Synchronization Phenomena on Asymmetrical Globally Coupled Chaotic Systems Using Circuits or Maps

Yasuteru Hosokawa
Dept. Information Science,
Shikoku University
Email: hosokawa@keiei.shikoku-u.ac.jp

Yoshifumi Nishio
Dept. Electrical and Electronic Eng.,
Tokushima University
Email: nishio@ee.tokushima-u.ac.jp

***Abstract*—In our previous study, we observed the interesting phenomena on asymmetrical chaotic coupled systems. It is that the ratio of synchronization time of the one subsystem group increases in spite of increasing parameter mismatches in the other subsystem.**

In this study, an asymmetrical globally coupled maps is proposed and investigated, in order to explain the these phenomena. Asymmetry of the system is realized by using two parameters. In the case of five maps, it was confirmed that ratios of synchronization time of maps using one parameter set are increased by increasing a parameter mismatch rate of the other maps group. We consider that this result is corresponding to results of previous study.

I. Introduction

Coupled chaotic systems generate various kinds of complex higher-dimensional phenomena such as spatio-temporal chaotic phenomena, clustering phenomena and so on. One of the most studied systems may be the coupled map lattice proposed by Kaneko[1]. The advantage of the coupled map lattice is its simplicity. However, many of nonlinear phenomena generated in nature would be not so simple. Therefore, it is important to investigate the complex phenomena observed in natural physical systems such as electric circuits systems[2]-[6].

One of useful tools for investigating these phenomena is electric circuits. There are many advantages of using electric circuits. For instance, circuits are natural physical systems, circuit elements is low price and high quality, getting easily, circuit experiments are easy to match theory, and so on.

In our previous study [7][8], some kinds of asymmetrical global coupled chaotic systems are investigated. Especially, we paid attention to relationships between synchronization phenomena and small parameter mismatches. In all systems, an interesting phenomena are observed. The phenomena are that a ratio of the synchronization time increases in spite of increasing parameter mismatches in the other system.

In this study, in order to explain the these phenomena, an asymmetrical globally coupled maps is proposed and investigated. Asymmetry of the system is realized by using two parameters.

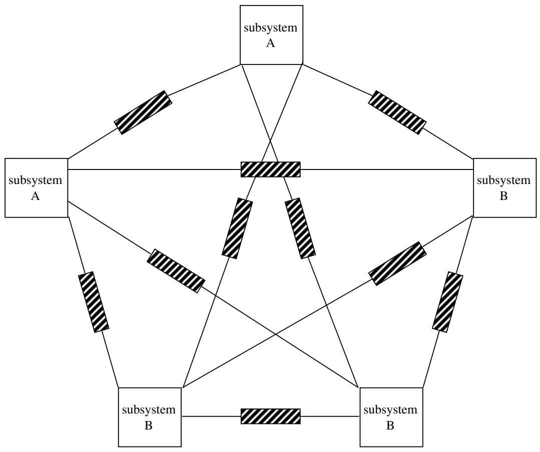


Fig. 1. Proposed system.

II. Proposed System

A proposed system is shown in Fig. 1. This system consists of two kinds of subsystems and coupling elements. Subsystems are coupled globally. An asymmetry of the system is realized by using two kinds of subsystems. Some kinds of chaotic circuits or autonomous oscillators were applied as subsystems in previous studies. For instance, two different coupling points of one chaotic circuit, two different parameter of one chaotic circuit, chaotic circuits and van der Pol oscillators and so on. In this study, the logistic map is applied as subsystems. The asymmetry is realized by using two parameters. The numbers of subsystem A and B are two and three. This system is based on globally coupled map (GCM) proposed by Kaneko [1]. GCM is shown as following model.

$$x_{n+1}(i) = (1-\varepsilon)f(x_n(i)) + \frac{\varepsilon}{N}\sum_{j=1}^{N} f(x_n(j)) \qquad (1)$$

where n is a discrete time step and i is the index of an element. $f(x)$ is applied one of chaotic maps.

In this study, the logistic map $f(x) = 1 - ax^2$ is chosen. The asymmetry of the system is realized by using two parameter.

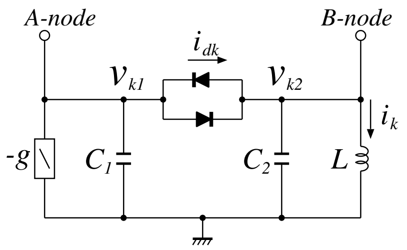


Fig. 2. Chaotic Circuit.

Therefore, function $f(x)$ is shown as follows.

Subsystem A ($1 \leq i \leq p$):

$$f(x_n(i)) = 1 - (1 + Q_a i) a_1 x_n(i)^2 \tag{2}$$

Subsystem B ($p+1 \leq k \leq p+q$):

$$f(x_n(i)) = 1 - \{1 + Q_b(i-p)\} a_2 x_n(i)^2 \tag{3}$$

where i is a number of maps. n is a number of iterations. a_1 and a_2 are corresponding to a parameter a of the logistic map. p and q are numbers of subsystem A and B. Q_a and Q_b are parameter mismatch rates of each subsystems.

III. Computer Simulations

A. Electrical Circuits

At first, we introduce the case of the system applied chaotic circuits. The applied chaotic circuit is shown in Fig. 2. This chaotic circuit is a simple three-dimensional autonomous circuit proposed by Shinriki et al.[9]. A-node is used as a coupling node. An asymmetry of the system is realized as a difference of parameters. Namely, parameters of subsystem A is different from subsystem B. Double scroll type attractors are observed on the each subsystems. Normalized circuit equations are described as follows:
Subsystem A ($1 \leq k \leq m$):

$$\begin{cases} \dot{x}_k = & \alpha\beta x_k - \alpha\gamma f(x_k - y_k) \\ & +\alpha\delta \left\{ \sum_{i=1}^{m+n} x_i - (m+n)x_k \right\}, \\ \dot{y}_k = & -z_k + \gamma f(x_k - y_k), \\ \dot{z}_k = & (1+p_k)y_k, \end{cases} \tag{4}$$

Subsystem B ($m+1 \leq k \leq m+n$):

$$\begin{cases} \dot{x}_k = & \varepsilon\beta x_k - \varepsilon\gamma f(x_k - y_k) \\ & +\varepsilon\delta \left\{ \sum_{i=1}^{m+n} x_i - (m+n)x_k \right\}, \\ \dot{y}_k = & \zeta \left\{ -z_k + \gamma f(x_k - y_k) \right\}, \\ \dot{z}_k = & \eta(1+q_k)y_k, \end{cases} \tag{5}$$

where,

$$f(x) = x + \frac{(|x-1| - |x+1|)}{2}.$$

Figure 3 shows the voltage differences between each subsystems. Vertical axes show voltage differences and horizontal axes show time. Namely, in the case of synchronizing two subcircuits, the amplitude becomes zero. First graph shows the voltage difference between the two subsystem A. Synchronizations and un-synchronized burst appear alternately in a random way. The second graph shows the voltage difference between subsystem A and subsystem B. These are not synchronized at all. The third and fourth graphs show the voltage differences between two subsystem B.

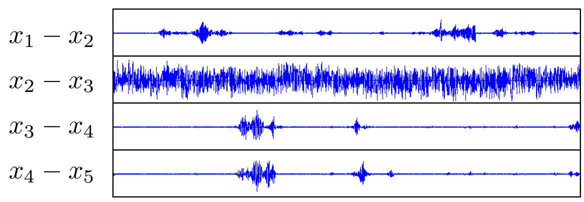


Fig. 3. Voltage differences between two subsystems in the case of chaotic circuit.

Here we define the synchronization as following equation and figure.

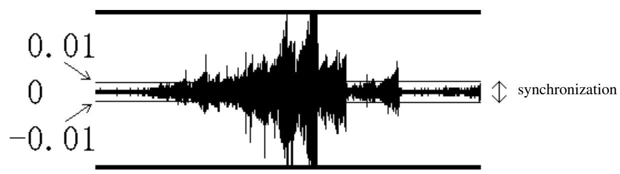


Fig. 4. Definition of the synchronization.

$$|x_k - x_{k+1}| < 0.01 \tag{6}$$

Figure 5 shows ratios of the synchronization time and total time. Q is shown as following equation.

$$q_k = Q(k-1) \tag{7}$$

Q is corresponding to small parameter mismatches q_k of subsystem B group. By increasing small parameter mismatch of subsystem B group, the synchronization time of subsystem

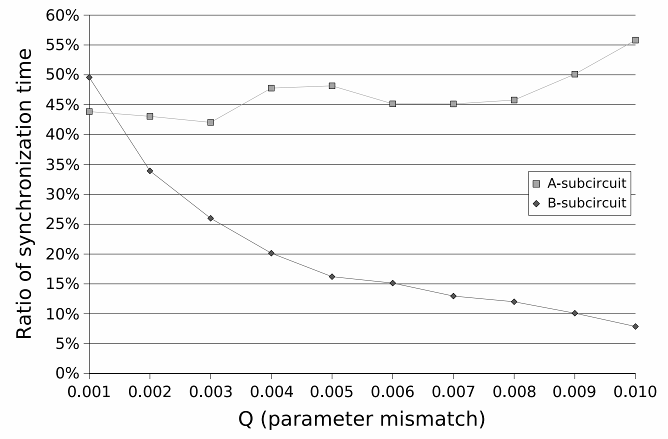


Fig. 5. Relationship of the ratio of the synchronization time and small parameter mismatches in the case of System 2. $m = 2$, $n = 3$, $p_k = 0.001(k-1)$, $\alpha = 0.600$, $\beta = 0.500$, $\gamma = 20.0$, $\delta = 0.070$, $\varepsilon = 0.6$, $\zeta = 1.5$ and $\eta = 0.5$.

A group is increased. Namely, in spite of increasing small parameter mismatches of the system, the synchronization time of subsystem A group is increased.

B. Chaotic Maps

We carry out computer simulations using Eq. 2 and Eq. 3. Here we define the synchronization as following conditional equations:

$$|x_{n-1}(i) - x_{n-1}(i+1)| < 0.05 \quad (8)$$

and

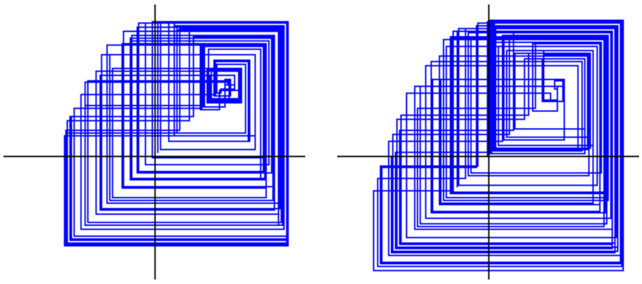
$$|x_n(i) - x_n(i+1)| < 0.05 \quad (9)$$

These equations show that two consecutive closer values than a given threshold value(0.05) define synchronization. Figures 6 and Figs. 7 are examples of the computer simulation results. In Figs. 6, horizontal axes show $x_n(i)$ and vertical axes show $x_{n+1}(i)$. Figures 6(a) shows x_1 and Fig. 6(b) shows x_3. Figures 6(a) is smaller than Fig. 6(b). We can see the differece between the two maps. In Figs. 7, horizontal axes show iterations and vertical axes show values of x_k. Read areas show asynchronous states and blue areas show synchronous states. From these results, we can see that subsystem A and subsystem B are not synchronized at all and increasing Q_b causes increasing a ratio of synchronization time of subsystem A. Figures 8 are corresponding to Figs. 5. We can also see the similar result as Figs. 5.

The chaotic circuit is continuous-time system. The logistic map is discrete-time system. In spite of this difference, we can observe the phenomena in both systems. Therefore, We consider that the phenomena are closely related to the structure of the system.

IV. Conclusions

In this study, an asymmetrical globally coupled maps is proposed and investigated. Asymmetry of the system is realized by using two parameters. In the case of five maps, it was confirmed that ratios of synchronization time of maps using one parameter set are increased by increasing a parameter mismatch rate of the other maps group. We consider that this result is corresponding to results of previous study.



(a) $x_n(1)$ vs. $x_{n+1}(1)$ (b) $x_n(3)$ vs. $x_{n+1}(3)$

Fig. 6. Maps of (a) subsystem A and (b) subsystem B. $p = 2$, $q = 3$, $Q_a = 0.050$, $a_1 = 1.70$, $a_2 = 1.98$ and $\varepsilon = 0.40$.

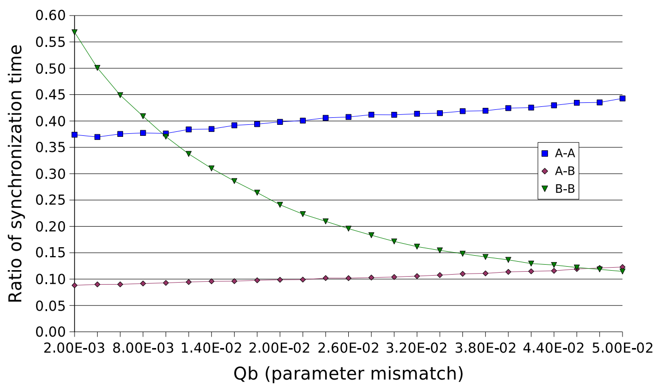


Fig. 8. Relationship of the ratio of the synchronization time and small parameter mismatches. $p = 2$, $q = 3$, $Q_a = 0.050$, $a_1 = 1.70$, $a_2 = 1.98$ and $\varepsilon = 0.40$. The number of iteration is 1000000.

Acknowledgments

This work was partly supported by Yazaki Memorial Foundation for Science and Technology.

References

[1] K. Kaneko, "Clustering, Coding, Switching, Hierarchical Ordering, and Control in a Network of Chaotic Elements," *Physica D*, vol. 41, pp. 137–172, 1990.
[2] L. M. Pecora and T. L. Carrol, "Synchronization in Chaotic Systems," *Physical Review Letters*, vol. 64, pp. 821–824, 1990.
[3] T. Endo, and S. Mori, "Mode Analysis of a Ring of a Large Number of Mutually Coupled van der Pol Oscillators," *IEEE Trans. Circuit and Systems*, vol. CAS-25, no. 5, pp. 7–18, 1978.
[4] D. A. Linkens, "Analytical solution of large numbers of mutually-coupled nearly-sinusoidal oscillators," *IEEE Trans. Circuits and Systems*, vol. CAS-21, pp.294–300, Mar, 1974.
[5] H. Sekiya, S. Moro, S. Mori, and I. Sasase, "Synchronization Phenomena on Four Simple Chaotic Oscillators Full-Coupled by Capacitors," *IEICE Trans. Fundamentals*, vol. J82-A, No. 3, pp.375–385, 1999.
[6] M. Miyamura, Y. Nishio and A. Ushida, "Clustering in Globally Coupled System of Chaotic Circuits," *IEEE Proc. ISCAS'02*, vol. 3, pp. 57–60, 2002.

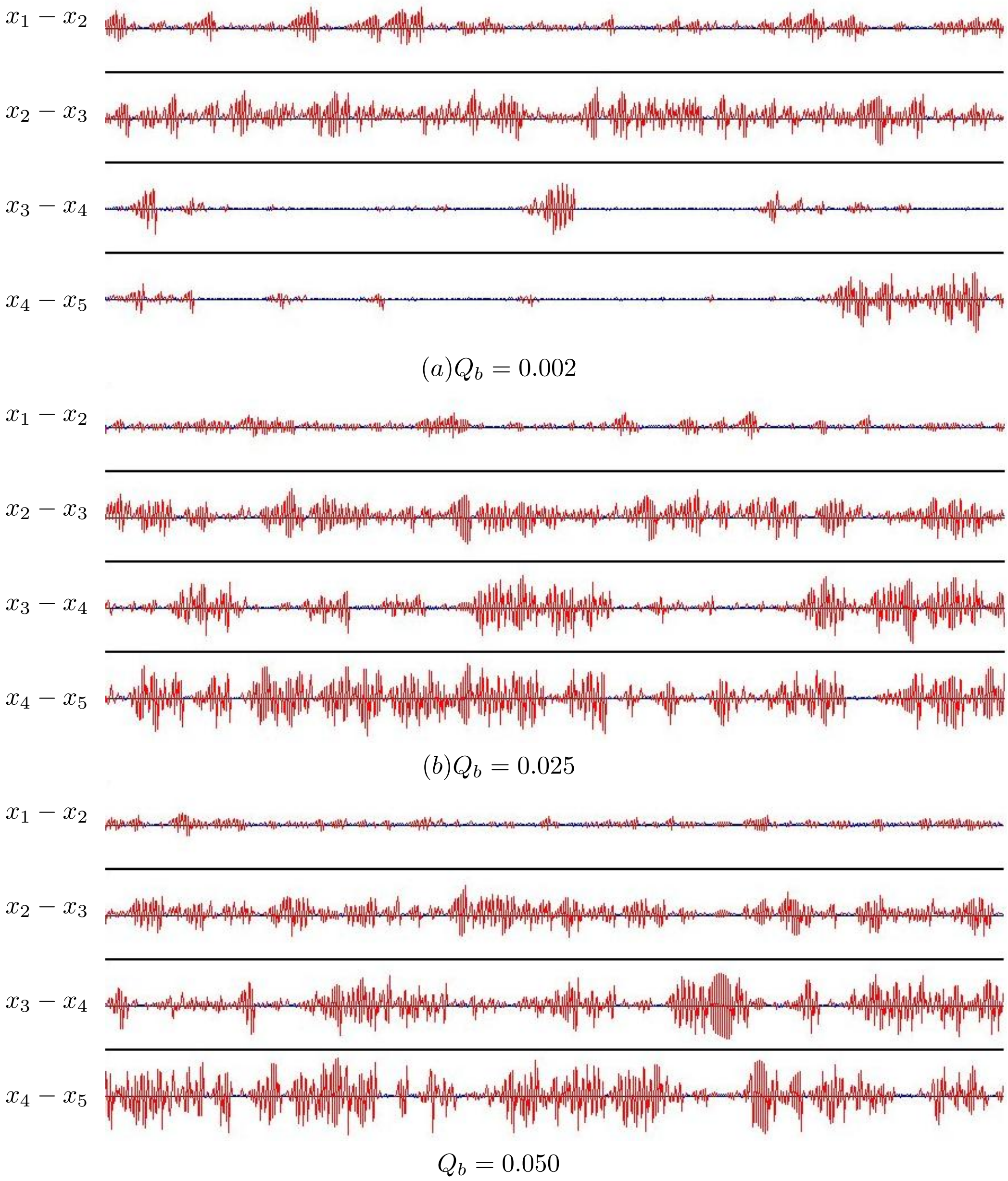


Fig. 7. Variable differences between two subsystems. $p = 2$, $q = 3$, $Q_a = 0.050$, $a_1 = 1.70$, $a_2 = 1.98$ and $\varepsilon = 0.40$.

[7] Y. Hosokawa, R. Tsujioka and Y. Nishio, "Relation between Synchronous Rate and Small Variations on an Asymmetrical Coupled Chaotic System," *Proc. NOLTA'05*, pp. 166–169, 2005.

[8] Y. Hosokawa and Y. Nishio, "Asymmetrical Chaotic Coupled System Using Two Different Parameter Sets," *Proc. NOLTA'06*, pp. 543–546, Sep. 2006.

[9] M. Shinriki, M. Yamamoto and S. Mori, "Multimode Oscillations in a Modified van der Pol Oscillator Containing a Positive Nonlinear Conductance," *Proc. IEEE*, vol. 69, pp. 394–395, 1981.