

Synchronization of Chaotic Circuits with Asymmetric Coupling by Nonlinear Mutual Inductors

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Abstract—In this research, synchronization phenomena observed from simple chaotic circuits with asymmetric coupling by nonlinear mutual inductors are investigated. A simple threedimensional autonomous circuit is considered as a chaotic subcircuit. By carrying out computer calculations for some cases, various kinds of synchronization phenomena of chaos are observed.

I. INTRODUCTION

Many nonlinear dynamical systems in various fields have been confirmed to exhibit chaotic oscillations. Recently applications of chaos to engineering systems are expected such as chaos noise generators, control of chaos, synchronization of chaos, and so on. In those applications, we are especially interested in synchronization of chaos. Synchronization and the related bifurcation in chaotic systems are good models to describe various high-dimensional nonlinear phenomena in the field of natural science and many excellent studies on synchronization of chaos have been reported. Now mechanisms of chaotic phenomena generated in low-dimensional systems have been elucidated theoretically, and complex phenomena observed from higher dimensional circuits represented by coupled plural chaotic circuits attract attentions [1]-[5].

In our previous research, quasi-synchronization phenomena observed from simple chaotic circuits coupled by linear mutual inductors were investigated [6]. We could observe various kinds of synchronization phenomena of chaos by carrying out computer calculations for two or three subcircuits cases. In the two subcircuits case, in-phase and anti-phase synchronization were observed. Moreover in-phase and three-phase synchronization were observed in the three subcircuit case.

In this research, synchronization phenomena observed from simple chaotic circuits with asymmetric coupling by nonlinear mutual inductors are investigated. A simple three-dimensional autonomous circuit is considered as a chaotic subcircuit. This subcircuit is a symmetric version of the chaotic circuit proposed by Inaba *et al.* [7]. They used an ideal piecewise linear model of diodes in [7], but in this research the i - vcharacteristics of the nonlinear resistor consisting of diodes are approximated by a smooth function. This is more real than piecewise linear approximation in the sense that every real elements in the natural field are not piecewise linear. By carrying out computer calculations, various kinds of synchronization phenomena of chaos are observed.

II. CIRCUIT MODEL



Fig. 1. Circuit model.

Figure 1 shows the circuit model. In the circuit, three identical chaotic circuits are coupled asymmetrically by nonlinear mutual inductors. Each chaotic subcircuit consists of three memory elements, one linear negative resistor and one nonlinear resistor, which is realized by connecting some diodes, and is one of the simplest autonomous chaotic circuits. First, we approximate the i - v characteristics of the nonlinear resistor by the following function.

$$v_d(i_k) = \sqrt[9]{r_d \, i_k}.\tag{1}$$

Further, the $\phi - i$ characteristics of the nonlinear inductor in



Fig. 2. $\phi - i$ charactristics of the nonlinear inductor.

Fig. 2 is described as following function.

$$I_k = \frac{\phi_k}{L_2} + \left(\frac{1}{L_1} - \frac{1}{L_2}\right) \frac{|\phi_k + \Phi| - |\phi_k - \Phi|}{2}.$$
 (2)

By changing the variables and parameters,

$$t = \sqrt{L_1 C} \tau, \quad a = \sqrt[8]{r_d \sqrt{\frac{C}{L_1}}}, \quad ``\cdot " = \frac{d}{d\tau},$$

$$\phi_k = a\sqrt{L_1 C} x_k, \quad i_k = a\sqrt{\frac{C}{L_1}} y_k, \quad v_k = a z_k, \qquad (3)$$

$$\alpha = \frac{L_1}{L_0}, \quad \beta = r\sqrt{\frac{C}{L_1}}, \quad \gamma = \frac{L_2}{L_0}, \quad \delta = \frac{\Phi}{a\sqrt{L_1 C}}.$$

the circuit equations are normalized and described as

$$\begin{split} \dot{x}_1 &= \frac{1}{1+m_1-2m_1m_2} \Big(\frac{1-m_1m_2}{1-m_1} \{\beta(X_1+y_1)-z_1\} \\ &- \frac{m_1(1-m_2)}{1-m_1} \{\beta(X_2+y_2)-z_2\} \\ &- m_2 \{\beta(X_3+y_3)-z_3\} \Big) \\ \dot{x}_2 &= \frac{1}{1+m_1-2m_1m_2} \Big(\frac{1-m_1m_2}{1-m_1} \{\beta(X_2+y_2)-z_2\} \\ &- \frac{m_1(1-m_2)}{1-m_1} \{\beta(X_1+y_1)-z_1\} \\ &- m_2 \{\beta(X_3+y_3)-z_3\} \Big) \\ \dot{x}_3 &= \frac{1}{1+m_1-2m_1m_2} \Big((1+m_1) \{\beta(X_3+y_3)-z_3\} \\ &- m_1 \{\beta(X_1+y_1)-z_1\} \\ &- m_1 \{\beta(X_2+y_2)-z_2\} \Big) \\ \dot{y}_k &= \alpha \{\beta(X_k+y_k)-z_k-f(y_k)\} \\ \dot{z}_k &= X_k+y_k \qquad (k=1,2,3) \end{split}$$

where

$$X_{k} = \frac{\alpha}{\gamma} x_{k} + \left(1 - \frac{\alpha}{\gamma}\right) \frac{|x_{k} + \delta| - |x_{k} - \delta|}{2}.$$

$$f(y_{k}) = \sqrt[9]{y_{k}}.$$
(5)

III. COMPUTER CALCULATED RESULTS

In this section, we perform computer calculations for some cases. First, we consider the case where parameter m_1 is much larger than parameter m_2 . In this case, one subcircuit is not easy to interact with the other subcircuits. Next, the case where parameter m_1 is almost equal to parameter m_2 is considered. All subcircuits dominantly interact in this case. The parameter values corresponding to the inductors are fixed as $\alpha = 20.0$, $\gamma = 10.0$ and (4) is calculated by using the Runge-Kutta method with step size $\Delta t = 0.001$.

A. Case for $m_1 \gg m_2$

We set the parameter values $m_1 = 0.2$ and $m_2 = 0.02$ in this case. Subcircuit 3 is not easy to interact with subcircuits 1 and 2. Therefore subcircuit 3 becomes asynchronous to the others.



Fig. 3. In-phase synchronization for case A. $m_1 = 0.2$. $m_2 = 0.02$. $\delta = 0.5$. (a) $\beta = 0.15$. (b) $\beta = 0.18$. (c) $\beta = 0.181$. (d) $\beta = 0.2$. (e) $\beta = 0.23$. (1) x_1 vs. z_1 . (2) x_3 vs. z_3 . (3) x_1 vs. x_2 . (4) x_1 vs. x_3 .

Figure 3 shows the in-phase synchronization and the asynchronous state. From Figs. 3(1) and (3), we can confirm that the attractors observed from subcircuit 1 and 2 bifurcate to chaotic attractors keeping in-phase synchronization (a)-(d). While, the attractor observed from subcircuit 3 bifurcates to chaotic attractor via period-doubling route Figs. 3(2)(a)-(e). In

(4)



Fig. 4. Bifurcation diagram for the in-phase synchronization. $m_1 = 0.2$. $m_2 = 0.02$. $\delta = 0.5$. (a) Horizontal: β . Vertical: x_1 . (b) Horizontal: β . Vertical: x_3 . (c) Horizontal: β . Vertical: $x_1 - x_2$.

order to investigate the bifurcation route in detail, we made one-parameter bifurcation diagrams. The poincare section is defined as $z_1 = 0$, $x_1 < 0$. Figure 4 shows the bifurcation diagram for in-phase synchronization. From these figures, we can confirm the bifurcation route via period-doubling. We can also confirm that breakdown of chaos synchronization around $\beta = 0.223$ from Fig. 4(c).

Next, we also investigate the anti-phase synchronization by changing the initial values. Figures 5(a) and (b) shows the anti-phase synchronization for subcircuit 1 and 2. While, the attractors observed from subcircuit 3 become asynchronous state. Figure 6 shows bifurcation diagram for anti-phase synchronization. As parameter β increases, the anti-phase synchronization becomes unstable and the in-phase synchronization is observed around $\beta = 0.2$ as Fig. 5(3)(c) and Fig. 6.

B. Case for $m_1 \approx m_2$

In this case, we set the parameter values $m_1 = 0.2$ and $m_2 = 0.19$. Therefore, subcircuit 3 is easy to interact subcircuit 1 and 2.

First, we investigate the in-phase synchronization. Figure 7 shows that all subcircuits are synchronized at in-phase. How-



Fig. 5. Anti-phase synchronization for case A. $m_1 = 0.2$. $m_2 = 0.02$. $\delta = 0.5$. (a) $\beta = 0.02$. (b) $\beta = 0.18$. (c) $\beta = 0.2$. (d) $\beta = 0.23$. (l) x_1 vs. z_1 . (2) x_3 vs. z_3 . (3) x_1 vs. x_2 . (4) x_1 vs. x_3 .

ever as β increases, the in-phase synchronization becomes unstable and breaks down (c),(d).

Next, the three-phase synchronization is observed by changing the initial values. Figure 8 shows the three-phase synchronization. From this figure, we can confirm that each one-periodic attractor (a) bifurcates to torus (b), and as β increases, the torus bifurcates to chaos (c) and the chaos grows as (d). Figure 9 shows bifurcation diagram of the three-phase synchronization. We can confirm that the bifurcation route of the three-phase synchronization from Fig. 9, namely bifurcation of the one-periodic solution to torus around $\beta = 0.203$, the generation of chaotic solution for β values more than 0.215 and the generation of periodic solution around $\beta = 0.225$.

IV. CONCLUSIONS

In this research, we investigated quasi-synchronization phenomena observed from simple chaotic circuits with asymmetric coupling by nonlinear mutual inductors. By carrying out the computer calculations, we confirmed that various quasisynchronization phenomena of chaos were observed.

In the future, we investigate phenomena observed from the case which four or more subcircuit are coupled asymmetrically by nonlinear mutual inductors.

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Fig. 6. Bifurcation diagram for the anti-phase synchronization. $m_1 = 0.2$. $m_2 = 0.02$. $\delta = 0.5$. (a) Horizontal: β . Vertical: x_1 . (b) Horizontal: β . Vertical: x_3 . (c) Horizontal: β . Vertical: $x_1 + x_2$.



Fig. 7. In-phase synchronization for case B. $m_1 = 0.2$. $m_2 = 0.19$. $\delta = 1.5$. (a) $\beta = 0.15$. (b) $\beta = 0.23$. (c) $\beta = 0.26$. (d) $\beta = 0.27$. (1) x_1 vs. z_1 . (2) x_3 vs. z_3 . (3) x_1 vs. x_2 . (4) x_1 vs. x_3 .



Fig. 8. Three-phase synchronization for case *B*. $m_1 = 0.2$. $m_2 = 0.19$. $\delta = 1.5$. (a) $\beta = 0.15$. (b) $\beta = 0.2$. (c) $\beta = 0.22$. (d) $\beta = 0.24$. (1) x_1 vs. z_1 . (2) x_3 vs. z_3 . (3) x_1 vs. x_2 . (4) x_1 vs. x_3 . (5) Time waveform for $\beta = 0.22$.



Fig. 9. Bifurcation diagram for the three-phase synchronization. $m_1 = 0.2$. $m_2 = 0.19$. $\delta = 1.5$. Horizontal: β . Vertical: x_1 .

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