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On Phase Synchronization of Simple Coupled Chaotic Circuits

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Abstract – The synchronization phenomena are the representative phenomena in the nonlinear science and can be often observed in nature world. Coupled oscillatory systems have attracted a great deal of attentions in various fields. In particular, synchronization in such systems is very important phenomenon and many researches have been reported.

In this study, we investigate synchronization phenomena in two coupled chaotic circuits. By computer simulations and circuit experiments, interesting synchronization phenomena are observed.

1. Introduction

Coupled oscillatory systems have attracted a great deal of attentions in various fields. Studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields, physics [1]-[3], biology [4], engineering [5]-[12] and so on.

In our past studies [13][14], we have reported that a certain class of coupled systems of the Wien-Bridge oscillators synchronizes with 0 or 120 degrees.

In [15], we have investigated synchronization phenomena in coupled Wien-Bridge oscillators by both circuit experiments and computer simulations using SPICE. We observed the synchronization state with 143 degrees phase difference. The phase difference of 143 degrees is very strange, because in-phase, anti-phase, or N-phase synchronization is typical.

In this study, phase synchronization phenomena in two coupled chaotic circuits are investigated. By computer simulations and circuit experiments, we confirm that the synchronization state with 143 degrees phase difference in two coupled chaotic circuit is observed.

In Sec. 2, the circuit model in this study and the circuit equations are given. In Sec. 3, simulation results and circuit experiment are shown. In Sec. 4, we conclude our study.

2. Circuit Model

In this study, we consider a simple coupled system of two chaotic circuits to investigate the generation of the 143 degrees synchronization. The circuit model is shown in Fig. 1. In this model, two chaotic circuits are coupled by one resistor R.

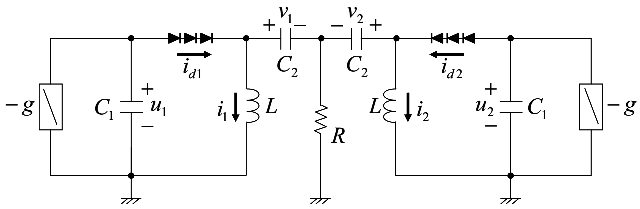


Fig. 1. Circuit model.

The characteristics of the nonlinear resistor consisting of diodes are approximated by the following function.

$$i_{dk} = G\left(u_k - L\frac{di_k}{dt} - E\right), (k = 1,2).$$

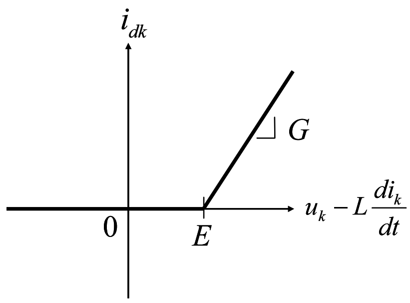


Fig. 2. The characteristics of the nonlinear resistor consisting of diodes.

The circuit equations are given as follows.

$$\begin{cases} C_1 \dfrac{du_k}{dt} = gu_k - i_{dk} \\ C_2 \dfrac{dv_k}{dt} = i_{dk} - i_k \\ L \dfrac{di_k}{dt} = v_k + R(i_{d1} - i_1 + i_{d2} - i_2) \end{cases}$$

$(k = 1, 2)$.

By changing variables and parameters,

$$\text{``}\cdot\text{''} = \frac{d}{d\tau},\ t = \sqrt{LC_1}\tau,\ u_k = Ex_k,\ v_k = Ey_k,\ i_k = \sqrt{\frac{C_2}{L}}Ez_k,$$

$$\alpha = \frac{C_2}{C_1},\ \beta = g\sqrt{\frac{L}{C_2}},\ \delta = R\sqrt{\frac{L}{C_2}},\ \gamma = G\sqrt{\frac{L}{C_2}},\ (k = 1,2),$$

The circuit equations are normalized as follows.

(i) D00

$$x_k - y_k + \delta(z_1 + z_2) \le 1,\ (k = 1,2).$$

$$\begin{cases} \dot{x}_k = \alpha\beta x_k \\ \dot{y}_k = -z_k \\ \dot{z}_k = y_k - \delta(z_1 + z_2) \end{cases}$$

$(k = 1, 2)$.

(ii) D+0

$$\begin{cases} \dfrac{x_1 - y_1 - 1}{1+\delta\gamma} + \dfrac{\delta(z_1 + z_2)}{1+\delta\gamma} > 0, \\ x_2 - y_2 - \dfrac{\delta\gamma(x_1 - y_1 - 1)}{1+\delta\gamma} + \delta(z_1 + z_2)\left(\dfrac{\delta\gamma}{1+\delta\gamma} - 1\right) \le 1 \end{cases}$$

$$\begin{cases} \dot{x}_1 = \alpha\left\{\beta x_1 - \dfrac{\gamma}{1+\delta\gamma}(x_1 - y_1 - 1) - \dfrac{\delta\gamma}{1+\delta\gamma}(z_1 + z_2)\right\} \\ \dot{y}_1 = \dfrac{\gamma}{1+\delta\gamma}(x_1 - y_1 - 1) + \dfrac{\delta\gamma}{1+\delta\gamma}(z_1 + z_2) - z_1 \\ \dot{z}_1 = \dfrac{y_1}{1+\delta\gamma} + \dfrac{\delta\gamma}{1+\delta\gamma}(x_1 - 1) + \dfrac{\delta}{1+\delta\gamma}(z_1 + z_2) \end{cases}$$

$$\begin{cases} \dot{x}_2 = \alpha\beta x_2 \\ \dot{y}_2 = -z_2 \\ \dot{z}_2 = y_2 + \dfrac{\delta\gamma}{1+\delta\gamma}(x_1 - y_1 - 1) + \delta(z_1 + z_2)\left(\dfrac{\delta\gamma}{1+\delta\gamma} - 1\right) \end{cases}$$

(iii) D0+

$$\begin{cases} x_1 - y_1 - \dfrac{\delta\gamma(x_2 - y_2 - 1)}{1+\delta\gamma} - \delta(z_1 + z_2)\left(\dfrac{\delta\gamma}{1+\delta\gamma} - 1\right) \le 1, \\ \dfrac{x_2 - y_2 - 1}{1+\delta\gamma} + \dfrac{\delta(z_1 + z_2)}{1+\delta\gamma} > 0 \end{cases}$$

$$\begin{cases} \dot{x}_1 = \alpha\beta x_1 \\ \dot{y}_1 = -z_1 \\ \dot{z}_1 = y_1 + \dfrac{\delta\gamma}{1+\delta\gamma}(x_2 - y_2 - 1) + \delta(z_1 + z_2)\left(\dfrac{\delta\gamma}{1+\delta\gamma} - 1\right) \end{cases}$$

$$\begin{cases} \dot{x}_2 = \alpha\left\{\beta x_2 - \dfrac{\gamma}{1+\delta\gamma}(x_2 - y_2 - 1) - \dfrac{\delta\gamma}{1+\delta\gamma}(z_1 + z_2)\right\} \\ \dot{y}_2 = \dfrac{\gamma}{1+\delta\gamma}(x_2 - y_2 - 1) + \dfrac{\delta\gamma}{1+\delta\gamma}(z_1 + z_2) - z_2 \\ \dot{z}_2 = \dfrac{y_2}{1+\delta\gamma} + \dfrac{\delta\gamma}{1+\delta\gamma}(x_2 - 1) + \dfrac{\delta}{1+\delta\gamma}(z_1 + z_2) \end{cases}$$

(iv) D++

$$\begin{cases} \dfrac{\delta\gamma}{1+2\delta\gamma}\{y_2 + \delta\gamma(x_1 + x_2 - 2) - \delta(z_1 + z_2)\} \\ \qquad - \dfrac{1+\delta\gamma}{1+2\delta\gamma}\{y_1 + \delta\gamma(x_1 + x_2 - 2) - \delta(z_1 + z_2)\} + (x_1 - 1) > 0 \\ \dfrac{\delta\gamma}{1+2\delta\gamma}\{y_1 + \delta\gamma(x_1 + x_2 - 2) - \delta(z_1 + z_2)\} \\ \qquad - \dfrac{1+\delta\gamma}{1+2\delta\gamma}\{y_2 + \delta\gamma(x_1 + x_2 - 2) - \delta(z_1 + z_2)\} + (x_2 - 1) > 0 \end{cases}$$

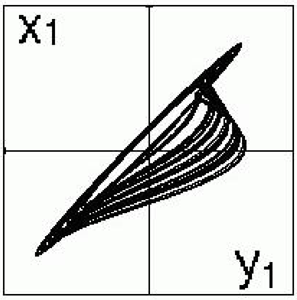
$$\begin{cases} \dot{x}_1 = \alpha\beta x_1 + \alpha\gamma\dfrac{1+\delta\gamma}{1+2\delta\gamma}\{y_1 + \delta\gamma(x_1 + x_2 - 2) - \delta(z_1 + z_2)\} \\ \qquad - \alpha\gamma\dfrac{\delta\gamma}{1+2\delta\gamma}\{y_2 + \delta\gamma(x_1 + x_2 - 2) - \delta(z_1 + z_2)\} - \alpha\gamma(x_1 - 1) \\ \dot{y}_1 = \gamma\dfrac{1+\delta\gamma}{1+2\delta\gamma}\{y_1 + \delta\gamma(x_1 + x_2 - 2) - \delta(z_1 + z_2)\} \\ \qquad - \gamma\dfrac{\delta\gamma}{1+2\delta\gamma}\{y_2 + \delta\gamma(x_1 + x_2 - 2) - \delta(z_1 + z_2)\} - \gamma(x_1 - 1) - z_1 \\ \dot{z}_1 = \dfrac{1+\delta\gamma}{1+2\delta\gamma}\{y_1 + \delta\gamma(x_1 + x_2 - 2) - \delta(z_1 + z_2)\} \\ \qquad - \dfrac{\delta\gamma}{1+2\delta\gamma}\{y_2 + \delta\gamma(x_1 + x_2 - 2) - \delta(z_1 + z_2)\} \end{cases}$$

$$\begin{cases} \dot{x}_2 = \alpha\beta x_2 + \alpha\gamma\dfrac{1+\delta\gamma}{1+2\delta\gamma}\{y_2 + \delta\gamma(x_1+x_2-2) - \delta(z_1+z_2)\} \\ \qquad - \alpha\gamma\dfrac{\delta\gamma}{1+2\delta\gamma}\{y_1 + \delta\gamma(x_1+x_2-2) - \delta(z_1+z_2)\} - \alpha\gamma(x_2-1) \\ \dot{y}_2 = \gamma\dfrac{1+\delta\gamma}{1+2\delta\gamma}\{y_2 + \delta\gamma(x_1+x_2-2) - \delta(z_1+z_2)\} \\ \qquad - \gamma\dfrac{\delta\gamma}{1+2\delta\gamma}\{y_1 + \delta\gamma(x_1+x_2-2) - \delta(z_1+z_2)\} - \gamma(x_2-1) - z_2 \\ \dot{z}_2 = \dfrac{1+\delta\gamma}{1+2\delta\gamma}\{y_2 + \delta\gamma(x_1+x_2-2) - \delta(z_1+z_2)\} \\ \qquad - \dfrac{\delta\gamma}{1+2\delta\gamma}\{y_1 + \delta\gamma(x_1+x_2-2) - \delta(z_1+z_2)\} \end{cases}$$

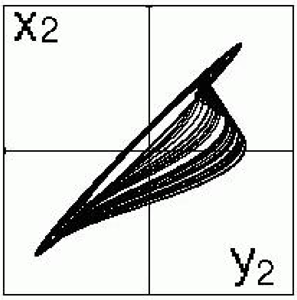
3. Simulation and Circuit Experiment Results

We investigate phase synchronization phenomena in two coupled chaotic circuits by computer simulations using the Runge-Kutta method. Figure 3 shows the simulated results. Figures 3(a) and (b) show the chaotic attractor of each subcircuit. The phase difference and the time waveforms are shown in Figs. 3(c) and (d), respectively. From these figures, the two coupled chaotic circuits are synchronized with around 143 degrees phase differences same as the two coupled RC oscillators.

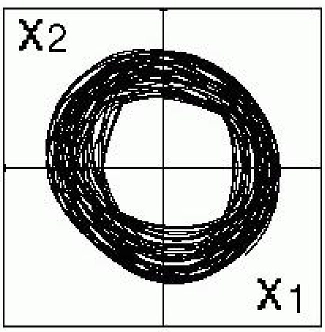
We also confirm the generation of the same synchronization states in circuit experiments as shown in Fig. 4.



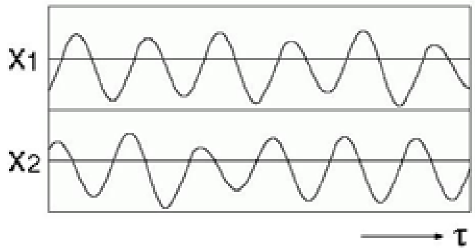
(a)



(b)

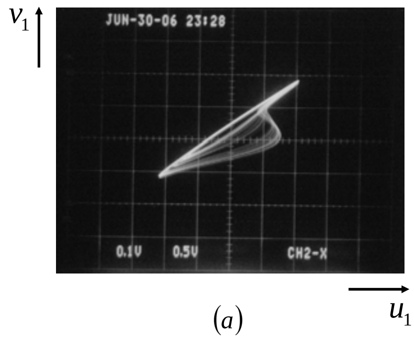


(c)

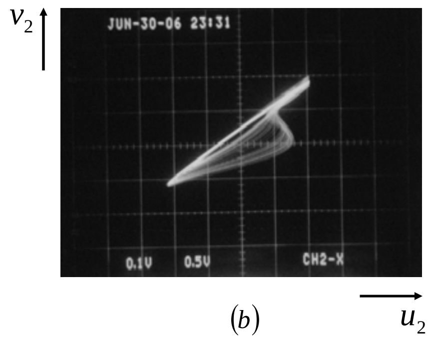


(d)

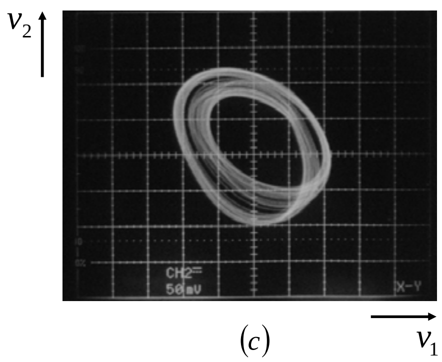
Fig. 3 Computer simulation result. (a) Chaotic attractor. y_1 vs. x_1. (b) Chaotic attractor. y_2 vs. x_2. (c) Phase difference. x_1 vs. x_2. (d) Time waveform. $\alpha = 0.33$, $\beta = 0.43$, $\gamma = 14.0$, and $\delta = 0.05$.



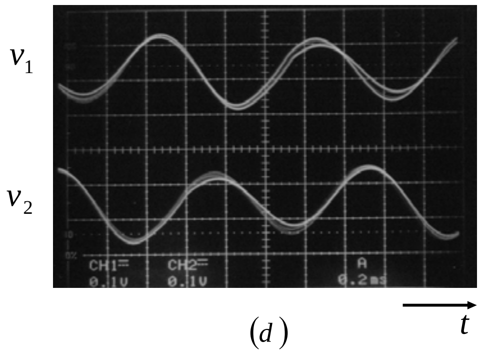
(a)



(b)



(c)



(d)

Fig.4 Circuit experimental result. (a) Chaotic attractor. u_1 vs. v_1. (b) Chaotic attractor. u_2 vs. v_2. (c) Phase difference. v_1 vs. v_2. (d) Time waveform. C_1 = 100nF, C_2 = 33nF, L = 100mH, and R = 100Ω.

4. Conclusions

In this study, we have investigated phase synchronization phenomena in two coupled chaotic circuits. By computer simulations and circuit experiments, interesting synchronization phenomena were observed.

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