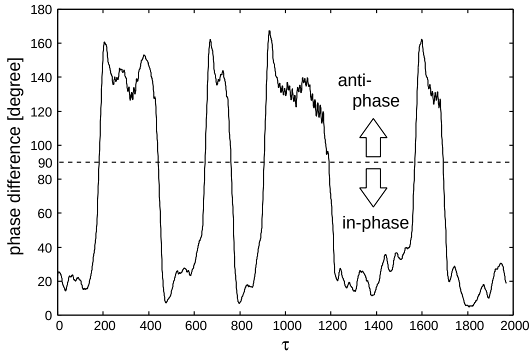


Fig. 8. Moving average of the phase states.

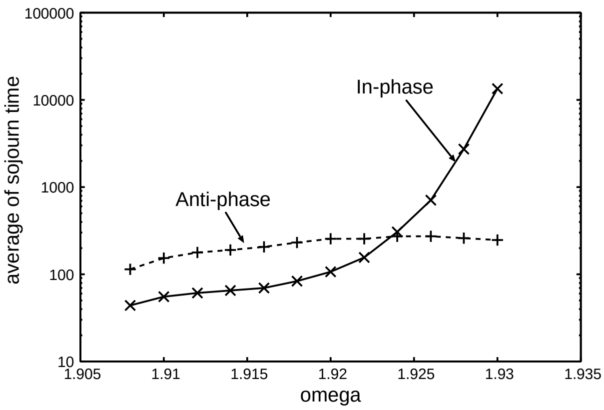


Fig. 9. Average sojourn time in dependence on ω.

Furthermore, we investigate the average sojourn time of the in-phase state and the anti-phase state when the frequency ω of the time-varying resistor is changed. The simulated result is shown in Fig. 9. The horizontal axis is frequency ω and the vertical axis is average sojourn time. The average sojourn time of the in-phase state increases by increasing ω. On the other hand, the average sojourn time of the anti-phase state is almost constant. From these results, we can see that the anti-phase state does not depend on the frequency ω of the time-varying resistor. When ω is smaller than 1.908, two chaotic circuits coupled by time-varying resistor do not synchronous neither the in-phase state nor the anti-phase state. And if in the case ω is larger than 1.930, only the in-phase state can occur.

IV. Effect of Parameter Mismatch

In this study, we investigate synchronization in dependence on the parameter mismatch. Namely, the two chaotic circuits with different characteristics are coupled. We give the parameter mismatch of β to the two chaotic circuits. The parameter β of one chaotic circuit is fixed as $\beta = 0.084$. We change the parameter β_2 of the other chaotic circuit from $\beta_2 = 0.084$ to $\beta_2 = 0.078$. The average sojourn times of the in-phase state and the anti-phase states are shown in Fig. 10. The average sojourn times of the in-phase state increase by increasing parameter mismatch. This synchronization phenomena is very interesting. This is because, generally we assume that if the parameter mismatch becomes large, the average sojourn time decreases. While, the average sojourn time of the anti-phase decreases by increasing parameter mismatch.

Some examples of the switching synchronization state and the frequency distribution when β_2 are set to 0.083 and 0.080 are shown in Figs. 11 and 12, respectively. In the case of $\beta_2 = 0.083$, the sojourn times of both in-phase state and anti-phase state are short (Fig. 11). In the case of $\beta_2 = 0.080$, the sojourn times of in-phase state becomes longer and anti-phase state becomes shorter (Fig. 12).

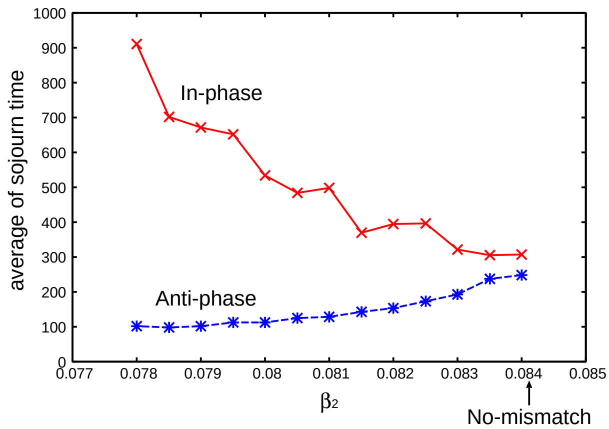


Fig. 10. Average sojourn time in dependence on parameter mismatch β.

V. Other Parameters Mismatch

Finally, we investigate synchronization in dependence on other parameters mismatch α and δ.

The parameter α of one chaotic circuit is fixed as $\alpha = 7.0$. We change the parameter α_2 of the other chaotic circuit from $\alpha_2 = 6.5$ to $\alpha_2 = 8.0$. The simulated result of average sojourn time is shown in Fig. 13. From this figure, we can confirm that the average sojourn time of in-phase and anti-phase is almost similar when the parameter α_2 is larger than 7.0. While, the

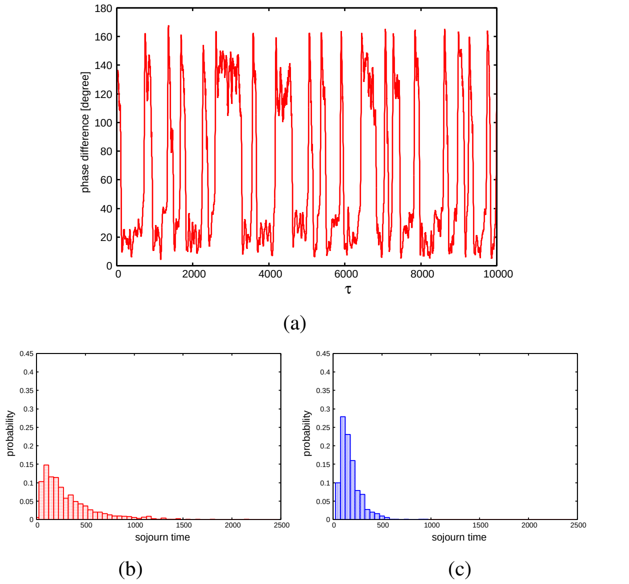


Fig. 11. Switching phase state for $\beta_2 = 0.083$. (a) An example of switching synchronization state. (b) Frequency distribution of sojourn time (in-phase). (c) Frequency distribution of sojourn time (anti-phase).

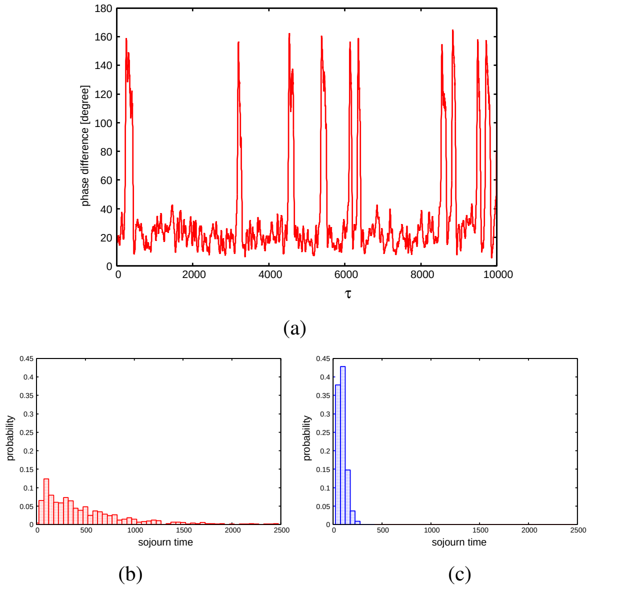


Fig. 12. Switching phase state for $\beta_2 = 0.080$. (a) An example of switching synchronization state. (b) Frequency distribution of sojourn time (in-phase). (c) Frequency distribution of sojourn time (anti-phase).

average sojourn time of both the in-phase and the anti-phase increases when the α_2 is smaller than 7.0.

Next, we investigate synchronization in dependence on the parameter mismatch δ. The parameter δ of one chaotic circuit is fixed as $\delta = 100$. We change δ_2 of the other chaotic circuit from $\delta_2 = 80$ to $\delta_2 = 120$. Figure 14 shows the simulated results of average sojourn time. When δ_2 is larger than 100, the average sojourn time of the anti-phase is longer than the in-phase state. While, when δ_2 is smaller than 100, the average sojourn time of the in-phase increase and the anti-phase decreases.

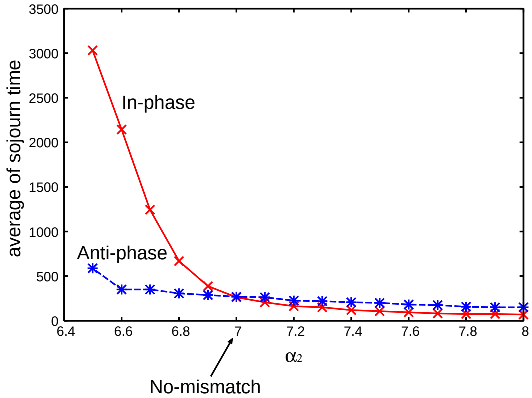


Fig. 13. Average sojourn time in dependence on parameter mismatch α.

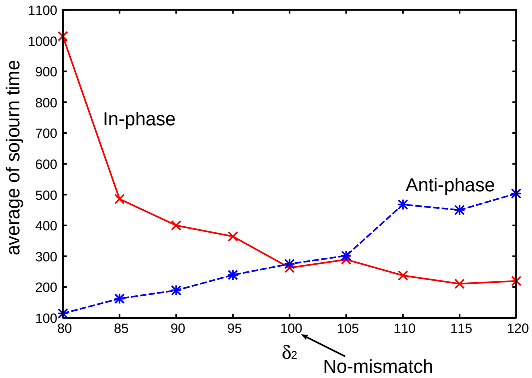


Fig. 14. Average sojourn time in dependence on parameter mismatch δ.

VI. Conclusions

In this study, we have investigated phase differences in two chaotic circuits coupled by a time-varying resistor when the parameter of the two chaotic circuits are changed. By carrying out computer calculations, we have observed very interesting synchronization phenomenon. Namely the sojourn times of in-phase and anti-phase states become longer when the parameter mismatch increases.

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