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Comparing Two-Layer CNN with van der Pol Oscillators Coupled by Inductors

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Abstract—In this work, we present that the 2-layer CNN is similar to van der Pol oscillators coupled by inductors, which can generate the phase-wave propagation phenomena. However, we cannot observe the phase-wave propagation phenomena in the original 2-layer CNN. Therefore, we introduce a modified 2-layer CNN. We clearly show the correspondence between the modified 2-layer CNN and the van der Pol oscillators coupled by inductors.

1. Introduction

Cellular Neural Networks (CNN) were invented by L.O.Chua and L.Yang in 1988 [1]. CNN are constructed by cells connected each other. The cell contains linear and nonlinear current sources controlled by voltage. Already a lot of applications and VLSI implementations of CNN were reported. Many nonlinear phenomena such as pattern formation and autowaves could be observed in CNN. Investigating the nonlinear phenomena is an important work for clarifying dynamics of CNNs. On the other hand, phase-wave propagation phenomena in van der Pol oscillators coupled by inductors were reported [2][3]. It is known that 2-layer CNNs can exhibit phase-wave propagation phenomena by choosing an appropriate set of the parameters [4][5].

In this work, we report the detailed investigation on the similarity between the 2-layer CNN and the van der Pol oscillators coupled by inductors. In particular we make clear the correspondence of the parameters of the two systems. Further, we confirm that slight difference between them prevents to generate some of phase-wave propagation phenomena.

2. van der Pol Oscillators Coupled by Inductors

The van der Pol oscillators coupled by inductors L_0 as a comparative object used in this study are shown in Fig. 1.

We assume the v - i characteristics of the nonlinear negative resistors in the circuit as the following function.

$$i_r(v_k) = -g_1 v_k + g_3 v_k^3 \quad (g_1, g_3 > 0). \tag{1}$$

The circuit equations governing the circuit in Fig. 1 are written as

$$\frac{du_k}{d\tau} = w_k \tag{2}$$

$$\frac{dw_k}{d\tau} = -u_k + \alpha(u_{k+1} - 2u_k + u_{k-1}) + \epsilon\left(w_k - w_k^3/3\right) \tag{3}$$

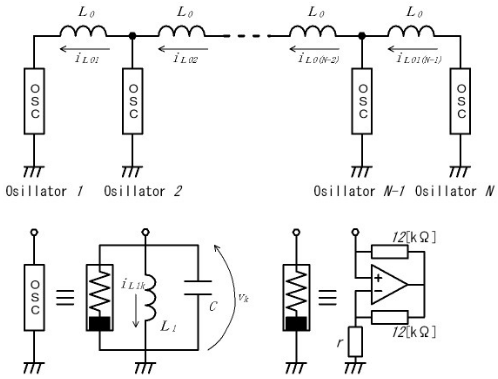


Figure 1: van der Pol oscillators coupled by inductors.

where

$$t = \sqrt{L_1 C}\tau,\ i_{L_1 k} = \sqrt{\frac{Cg_1}{3L_1 g_3}}u_k,\ v_k = \sqrt{\frac{g_1}{3g_3}}w_k,$$

$$\alpha = \frac{L_1}{L_0},\ \epsilon = g_1\sqrt{\frac{L_1}{C}}. \tag{4}$$

And, we consider the boundary conditions as follows:

$$u_0 = u_1, \quad u_{(N+1)} = u_N. \tag{5}$$

It should be noted that α corresponds to the coupling of the oscillators and ϵ corresponds to the nonlinearity of the oscillators. In this study, we calculate (2) and (3) by using the Runge-Kutta-Gill method.

In this study, we use $N = 8$, $\alpha = 0.050$ and $\epsilon = 0.30$ for numerical analysis. And the initial conditions are given as follows:

1. Setting the initial conditions of all oscillators as the same.
2. Putting the arbitrary phase difference of the voltage and the current to one oscillator.

Putting the phase difference +180 [deg] to the 1st oscillator, simulation result is shown in Fig. 2. In Fig. 2, the vertical axis is the sum of voltages of adjacent oscillators, and horizontal axis is time. If the sum of voltages of adjacent oscillators is zero, the phase difference between adjacent oscillators is +180 [deg]. At first, only the 1st oscillator has phase difference +180[deg], phase difference propagates to the adjacent oscillator as time goes.

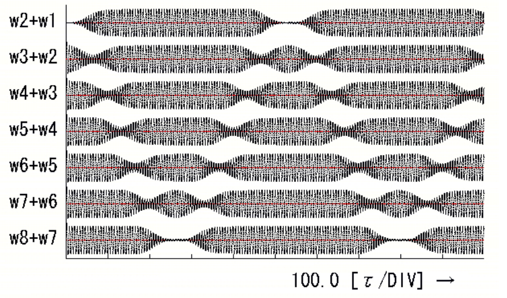


Figure 2: Phase-wave propagation phenomenon in coupled van der Pol oscillators.

3. Two-Layer CNN

In this study, we use 1-dimensional 2-layer CNN as shown in Fig. 3.

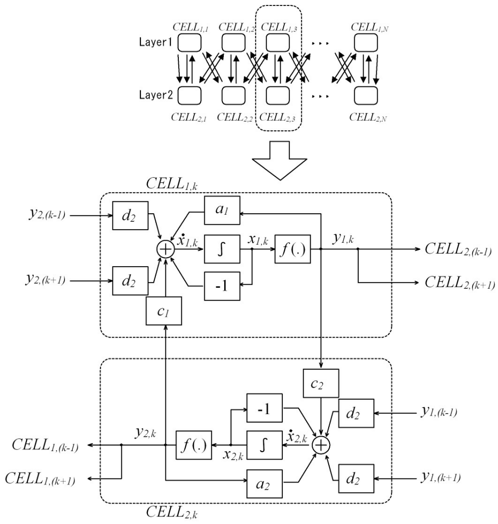


Figure 3: One-dimensional 2-layer CNN.

The circuit equations governing the CNN in Fig. 3 are written as

$$
\begin{aligned}
\dot{x_{1,k}} &= -x_{1,k} + a_1 y_{1,k} + c_1 y_{2,k} \\
&\quad + d_1 y_{2,(k-1)} + d_1 y_{2,(k+1)} && (6) \\
\dot{x_{2,k}} &= -x_{2,k} + a_2 y_{2,k} + c_2 y_{1,k} \\
&\quad + d_2 y_{1,(k-1)} + d_2 y_{1,(k+1)} && (7) \\
y_{\ell,k} &= f(x_{\ell,k}) = 0.5\left(|x_{\ell,k}+1| - |x_{\ell,k}-1|\right) && (8)
\end{aligned}
$$

$$(k = 1, 2, \cdots, N, \quad \ell = 1, 2)$$

where $x_{\ell,k}$ is the state, $y_{\ell,k}$ is the output of $\mathrm{CELL}_{\ell,k}$, a_ℓ, c_ℓ, and d_ℓ are the feedback parameters from the output of its own cell, from the output of the cell which is at the same position in the other layer, and from the output of the neighborhood cell in the other layer, respectively.

In order to investigate the correspondence between the 2-layer CNN and the van der Pol oscillators, we replace the output feedbacks $y_{\ell,k}$ in Eqs. (6) and (7) by the state feedbacks $x_{\ell,k}$, except $y_{2,k}$, as follows;

$$
\begin{aligned}
\dot{x_{1,k}} &= -x_{1,k} + a_1 x_{1,k} + c_1 x_{2,k} \\
&\quad + d_1 x_{2,(k-1)} + d_1 x_{2,(k+1)} && (9) \\
\dot{x_{2,k}} &= -x_{2,k} + a_2 y_{2,k} + c_2 x_{1,k} \\
&\quad + d_2 x_{1,(k-1)} + d_2 x_{1,(k+1)} && (10)
\end{aligned}
$$

By comparing Eq. (9) with Eq. (2), some of the template values are decided as follows;

$$a_1 = -1, \quad c_1 = 1, \quad d_1 = 0. \qquad (11)$$

Further, we approximate the piecewise linear function $f(\cdot)$ of the output feedback from its own cell by the 3rd order polynomial expression.

$$
\begin{aligned}
& -x_{2,k} + a_2 y_{2,k} \\
=\; & -x_{2,k} + 0.5 a_2 \left(|x_{2,k}+1| - |x_{2,k}-1|\right) && (12) \\
\sim\; & \epsilon\left(x_{2,k} - x_{2,k}^3/3\right) && (13)
\end{aligned}
$$

Equation (13) with $\epsilon = 0.30$ is shown by a dotted line in Fig. 4. We approximate this curve by Eq. (12) with $a_2 = 1.2$, whose curve is shown by a solid line in Fig. 4.

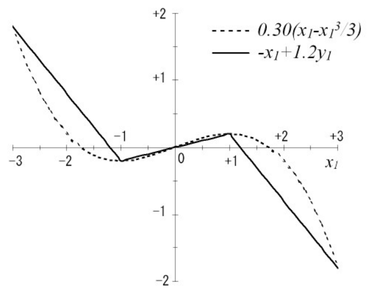


Figure 4: Approximation of piecewise linear function.

Equation (10) can be rewritten using the Eqs. (12) and (13) as follows;

$$
\begin{aligned}
\dot{x_{2,k}} &= \epsilon\left(x_{2,k} - x_{2,k}^3/3\right) + c_2 x_{1,k} \\
&\quad + d_2 x_{1,(k-1)} + d_2 x_{1,(k+1)} && (14)
\end{aligned}
$$

By comparing Eq. (14) with Eq. (3), the other template values are decided as follows;

$$a_2 = 1.2, \quad c_2 = -(1+2\alpha), \quad d_2 = \alpha \qquad (15)$$

Using the template values in Eqs. (11) and (15), Eqs. (9) and (10) are rewritten as follows;

$$
\begin{aligned}
\dot{x_{1,k}} &= x_{2,k} && (16) \\
\dot{x_{2,k}} &= -x_{1,k} + \alpha(x_{1,(k-1)} - 2x_{1,k} + x_{1,(k+1)}) \\
&\quad + \epsilon\left(x_{2,1} - x_{2,1}^3/3\right) && (17)
\end{aligned}
$$

Now, we consider the boundary conditions as follows:

$$x_{1,0} = x_{1,1} \quad (18)$$
$$x_{1,(N+1)} = x_{1,N} \quad (19)$$

Equations (16)-(19) are completely the same as the equations of the van der Pol Oscillators coupled by the inductors (2), (3), and (5).

We expect that the phase-wave propagation phenomena, which are observed in coupled van der Pol oscillators, can be generated in the 2-layer CNN (6) and (7) with the template values (11) and (15). Namely,

$$\dot{x}_{1,k} = y_{2,k} \quad (20)$$
$$\dot{x}_{2,k} = -y_{1,k} + \alpha(y_{1,(k-1)} - 2y_{1,k} + y_{1,(k+1)}) + \epsilon\left(x_{2,1} - x_{2,1}^3/3\right) \quad (21)$$

However, the simulated results of Eqs. (20) and (21) with $\alpha = 0.050$ and $\epsilon = 0.30$ did not show the continuously existing phase-wave propagation phenomena. Figure 5 shows an example of the observed phenomena for $N = 8$ obtained by using the Runge-Kutta-Gill Method.

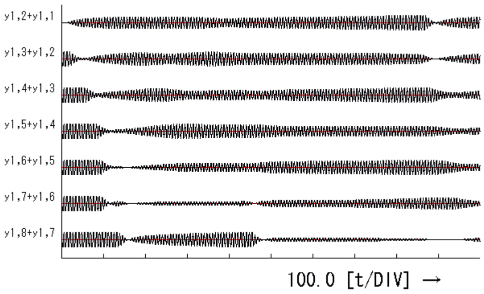


Figure 5: Simulated result of 2-layer CNN ($N = 8$).

The difference between Eqs. (20) and (21) and Eqs. (2) and (3) is only the feedbacks. Namely, the output feedbacks in the 2-layer CNN and the state feedbacks in the coupled van der Pol oscillators.

4. Modified 2-Layer CNN

We introduce the modified 2-layer CNN that the output feedbacks from the cell at the same position in the other layer are replaced by the state feedback. The equations governing the modified 2-layer CNN are written as follows;

$$\dot{x}_{1,k} = -x_{1,k} + a_1 y_{1,k} + c_1 x_{2,k} \quad (22)$$
$$\dot{x}_{2,k} = -x_{2,k} + a_2 y_{2,k} + c_2 x_{1,k} + d_2 y_{1,(k-1)} + d_2 y_{1,(k+1)} \quad (23)$$

Equations (22) and (23) are rewritten using the templates (11) and (15);

$$\dot{x}_{1,k} = x_{2,k} \quad (24)$$
$$\dot{x}_{2,k} = -x_{1,k} + \alpha(y_{1,(k-1)} - 2x_{1,k} + y_{1,(k+1)}) + \epsilon\left(x_{2,1} - x_{2,1}^3/3\right) \quad (25)$$

The simulation result of Eqs. (24) and (25) is shown in Fig. 6. In Fig. 6, we can observe the phase-wave propagation phenomena similar to Fig.2.

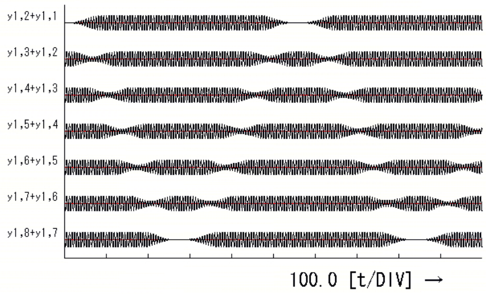


Figure 6: Simulated result of modified 2-layer CNN ($N = 8$).

From this result we consider that the cells at the layer 1 correspond to the differentiation of u_k. The output feedbacks from their own cells at the layer 2 correspond to the nonlinear resistors, and the output feedbacks from the cells at the layer 1 correspond to the couplings between the oscillators.

5. Phase-Wave Propagation Phenomena

The simulated result of the coupled van der Pol oscillators with different coupling parameter $\alpha = 0.10$ is shown in Fig. 7. The speed of the phase-wave propagation is faster than Fig. 2 with $\alpha = 0.05$. Namely, the propagation speed can be controlled by changing the coupling inductor L_0 in the coupled oscillators.

We can confirm similar effect in the modified 2-layer CNN by changing the template values. From $\alpha = 0.10$,

$$c_2 = -(1 + 2\alpha) = -1.2, \quad d_2 = \alpha = 0.10.$$

The simulated result of the modified 2-layer CNN with the calculated template values is shown in Fig. 8. We can see that the speed of the phase-wave propagation is faster than Fig. 6 similar to Fig. 7.

Figure 9 shows some typical examples of the phase-wave propagation phenomena in the van der Pol oscillators coupled by the inductors.

The corresponding simulated results by the modified 2-layer CNN are shown in Fig. 10. We can observe the reflection of the waves, but cannot observe the extinction by the collision.

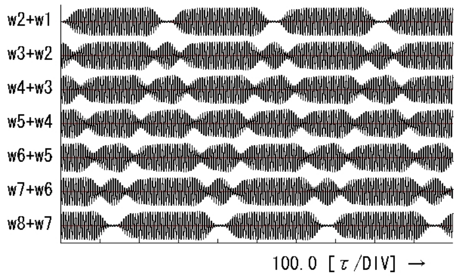


Figure 7: Simulated result of van der Pol oscillators with different $\alpha = 0.10$.

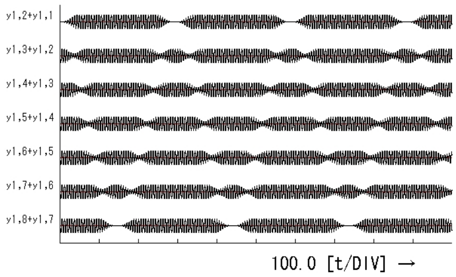
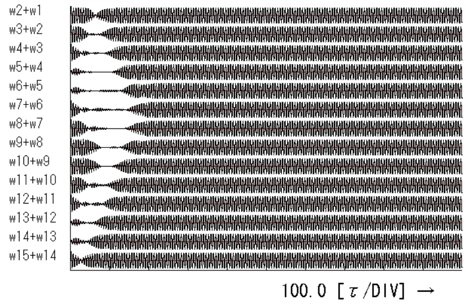


Figure 8: Simulated result of modified 2-layer CNN with different template values. $a_1 = 1$, $c_1 = 1$, $d_1 = 0$, $a_2 = 1.2$, $c_2 = -1.2$, and $d_2 = 0.10$.

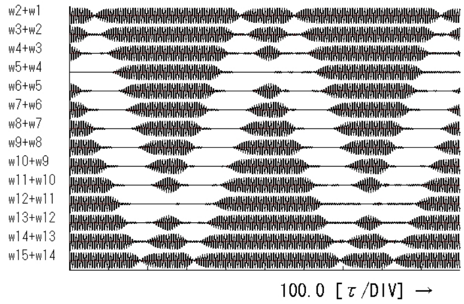
6. Conclusions

In this study, we clearly explained the correspondence between an array of van der Pol oscillators coupled by inductors and 1-dimensional 2-layer CNN. By investigating the generated phase-wave propagation phenomena, we found that the difference between the output feedbacks and the state feedbacks played an important role to make difference in the observed phenomena.

In future works, we will try to clarify the mechanism of the phase-wave propagation phenomena by investigating the modified 2-layer CNN, so that we can control the phase-wave propagation for applications.

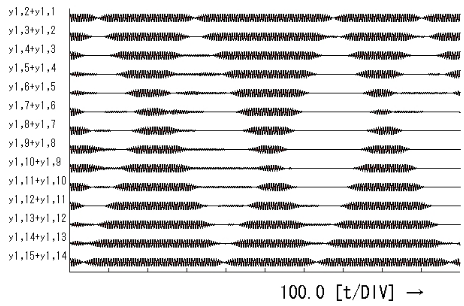


(a) Extinction by collision of two waves.

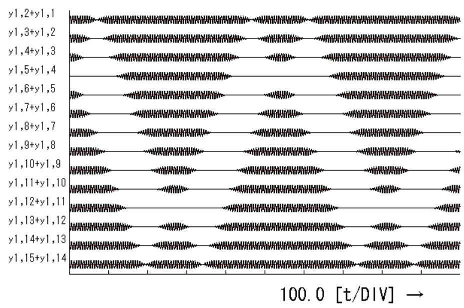


(b) Reflection by collision of two waves.

Figure 9: Examples of phase-wave propagation phenomena in van der Pol oscillators coupled by the inductors. $N = 15$, $\alpha = 0.10$ and $\epsilon = 0.30$.



(a) Extinction by collision can not be observed.



(b) Reflection by collision of two waves.

Figure 10: Example of phase-wave propagation phenomena in modified 2-layer CNN, $N = 15$, $a_1 = 1$, $c_1 = 1$, $d_1 = 0$, $a_2 = 1.2$, $c_2 = -1.2$, and $d_2 = 0.10$.

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