Spice-Oriented Intermodulation Analysis Combining with MATLAB

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***Abstract*— It is very important to analyze the mixer and modulator circuits which are driven by multiple input frequencies. Volterra series methods are widely used for the frequency-domain analysis of nonlinear circuits, which can be applied to relatively strong nonlinear circuits. We compare properties of the above two methods using illustrative examples. In this paper, we propose a Spice-oriented HB method combining with MATLAB. Firstly, for the circuits containing nonlinear devices characterized by the special function, they are approximated by the Taylor series. Then, the Fourier coefficients can be calculated by MATLAB in the *symbolic forms*. Thus, the determining equations of HB method can be formulated by net-list in Spice simulator. It can be efficiently solved by the DC analysis of Spice, and the frequency-domain solutions are obtained. We found from examples that, although Volterra series method can be efficiently applied to weakly nonlinear circuits, it becomes erroneous to strong nonlinear circuits.**

I. INTRODUCTION

Frequency-domain analysis of nonlinear electronic circuits driven by multiple inputs is very important for designing integrated circuits and communication systems. Volterra series methods are widely used for the analysis [1-3] because they give the solutions in analytical forms. The algorithms are based on bilinear theorem [1], and they can be effectively applied to the intermodulation analysis, where nonlinear elements must be described by the polynomial functions.

HB (harmonic balance) method is also well-known in the frequency-domain analysis, which will give good results even for relatively strong nonlinear circuits. The classical HB methods [4-6] can be applied to relatively small scale circuits and find many interesting nonlinear phenomena such as bifurcations and chaos. Note that, in order to obtain the Fourier coefficients, nonlinear elements must be also described by polynomial functions in HB method. It complicate the applications of HB method for large scale circuits. For example, if the circuit equation is composed of “n” functions and “K” frequency components are considered in the analysis, then the determining equation consist of a number of “n(2K+1)” functions.

Therefore, we propose here another type of *Spice-oriented HB method combining with MATLAB*. Firstly, bipolar transistors and MOSFET modeled by the special functions are approximated by the Taylor expansions in the polynomial forms. Thus, in this case, the Fourier coefficients can be described in the symbolic forms if we use MATLAB. It is further possible to formulate the determining equation of HB method in the form of net-list of Spice. It can be easily solved by DC analysis of Spice, and we can get the frequency-domain solutions, efficiently. Thus, we need not any troublesome tasks such as formulation of the circuit equations and many transformations required to get the determining equations of HB method.

Using an example, we will compare the Volterra series method with our HB method in section 2. In section 3, we show interesting examples of mixer circuit.

II. COMPARISON BETWEEN VOLTERRA SERIES AND HB METHODS

A. Taylor Approximation of Nonlinear Devices

Transistors are usually modeled by Ebers-Moll model consisted of two diodes, where the nonlinear diode (i_d, v_d) is described by

$$i_d = I_s \exp v_d / V_T. \tag{1}$$

where $I_s = 10^{-12}$, $V_T = 0.026$. Firstly, we estimate the Fourier coefficients DC, $\cos\omega t$, $\cos 2\omega t$, $\cos 3\omega t$ driven by

$$v_d = V_{d0} + V_d \cos\omega t, \quad \text{for} \quad V_{d0} = 0.6, \tag{2}$$

and compare them with those obtained from 3rd order approximation with Taylor expansion. Then, its approximation is described as follows:

$$i_d = k_0 + k_1 v_d + k_2 v_d^2 + k_3 v_d^3, \tag{3}$$

Then, the Fourier coefficients of DC, $\cos\omega t$, $\cos 2\omega t$, $\cos 3\omega t$ are shown in Fig.1, where the curves are the coefficients obtained from the 3rd order approximation given by (3), and the bar graphs are those of the exponential function (1), where horizontal axis is V_d in (2).

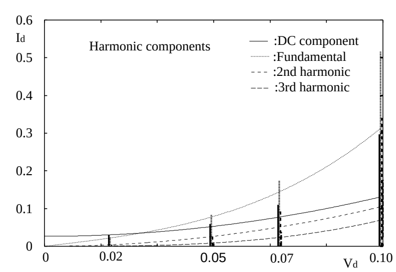

Fig.1 Fourier coefficients obtained from (1) and (3).

Observe that when we apply 3rd order approximation of Taylor expansion to (2), the errors increase rapidly beyond $V_d \approx 0.05$.

B. Volterra Series Method

In order to understand the Volterra series method driven by 2 inputs [1-3], we consider a simple circuit shown in Fig.2, where it has one nonlinear resistive element given by (3).

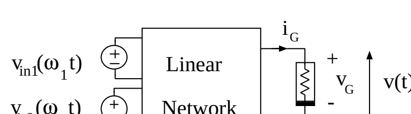

Fig.2 Nonlinear circuit driven by two inputs.

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Now, let us derive the Volterra kernels in the complex frequency domain "s"[3]. The 1st order kernel corresponds to the output voltages at the resistor "r" for the unit inputs $U_1(s_1)$ and $U_2(s_1)$ as shown in the linear circuits of Fig.3(a). Thus, 1st order kernel is given by

$$H_{1k}(s_1) = H_1(s_1), \tag{4}$$

for two inputs. The nonlinear current source for 2nd order is given by [3]

$$I_{2k}(s_1, s_2) = k_2 H_{1k}(s_1) H_{1k}(s_2). \tag{5}$$

Let the admittance from the output terminal be $Y(s)$. Then, the 2nd order kernel $H_{2k}(s_1, s_2)$ is derived by

$$Y(s_1 + s_2) H_{2k}(s_1, s_2) = -I_{2k}(s_1, s_2). \tag{6}$$

In the same way, the nonlinear current source for the 3rd order is given by

$$I_{3k}(s_1, s_2, s_3) = k_3 H_{1k}(s_1) H_{1k}(s_2) H_{3k}(s_3) + \frac{2}{3} k_2 (H_1(s_1) H_2(s_2, s_3) + H_1(s_2) H_2(s_1, s_3) + H_1(s_3) H_2(s_1, s_2)) \tag{7}$$

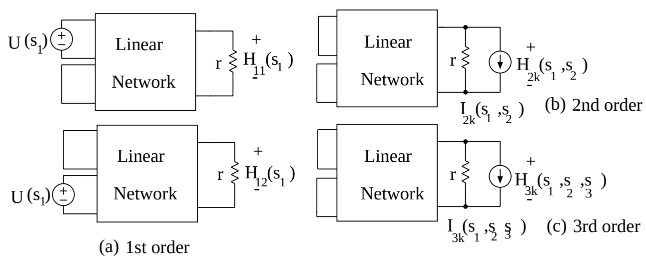

Fig.3 Computations of Volterra kernels.

and the 3rd order kernel $H_{3k}(s_1, s_2, s_3)$ is derived by

$$Y(s_1 + s_2 + s_3) H_{3k}(s_1, s_2, s_3) = -I_{3k}(s_1, s_2, s_3). \tag{8}$$

Thus, the output in the complex frequency domain is given by

$$H_{out}(s_1, s_2, s_3) = H_{1k}(s_1) + H_{2k}(s_1, s_2) + H_{3k}(s_1, s_2, s_3). \tag{9}$$

The output voltage for the following 2 inputs

$$v_{in1}(t) = A_1 \sin \omega_1 t, \qquad v_{in2}(t) = A_2 \sin \omega_2 t \tag{11}$$

are calculated by setting { $s_i = \{\pm\omega_1, \pm\omega_2\}, i = 1, 2$ }in (9). Thus, the output amplitudes are given in the Table 1 [3]. Note that the solutions from the Volterra series method can be easily solved by MATLAB.

Table 1 The output voltage in complex domain [3]

Frequency	Amplitude of response
DC	$\vert\frac{1}{2}A_1^2 H_{2k}(j\omega_1, -j\omega_1) + \frac{1}{2}A_2^2 H_{2k}(j\omega_2, -j\omega_2)\vert$
ω_1	$\vert A_1 H_{1k}(j\omega_1) + \frac{3}{2}A_1 A_2^2 H_{3k}(\omega_1, j\omega_2, -j\omega_2)$ $+\frac{3}{4}A_1^3 H_{3k}(\omega_1, j\omega_1, -j\omega_1)\vert$
ω_2	$\vert A_2 H_{1k}(j\omega_2) + \frac{3}{2}A_1^2 A_2 H_{3k}(\omega_1, -j\omega_1, j\omega_2)$ $+\frac{3}{4}A_2^3 H_{3k}(\omega_2, j\omega_2, -j\omega_2)\vert$
$\omega_1 + \omega_2$	$\vert A_1 A_2 H_{2k}(j\omega_1, j\omega_2)\vert$
$\vert\omega_1 - \omega_2\vert$	$\vert A_1 A_2 H_{2k}(j\omega_1, -j\omega_2)\vert$
$2\omega_1$	$\vert\frac{1}{2}A_1^2 H_{2k}(j\omega_1, j\omega_1)\vert$
$2\omega_2$	$\vert\frac{1}{2}A_2^2 H_{2k}(j\omega_2, j\omega_2)\vert$
$2\omega_1 + \omega_2$	$\frac{3}{4}A_1^2 A_2 H_{3k}(\omega_1, j\omega_1, j\omega_2)\vert$
$\vert 2\omega_1 - \omega_2\vert$	$\frac{3}{4}A_1^2 A_2 H_{3k}(\omega_1, j\omega_1, -j\omega_2)\vert$
$\omega_1 + 2\omega_2$	$\frac{3}{4}A_1 A_2^2 H_{3k}(\omega_1, j\omega_2, j\omega_2)\vert$
$\vert\omega_1 - 2\omega_2\vert$	$\frac{3}{4}A_1 A_2^2 H_{3k}(\omega_1, -j\omega_2, -j\omega_2)\vert$
$3\omega_1$	$\vert\frac{1}{4}A_1^3 H_{3k}(\omega_1, j\omega_1, j\omega_1)\vert$
$3\omega_2$	$\vert\frac{1}{4}A_2^3 H_{3k}(\omega_2, j\omega_2, j\omega_2)\vert$

C. Spice-Oriented HB Method

We propose here a Spice-oriented HB (harmonic balance) method combining with MATLAB, such that the *determining equation* is given by the equivalent circuit and/or net-list of Spice. To use MATLAB for the Fourier expansion of nonlinear devices, we need to describe the nonlinear characteristics in the polynomial forms. The schematic diagram of diode is shown in Fig.4(a), where it is driven by the two inputs

$$v_1(t) = V_1 \cos \omega_1 t, \quad v_2(t) = V_2 \cos \omega_2 t. \tag{12}$$

Firstly, it is approximated by 3rd order Taylor expansion as follows:

$$i_d = k_0(V_{d0}) + k_1(V_{d0}) v_d + k_2(V_{d0}) v_d^2 + k_3(V_{d0}) v_d^3 \tag{13}$$

with an unknown operating point V_{d0}. Now, we consider 13 intermodulation frequency components:

$$\left.\begin{array}{c} DC, \ \ \omega_1, \ \ \omega_2, \ \ 2\omega_1, \ \ 2\omega_2, \ \ 3\omega_1, \ \ 3\omega_2 \\ \omega_1 + \omega_2, \ \ |\omega_1 - \omega_2|, \ \ 2\omega_1 + \omega_2, \ \ |2\omega_1 - \omega_2| \\ \omega_1 + 2\omega_2, \ \ |\omega_1 - 2\omega_2| \end{array}\right\}. \tag{14}$$

Thus, the input voltage and output current waveforms are assumed as follows:

$$\left.\begin{array}{l} v_d(t) = V_{d0} + \sum_{k=1}^{K} (V_{d,2k-1} \cos \nu_k t + V_{d,2k} \sin \nu_k t) \\ i(t) = I_{d0} + \sum_{k=1}^{K} \{I_{d,2k-1} \cos \nu_k t + I_{d,2k} \sin \nu_k t) \end{array}\right\}, \tag{15}$$

Substituting the first equation of (15) into (13), and neglecting the terms except for (14), we have second relation of (15).

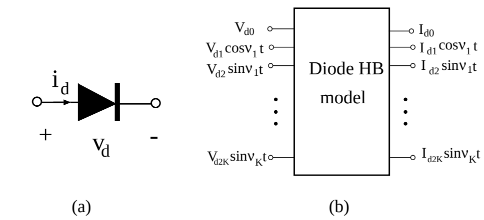

Fig.4 (a) Diode model, (b) HB module with MATLAB.

Note that the number of Fourier coefficients arisen by the above arithmetic calculations is

$$1 + 27 + 27^2 + 27^3 = 20440. \tag{16}$$

We choose only the frequency components corresponding to frequencies (14) using MATLAB, and formulate the net-list in the Spice as shown in Fig.4(b). Thus, we can formulate "Sine-Cosine circuit"[7,8], which corresponds to the determining equation of HB. The circuit is resistive, and it can be easily solved with DC analysis of Spice. Thus, we can get the frequency-domain solutions such as frequency response curves and so on.

It is convenient to develop the HB modules for bipolar transistors. In this case, if we model the transistor with Ebers-Moll model, the emitter, base and corrector currents are given as follows:

$$\left.\begin{array}{l} i_E = I_s \exp(\frac{v_{BE}}{V_T}) - \alpha_R I_s \exp(\frac{v_{BC}}{V_T}) \\ i_C = \alpha_F I_s \exp(\frac{v_{BE}}{V_T}) - I_s \exp(\frac{v_{BC}}{V_T}) \\ i_B = (1 - \alpha_F) I_s \exp(\frac{v_{BE}}{V_T}) + (1 - \alpha_R) I_s \exp(\frac{v_{BC}}{V_T}) \end{array}\right\} \tag{17}$$

for $I_S = 10^{-12}, \ \ V_T = 0.026, \ \ \alpha_F = 0.99, \ \ \alpha_R = 0.3.$
They have some exponential functions, so that HB transistor module can be developed by the use of the diode HB module shown by Fig.4(b).

D. Comparisons between Volterra Series and HB Methods

We consider a simple base-modulator circuit as shown in Fig.5(a) [11]. Firstly, let us solve it with the Volterra series method mentioned in subsection 2.1.

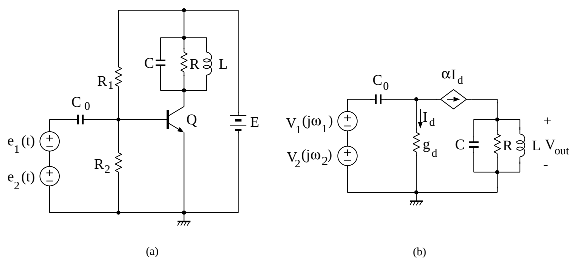

Fig.5(a) Base modulation circuit, (b) 1st order Volterra equivalent circuit.
$R_1 = 50[\mathrm{k}\Omega], \quad R_2 = 10[\mathrm{k}\Omega], \quad R = 0.2[\mathrm{k}\Omega]$
$C = 1[\mathrm{nF}], \quad C_0 = 1[\mathrm{nF}], \quad C_0 = 1[\mathrm{nF}], \quad L = 10[\mu\mathrm{F}], \quad E = 10[\mathrm{V}]$
$v_1 = 0.02 \sin \omega_1 t, \quad v_2 = 0.02 \sin \omega_1 t, \quad \text{for} \quad \omega_2 = 8.2 \times 10^6[\text{rad/sec}]$

The transistor is replaced by the Ebers-Moll model, and the operating point is calculated. Thus, we have $v_{d0} = 0.5822[\mathrm{V}]$. Next, the nonlinear characteristic is approximated by the 3rd order Taylor expansion at the point v_{d0} as follows:

$$i_d = 0.0130 + 0.52 v_d + 10.40 v_d^2 + 138.60 v_d^3. \qquad (18)$$

Thus, we have

$$k_1 = 0.52, \quad k_2 = 10.40, \quad k_3 = 138.60 \qquad (19)$$

in (3). Note that we have $v_{BC} < 0$ and $I_s \exp(v_{BC}/V_T) = 0$ for these parameters, so that 1st order Volterra kernel is given by Fig.5(b). The 2nd and 3rd Volterra kernels are calculated by methods shown in subsection 2.2. The frequency-domain intermodulation phenomena as shown in Fig.5(c) is easily obtained by the application of MATLAB to the Volterra kernels.

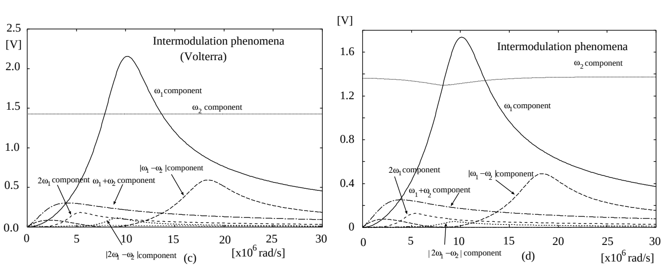

Fig.5(c),(d) The intermodulation phenomena obtained by Volterra series method and HB method.

Next, we analyze the circuit with HB method. Firstly, the transistor is modeled by Ebers-Moll model and two diodes are replaced by HB diode modules shown in Fig.4(b), where the diodes are approximated by the 3rd order Taylor expansion. And then, we formulated the "Sine-Cosine circuits"[7,8] corresponding to the frequency components in (14). These sub-circuits are coupled each other with controlled sources which correspond to the determining equation of Spice. Therefore, the frequency-domain intermodulation can be calculated by the DC analysis of Spice as shown in Fig.5(d).

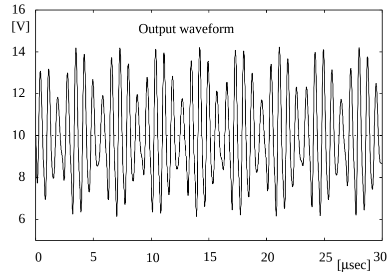

Fig.5(e) Steady-state waveform of base modulator circuit.

Note that although the response curves obtained by Volterra series and HB methods are similar, the amplitudes are largely different around the resonant point. We have also calculated the transient responses, where transistors are modeled by (17), instead of 3rd order Taylor approximation in Volterra and HB methods. The ω_1 and ω_2 frequency-components are calculated with DFT from the steady-state waveforms and shown in Table 2.

Table 2 Comparisons of amplitudes.
($\omega_1 = 10^7$[rad/s], $\omega_2 = 8.2 \times 10^6$[rad/s])

	Volterra		HB		DFT	
$V_1 = V_2$	ω_1	ω_2	ω_1	ω_2	ω_1	ω_2
0.01	0.950	0.693	0.929	0.691	0.913	0.698
0.02	2.20	1.41	1.72	1.31	1.68	1.29
0.03	3.91	2.32	2.50	1.92	2.17	1.72
0.04	6.25	3.35	3.35	2.59	2.50	2.11
0.05	9.80	4.55	4.31	3.34	2.75	2.38

Note that the results obtained from transient analysis will be most accurate as the results of the base modulation circuit, because the transistor is only modeled by Ebers-Moll model. On the other hand, the Volterra method approximates the Ebers-Moll model at fixed operating point. Our HB method approximates it at unknown operating point decided by HB analysis. Other differences are come from the analytical methods. For smaller input, the intermodulation phenomena obtained by Volterra and HB methods have almost same features. However, the amplitudes from the Volterra method are much larger than the others, especially around the resonant point. Therefore, it seems from the results in Table 2 that the Volterra series method becomes erroneous for strong nonlinear circuits with large inputs.

III. Illustrative Examples

We consider a mixer circuit containing 3 transistors as shown in Fig.6(a) [11]. Note that it becomes a modulator if we set $v_1(t)$ carrier signal, $v_2(t)$ input signal and choose the LC resonant frequency equal to the carrier frequency. Now, we analyze the intermodulation phenomena with our HB method. Firstly, the transistors are modeled by the Ebers-Moll model, where

$$i_d = I_s e^{\lambda v_d}, \quad \text{for} \quad I_s = 10^{-12}[\mathrm{A}], \quad \lambda = 40, \quad \text{and} \quad \alpha = 0.99 \qquad (20)$$

and they are approximated by Taylor series as follows:

$$\equiv k_0(v_{d0}) + k_1(v_{d0})v + k_2(v_{d0})v^2 + k_3(v_{d0})v^3 \quad \text{for} \quad v_d = v_{d0} + v \qquad (21)$$

In this case, we consider the 13 intermodulation frequencies given by (14). Firstly, all the transistors are replaced by the corresponding HB transistor modules, and RLC circuit is transformed into "Sine-Cosine circuits" [7-8]. Thus, the circuit is transformed into the 13 sub-circuits coupled with controlled sources, it can be solved by the DC analysis of Spice.

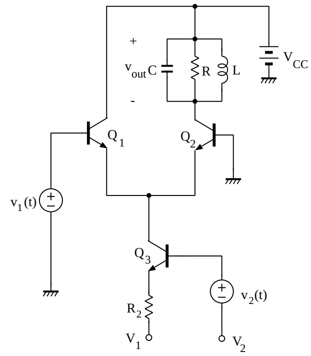

Fig.6(a) Mixer circuit.
$R_2 = 10[\text{k}\Omega],\quad R = 100[\Omega],\quad L = 10[\mu\text{H}],$
$V_{CC} = 10[\text{V}],\quad V_1 = -10[\text{V}],\quad V_2 = -5[\text{V}]$

For the mixer circuit, we had the intermodulation phenomena as shown in Fig.6(b) and the steady-state waveforms of Fig.6(c)

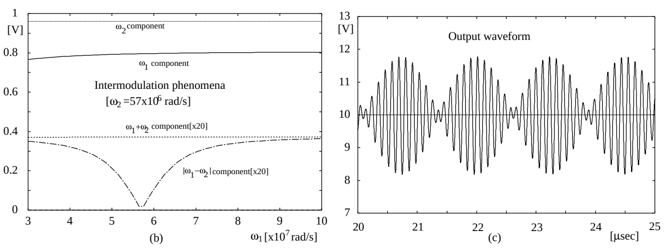

Fig.6(b) Frequency response of mixer circuit. $C = 1[\text{pF}]$,
$v_1 = 0.002 \sin \omega_1 t, \quad v_2 = 0.2 \sin \omega_2 t, \quad \text{for} \quad \omega_2 = 57 \times 10^6[\text{rad/s}].$
(c) Steady-state waveform for $\omega_1 = 50 \times 10^6$.

From DFT analysis of the waveform, we have the following results:

$$\left.\begin{array}{l} V_{out,1} = 0.7919[\text{V}](\text{HBmethod}) \quad V_1 = 0.8311[\text{V}](\text{DFT}) \\ \qquad\qquad \text{for component} \quad \omega_1 = 57 \times 10^6[\text{rad/s}] \\ V_{out,2} = 0.9603[\text{V}](\text{HBmethod}) \quad V_2 = 0.9585[\text{V}](\text{DFT}) \\ \qquad\qquad \text{for component} \quad \omega_2 = 50 \times 10^6[\text{rad/s}] \end{array}\right\}$$

The intermodulation components of $\omega_1 + \omega_2$ and $|\omega_1 - \omega_2|$ are small enough, and the $|\omega_1 - \omega_2|$-component becomes zero at $\omega_1 = \omega_2$. The computation time is 102.05[s].

Next, we analyze the circuit as modulator. In this case, the resonant frequency is equal to 10^7[rad/s]. Figure 6 (d) and (e) shows the intermodulation phenomena and the steady-state waveform, respectively.

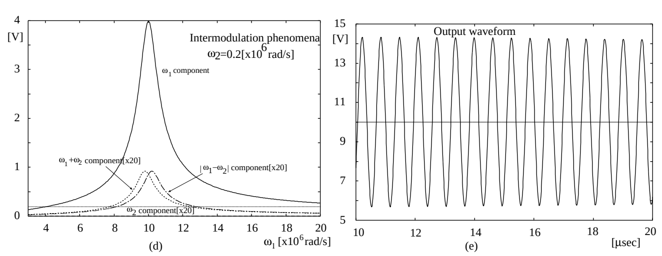

Fig.6(d) Frequency response of modulator. $C = 1[\text{nF}]$,
$v_1 = 0.001 \sin \omega_1 t, \quad v_2 = 0.1 \sin \omega_2 t, \quad \text{for} \quad \omega_2 = 0.2 \times 10^6[\text{rad/s}].$
(e) Steady-state waveform for $\omega_1 = 10^7$.

We have the following results:

$$\left.\begin{array}{l} V_{out,1} = 3.987[\text{V}](\text{HBmethod}) \quad V_1 = 4.218[\text{V}](\text{DFT}) \\ \qquad\qquad \text{for component} \quad \omega_1 = 10^7[\text{rad/s}] \\ V_{out,2} = 0.307[\text{V}](\text{HBmethod}) \quad V_2 = 0.337[\text{V}](\text{DFT}) \\ \qquad\qquad \text{for component} \quad \omega_1 = 0.2 \times 10^6[\text{rad/s}] \\ V_{out} = 85.5[\text{mV}] \quad \text{at} \quad \omega_1 - \omega_2 \end{array}\right\}$$

The computation time is 209.30[s].

Note that we can get good result from our HB method.

IV. Conclusions and Remarks

In this paper, we compared Volterra series method with our Spice-oriented HB method for the analysis of intermodulation phenomena, and we found the following results:

1) Both methods belong to the symbolical analysis, where the nonlinear devices must be described by the power series. In this case, Volterra method must found the operating point used in Taylor expansion. Our HB method need only to describe it as Taylor expansion with unknown operating point which is decided by HB analysis.
2) Volterra series method finds the solution by solving the kernels with MATLAB, efficiently.
3) Our HB method transforms the circuit into "Sine-Cosine circuit" corresponding to the determining equation of HB method, where we use HB modules. The circuit is solved by DC analysis of Spice.
4) Volterra series method can be only applied to weakly nonlinear circuits. Our HB method can be applied to relatively strong nonlinear circuits with a few harmonics.

As future works, we need to apply our HB method to strong nonlinear elements, and develop of HB modules of MOSFETs.

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