

# Comparison of Synchronization Phenomena on Asymmetrical Global Coupled Chaotic Systems

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Abstract—In this paper, three kinds of asymmetrical global chaotic coupled systems are investigated. The asymmetries are realized by different coupling nodes or different parameters sets. In these systems, an interesting phenomenon about synchronization phenomena is observed. The phenomenon is that a ratio of the synchronization time increases in spite of increasing parameter mismatches in the system.

#### I. INTRODUCTION

Many researchers have focused on engineering applications of chaos, for instance, chaotic communication systems, chaotic control, chaotic synchronization and so on. Especially, chaotic synchronization is very interesting phenomenon that chaotic subsystems are synchronized in spite of different initial values[1]. Additionally, coupled chaotic systems generate various kinds of complex higher-dimensional phenomena such as spatio-temporal chaotic phenomena, clustering phenomena and so on. One of the most studied systems may be the coupled map lattice proposed by Kaneko[2]. The advantage of the coupled map lattice is its simplicity. However, many of nonlinear phenomena generated in nature would be not so simple. Therefore, it is important to investigate the complex phenomena observed in natural physical systems such as electric circuits systems[3]-[4].

In this study, three kinds of asymmetrical global coupled chaotic systems are investigated. Especially, we paid attention to relationships between synchronization phenomena and small parameter mismatches. In all systems, an interesting phenomenon is observed. The phenomenon is that a ratio of the synchronization time increases in spite of increasing parameter mismatches in the system.

In the Sect. 2, proposed systems and its circuit equations are shown. In the Sect. 3, computer simulation results of each systems are shown. Relationships between synchronization phenomena and small parameter mismatches are also shown. Some concluding remarks is presented in the Sect. 5.

## **II. PROPOSED SYSTEMS**

A proposed system is shown in Fig. 1. This system consists of two kinds of subcircuits and resistors as coupling elements. Subcircuits are coupled globally. An asymmetry of the system is realized by using two kinds of subcircuits ( subcircuit A and B ). In this study, the number of subcircuit A and B are shown Yoshifumi Nishio

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Fig. 2. Diode model.

as m and n, respectively. The numbers of subcircuit A and B are two and three. Actual systems are shown in following three subsections. Additionally, in order to carry out the computer simulations, the circuit equations are derived. In all systems, the diode model is a piecewise linear function shown in Fig. 2. Small parameter mismatches of Subcircuit A and B are  $p_k$  and  $q_k$ , respectively.

#### A. System 1

The subcircuit of System 1 is shown in Fig. 3. This chaotic circuit is a simple three-dimensional autonomous circuit proposed by Shinriki et al.[6]. By selecting one of two coupling nodes(A-node and B-node), an asymmetry of the system is realized. Using following variables and parameters,



Fig. 3. Subcircuit 1.

$$x_{k} = \frac{v_{k1}}{V_{th}}, \qquad y_{k} = \frac{v_{k2}}{V_{th}}, \qquad z_{k} = \frac{1}{V_{th}}\sqrt{\frac{L}{C_{2}}}i_{k},$$

$$t = \sqrt{LC_{2}}\tau, \quad ``\cdot `` = \frac{d}{d\tau}, \qquad \alpha = \frac{C_{2}}{C_{1}}, \qquad (1)$$

$$\beta = g\sqrt{\frac{L}{C_{2}}}, \quad \gamma = G_{d}\sqrt{\frac{L}{C_{2}}}, \quad \delta = \frac{1}{R}\sqrt{\frac{L}{C_{2}}},$$

Normalized circuit equations are described as follows: Subcircuit A ( $1 \le k \le m$ ):

$$\dot{x}_{k} = \alpha \beta x_{k} - \alpha \gamma f(x_{k} - y_{k})$$

$$+ \alpha \delta \{ \sum_{i=n+1}^{m+n} x_{i} + \sum_{j=1}^{n} y_{j} - (m+n)x_{k} \},$$

$$\dot{y}_{k} = \gamma f(x_{k} - y_{k}) - z_{k},$$

$$\dot{z}_{k} = (1+p_{k})y_{k}.$$
(2)

Subcircuit B  $(m+1 \le k \le m+n)$ :

$$\begin{cases}
\dot{x}_k = \alpha \beta x_k - \alpha \gamma f(x_k - y_k), \\
\dot{y}_k = \delta \{\sum_{i=n+1}^{m+n} x_i + \sum_{j=1}^n y_j \\
-(m+n)y_k\} + \gamma f(x_k - y_k) - z_k, \\
\dot{z}_k = (1+q_k)y_k,
\end{cases}$$
(3)

The nonlinear function f(x) corresponding to the characteristics of the diodes is described as follows:

$$f(x) = x + \frac{(|x-1| - |x+1|)}{2}.$$

### B. System 2

The subcircuit of System 2 is same as the subcircuit of System 1. However, only A-node is used in this system. An asymmetry of the system is realized as a difference of parameters. Namely, parameters of subcircuit A in Fig. 1 is different from subcircuit B. Using the following parameters



Fig. 4. Subcircuit 2.

and variables,

$$x_{k} = \frac{v_{k1}}{V_{th}}, \qquad y_{k} = \frac{v_{k2}}{V_{th}}, \qquad z_{k} = \frac{1}{V_{th}} \sqrt{\frac{L_{a}}{C_{2a}}},$$

$$t = \sqrt{L_{a}C_{2a}}\tau, \quad ``\cdot `' = \frac{d}{d\tau}, \qquad \alpha = \frac{C_{2a}}{C_{1a}},$$

$$\beta = g\sqrt{\frac{L_{a}}{C_{2a}}}, \qquad \gamma = G_{d}\sqrt{\frac{L_{a}}{C_{2a}}}, \qquad \delta = G\sqrt{\frac{L_{a}}{C_{2a}}},$$

$$\varepsilon = \frac{C_{2a}}{C_{1b}}, \qquad \zeta = \frac{C_{2a}}{C_{2b}} \quad \text{and} \quad \eta = \frac{L_{a}}{L_{b}}.$$

$$(4)$$

Normalized circuit equations are described as follows: Subcircuit A  $(1 \le k \le m)$ :

$$\begin{cases}
\dot{x}_{k} = \alpha \beta x_{k} - \alpha \gamma f(x_{k} - y_{k}) \\
+ \alpha \delta \left\{ \sum_{i=1}^{m+n} x_{i} - (m+n)x_{k} \right\}, \\
\dot{y}_{k} = -z_{k} + \gamma f(x_{k} - y_{k}), \\
\dot{z}_{k} = (1+p_{k})y_{k},
\end{cases}$$
(5)

B) Subcircuit B  $(m+1 \le k \le m+n)$ :

$$\begin{cases}
\dot{x}_{k} = \varepsilon \beta x_{k} - \varepsilon \gamma f(x_{k} - y_{k}) \\
+ \varepsilon \delta \left\{ \sum_{i=1}^{m+n} x_{i} - (m+n)x_{k} \right\}, \\
\dot{y}_{k} = \zeta \left\{ -z_{k} + \gamma f(x_{k} - y_{k}) \right\}, \\
\dot{z}_{k} = \eta (1 + q_{k})y_{k},
\end{cases}$$
(6)

where,

$$f(x) = x + \frac{(|x-1| - |x+1|)}{2}.$$

### C. System 3

The subcircuit of System 3 is shown in Fig. 4. This chaotic circuit was proposed by Inaba et al.[7]. Using the following

parameters and variables,

$$\begin{aligned} x_k &= \sqrt{\frac{L_{1a}}{C_a}} \frac{i_{k1}}{V_{th}}, \quad y_k &= \sqrt{\frac{L_{1a}}{C_a}} \frac{i_{k2}}{V_{th}}, \quad z_k &= \frac{v_k}{V_{th}}, \\ t &= \sqrt{L_{1a}C_a}\tau, \qquad ``\cdot `' &= \frac{d}{d\tau}, \qquad \alpha &= \frac{L_{1a}}{L_{2a}}, \\ \beta &= g_a \sqrt{\frac{L_{1a}}{C_a}}, \qquad \gamma &= r_d \sqrt{\frac{C_a}{L_{1a}}}, \qquad \delta &= G \sqrt{\frac{L_{1a}}{C_a}}, \quad (7) \\ \varepsilon &= \frac{L_{1a}}{L_{1b}}, \qquad \zeta &= \frac{L_{1a}}{L_{2b}}, \qquad \eta &= \frac{C_a}{C_b}, \\ \text{and} \quad \theta &= \frac{g_b}{g_a}. \end{aligned}$$

Normalized circuit equations are described as follows: Subcircuit A  $(1 \le k \le m)$ :

$$\begin{cases} \dot{x}_{k} = (1+p_{k})z_{k} \\ \dot{y}_{k} = \alpha \{z_{k} - f(y_{k})\}, \\ \dot{z}_{k} = -x_{k} - y_{k} + \beta z_{k} \\ & +\delta \{\sum_{i=1}^{m+n} z_{i} - (m+n)z_{k}, \} \end{cases}$$
(8)

Subcircuit B  $(m+1 \le k \le m+n)$ :

$$\begin{cases} \dot{x}_{k} = (1+q_{k})z_{k} \\ \dot{y}_{k} = \zeta \{z_{k} - f(y_{k})\}, \\ \dot{z}_{k} = \eta[-x_{k} - y_{k} + \beta\theta z_{k} \\ +\delta\{\sum_{i=1}^{m+n} z_{i} - (m+n)z_{k}, \}] \end{cases}$$
(9)

where,

$$f(y_k) = \frac{\gamma y_k + 1 - |\gamma y_k - 1|}{2}.$$
  
III. Computer Simulation

At first, each results of the computer simulations are shown. Figures 5 are examples of the computer simulation results on System 2. Double scroll type attractors are observed on the each subcircuits. In the case of System 1, similar attractors can be observed. Figures 6 are examples of the computer simulation results on System 3. Rossler type attractors are observed. Figure 7 shows the voltage differences between each



Fig. 5. Attractors of System 1. Horizontal axes are  $x_k$  and vertical axes are  $z_k$ .



Fig. 6. Attractors of System 1. Horizontal axes are  $x_k$  and vertical axes are  $z_k$ .

subcircuits in the case of System 2. Vertical axes show voltage differences and horizontal axes show time. Namely, in the case of synchronizing two subcircuits, the amplitude becomes zero. First graph shows the voltage difference between the two subcircuit A. Synchronizations and un-synchronized burst appear alternately in a random way. The second graph shows the voltage difference between subcircuit A and subcircuit B. These are not synchronized at all. The third and fourth graphs show the voltage differences between two subcircuit B. In the Systems 1 and 3, similar results are observed.



Fig. 7. Voltage differences between two subcircuits in the case of System 2.

Next, the relationship between the synchronization and small parameter mismatches are investigated. In order to investigate it, the synchronization is defined as following equation and figure.



Fig. 8. Definition of the synchronization.

$$|x_k - x_{k+1}| < 0.01 \tag{10}$$

Figure 9 shows ratios of the synchronization time and total time in the case of System 1. Q is shown as following equation.

$$q_k = Q(k-1) \tag{11}$$

Q is corresponding to small parameter mismatches  $q_k$  of subcircuit B group. By increasing small parameter mismatch



Fig. 9. Relationship of the ratio of the synchronization time and small parameter mismatches in the case of System 1. m = 2, n = 3,  $p_k = 0.001(k-1)$ ,  $\alpha = 0.400$ ,  $\beta = 0.500$ ,  $\gamma = 20.0$  and  $\delta = 0.070$ 



Fig. 10. Relationship of the ratio of the synchronization time and small parameter mismatches in the case of System 2. m = 2, n = 3,  $p_k = 0.001(k-1)$ ,  $\alpha = 0.600$ ,  $\beta = 0.500$ ,  $\gamma = 20.0$ ,  $\delta = 0.070$ ,  $\varepsilon = 0.6$ ,  $\zeta = 1.5$  and  $\eta = 0.5$ .

of subcircuit B group, the synchronization time of subcircuit A group is increased. Namely, in spite of increasing small parameter mismatches of the system, the synchronization time of subcircuit A group is increased. Figure 10 and Figure 11 show the case of System 2 and 3, respectively. In these case, periodic orbits are observed on some value Q. In particular, Q = 0.004 and Q = 0.005 in the case of System 2 and Q =0.08 in the case of System 3. Excepting periodic orbits, the synchronization time of subcircuit A group is also increased by increasing small parameter mismatch of subcircuit B group. On all systems, we can also observe the case of decreasing the synchronization time when increasing small parameter mismatches of subcircuit B group in other parameters of subcircuit A group and B group. We suppose that the phenomenon can be explained as follows. The synchronizations of the one subcircuit group and the other subcircuit group are constricted each other. Therefore, in the case of decreasing the synchronization of one group, the synchronization of the other group increases.



Fig. 11. Relationship of the ratio of the synchronization time and small parameter mismatches in the case of System 3. m = 2, n = 3,  $p_k = 0.001(k-1)$ ,  $\alpha = 0.600$ ,  $\beta = 0.400$ ,  $\gamma = 100.0$ ,  $\delta = 0.060$ ,  $\varepsilon = 1.4$ ,  $\zeta = 7$ ,  $\eta = 0.5$ , and  $\theta = 1.2$ .

# IV. CONCLUSION

In this study, three kinds of asymmetrical global chaotic coupled systems are proposed and investigated. In the case of five subcircuits, we confirmed synchronization phenomena. Additionally, It was confirmed that the synchronization time ratio of one subcircuit group are increased by decreasing the synchronization time ratio of the other subcircuit group. We suppose that the phenomenon can be explained as follows. The synchronizations of the one subcircuit group and the other subcircuit group are constricted each other. Therefore, in the case of decreasing the synchronization of one group, the synchronization of the other group increases.

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