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Asymmetrical Chaotic Coupled System Using Two Different Parameter Sets.

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Abstract—In our previous study, we observed the interesting phenomenon in an asymmetrical chaotic coupled system. It is that the synchronous rate of the one subcircuits group increases in spite of increasing parameter mismatches in the system.

In this study, in order to verify the this phenomenon using another system, an asymmetrical chaotic coupled system is proposed and investigated. Asymmetry of the system is realized by using two parameter sets. In the case of five subcircuits, we confirmed synchronization phenomena in computer calculations. Additionally, It was confirmed that synchronous rates of subcircuits using one parameter set are increased by increasing a parameter mismatch rate of the other subcircuits. We consider that this result is corresponding to results of previous study.

1. Introduction

Coupled systems of chaotic subsystems generate various kinds of complex higher-dimensional phenomena such as spatio-temporal chaotic phenomena, clustering phenomena, and so on. One of the most studied systems may be the coupled map lattice proposed by Kaneko[1]. The advantage of the coupled map lattice is its simplicity. However, many of nonlinear phenomena generated in nature would be not so simple. Therefore, it is important to investigate the complex phenomena observed in natural physical systems such as electric circuits systems[2]-[6].

In our previous study[7], synchronization phenomena in an asymmetrical coupled system of chaotic circuits were investigated. The system is coupled globally and the coupling elements are resistors. Each subcircuit has two coupling nodes and the asymmetrical coupled system is realized by selecting one of two coupling nodes. This system was investigated as non-ideal system. The small parameter mismatches were given to the subcircuits as an mismatch of the oscillation frequency. As a result of investigating this system, interesting phenomenon was observed. The phenomenon is that the synchronous rate of the one subcircuits group increases in spite of increasing parameter mismatches in the system. We suppose that the phenomenon can be explained as follows. The synchronizations of the one subcircuits group and other subcircuits group are constricted each other. Therefore, in the case of decreasing the synchronization of one group decreases, the synchronization of the other group increases.

In this study, in order to verify the this phenomenon using another system, an asymmetrical chaotic coupled system is proposed and investigated. Asymmetry of the system is realized by using two parameter sets. In the case of five subcircuits, we confirm synchronization phenomena in computer calculations. Additionally, It is confirm that synchronous rates of subcircuits using one parameter set are increased by increasing a parameter mismatch rate of another subcircuits.

2. Asymmetrical Coupled Systems

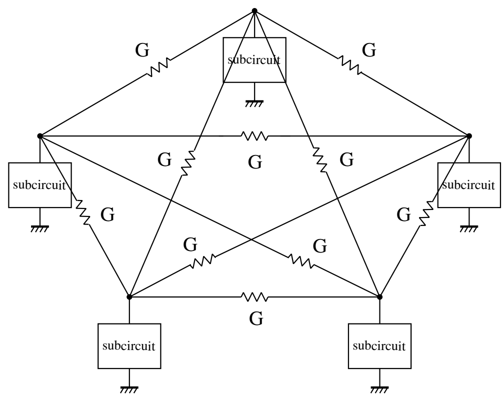


Figure 1: Asymmetrical coupled chaotic system.

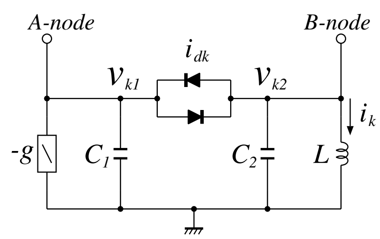
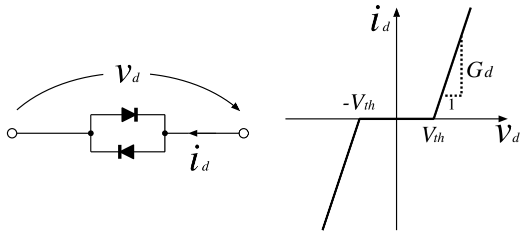


Figure 2: Chaotic subcircuit.

The system is coupled globally and the coupling elements are resistors as shown in Fig. 1. The coupled subcircuit is a chaotic circuit[8] as shown in Fig. 2. In our previous study, asymmetry is realized as selecting one of two nodes in Fig. 2. In this study, only one node are used and asymmetry is realized by using two parameter sets. The circuit using one parameter set is called as "A-subcircuits", and using the other parameter sets is called as "B-subcircuit". Now, in order to carry out computer calculation, circuit equations are derived. Figure 3 shows a nonlinear resistor which consist of two diodes. Circuit equations are derived using Figure 3(b). We define m and n as the number of circuits. The number of A-subcircuits is m, and the number of B-subcircuits is n.



(a)circuit model (b)characteristics

Figure 3: Nonlinear resistor which consist of two diodes.

A-subcircuits($1 \le k \le m$):

$$\begin{cases} C_{1a}\dfrac{dv_{k1}}{dt} &= gv_{k1} - G_d f(v_{k1} - v_{k2}) \\ &\quad +G\left\{\displaystyle\sum_{i=1}^{m+n} v_{i1} - (m+n)v_{k1}\right\}, \\ C_{2a}\dfrac{dv_{k2}}{dt} &= -i_{k3} + G_d f(v_{k1} - v_{k2}), \\ L_a\dfrac{di_{k3}}{dt} &= (1+p_k)v_{k2}, \end{cases} \tag{1}$$

B-subcircuits($m \le k \le m+n$):

$$\begin{cases} C_{1b}\dfrac{dv_{k1}}{dt} &= gv_{k1} - G_d f(v_{k1} - v_{k2}) \\ &\quad +G\left\{\displaystyle\sum_{i=1}^{m+n} v_{i1} - (m+n)v_{k1}\right\}, \\ C_{2b}\dfrac{dv_{k2}}{dt} &= -i_{k3} + G_d f(v_{k1} - v_{k2}), \\ L_b\dfrac{di_{k3}}{dt} &= (1+q_k)v_{k2}, \end{cases} \tag{2}$$

where,

$$f(v) = v + \frac{|v - V_{th}|}{2} - \frac{|v + V_{th}|}{2}.$$

Function $f(v)$ is characteristics of the nonlinear resistor shown in Fig. 3. Parameters p_k and q_k shows small parameter mismatches. Using the following parameters and variables,

$$\begin{aligned} &x_k = \frac{v_{k1}}{V_{th}}, \quad y_k = \frac{v_{k2}}{V_{th}}, \quad z_k = \frac{1}{V_{th}}\sqrt{\frac{L_a}{C_{2a}}}, \\ &t = \sqrt{L_a C_{2a}}\tau, \quad \text{“ . ”} = \frac{d}{d\tau}, \quad \alpha = \frac{C_{2a}}{C_{1a}}, \\ &\beta = g\sqrt{\frac{L_a}{C_{2a}}}, \quad \gamma = G_d\sqrt{\frac{L_a}{C_{2a}}}, \quad \delta = G\sqrt{\frac{L_a}{C_{2a}}}, \\ &\varepsilon = \frac{C_{2a}}{C_{1b}}, \quad \zeta = \frac{C_{2a}}{C_{2b}} \quad \text{and} \quad \eta = \frac{L_a}{L_b}. \end{aligned} \tag{3}$$

Normalized equations are described as follows.
A-subcircuits($1 \le k \le m$):

$$\begin{cases} \dot{x}_k &= \alpha\beta x_k - \alpha\gamma f'(x_k - y_k) \\ &\quad +\alpha\delta\left\{\displaystyle\sum_{i=1}^{m+n} x_i - (m+n)x_k\right\}, \\ \dot{y}_k &= -z_k + \gamma f'(x_k - y_k), \\ \dot{z}_k &= (1+p_k)y_k, \end{cases} \tag{4}$$

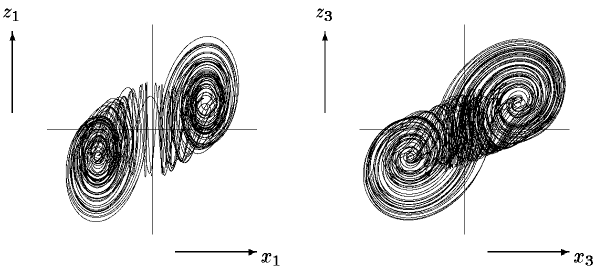
B-subcircuits($m+1 \le k \le m+n$):

$$\begin{cases} \dot{x}_k &= \varepsilon\beta x_k - \varepsilon\gamma f'(x_k - y_k) \\ &\quad +\varepsilon\delta\left\{\displaystyle\sum_{i=1}^{m+n} x_i - (m+n)x_k\right\}, \\ \dot{y}_k &= \zeta\left\{-z_k + \gamma f'(x_k - y_k)\right\}, \\ \dot{z}_k &= \eta(1+q_k)y_k, \end{cases} \tag{5}$$

where,

$$f'(x) = x + \frac{(|x-1| - |x+1|)}{2}.$$

We carry out computer calculations in the case of five subcircuits. The system consists of two A-subcircuits and three B-subcircuits. Figures 4 and Figure 5 show computer calculated results using following initial values and parameters. Figure 4(a) show a attractor of A-subcircuit ($k = 1$). Figure 4(b) show a attractor of B-subcircuit ($k = 3$). Horizontal axes are x_k and vertical axes are z_k. Double scroll type attractors are observed. Figures 5 show voltage differences between each subcircuits. Vertical axes show voltage differences and horizontal axes show time. Namely, in the case of synchronizing two subcircuits, the amplitude becomes zero. Value Q is corresponding to parameter mismatch rate of B-subcircuits.



(a)A-subcircuit (b)B-subcircuit

Figure 4: Attractors of A-subcircuit ($k = 1$) and B-subcircuit ($k = 3$). Horizontal axes are x_k and vertical axes are z_k. In the case of Eq. (6) and $Q = 0.05$.

$$\begin{cases} x_k(0) = y_k(0) = z_k(0) = 0.100,\ (k = 1, 2, 3, 4, 5), \\ m = 2,\ n = 3,\ \alpha = 0.600,\ \beta = 0.500,\ \gamma = 20.0, \\ \delta = 0.070,\ \varepsilon = 0.6,\ \zeta = 1.5,\ \eta = 0.5 \\ p_k = 0.001(k-1) \text{ and } q_k = Q(k-1). \end{cases} \tag{6}$$

The first graph shows the voltage difference between two

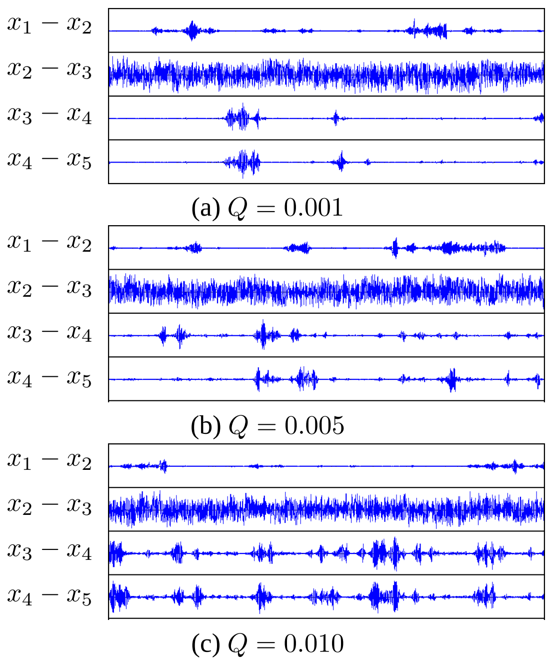


Figure 5: Voltage difference between two subcircuits. In the case of Eq. (6).

A-subcircuits. We can see that synchronization and unsynchronized burst appear alternately in a random way. The second graph shows the voltage difference between an A-subcircuit and a B-subcircuit. These are not synchronized at all. The third and fourth graphs show the voltage differences between two B-subcircuits. These results are similar than first graph. However these are not the same. In the case of previous study, corresponding results are shown in Fig. 6. From comparing two results, we can see as follows: Fig. 5(a) is similar to the case of lower Q value than Fig. 6. Fig. 5(b) is similar to Fig. 6. In Fig. 5(c), synchronous rate is higher than Fig. 6. Now, in order to in-

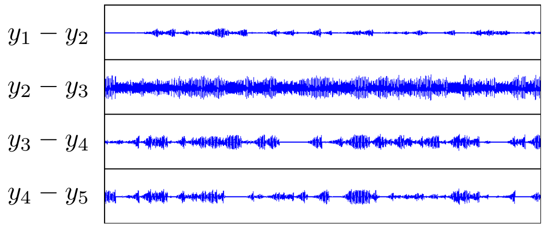


Figure 6: One of the computer calculated result of previous study.

vestigate the relationship between synchronous rates and parameter mismatches, we define the synchronization as shown in Fig. 7 and follows:

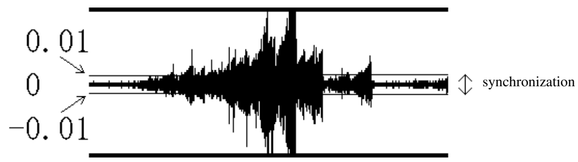


Figure 7: Definition of the synchronization.

$$|x_k - x_{k+1}| < 0.01 \tag{7}$$

where k is number of subcircuits.

Fig. 8 shows synchronous rate averages between two subcircuits to Q which is corresponding to parameter mismatch rate. The synchronous rate is defined as a ratio of synchronous time and total time. Fig. 9 shows a result of previous study corresponding to Fig. 8. A-node of Fig. 9 is corresponding to A-subcircuit, and B-node is corresponding to B-subcircuit. In Fig. 8 and Fig. 9, the synchronous rates of one group are increasing by increasing Q. We can consider to obtain the same phenomena. Additionally, thirty subcircuits case is investigated. Figures 10 and 11 shows the result. We can also observed the same result. Namely, in spite of increasing parameter mismatch rate, synchronous rate of A-subcircuit increasing.

3. Conclusions

In this study, in order to verifier the phenomenon of previous study, an asymmetrical coupled system is proposed and investigated. Asymmetry of the system is realized by using two parameter sets.

In the case of five subcircuits, we confirmed similar phenomena in computer calculations. Additionally, It was confirmed that synchronous rates between circuits using one parameter set are increased by increasing a parameter mismatch rate of the other parameter set. We consider that this

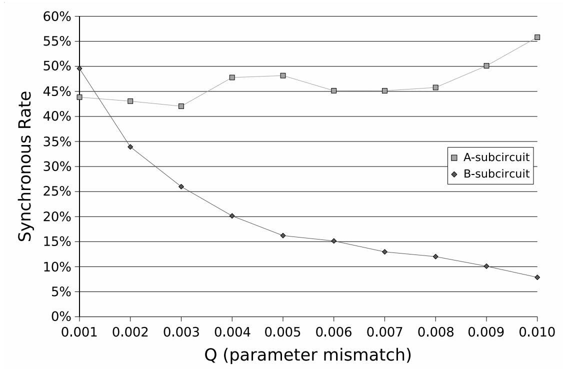


Figure 8: Synchronous rate of chaotic circuits to the parameter mismatch of B-subcircuit in the case of five subcircuits. $m = 2$, $n = 3$, $\alpha = 0.600$, $\beta = 0.500$, $\gamma = 20.0$, $\delta = 0.070$, $\varepsilon = 0.6$, $\zeta = 1.5$ and $\eta = 0.5$.

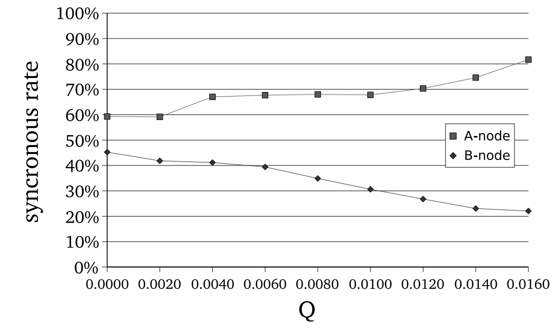


Figure 9: Synchronous rate of previous study corresponding to the parameter mismatch of B-subcircuit in the case of five subcircuits. $m = 2$, $n = 3$, $\alpha = 0.400$, $\beta = 0.500$, $\gamma = 20.0$, and $\delta = 0.07$.

result is corresponding to results of previous study. Furthermore, we also confirmed the same phenomena in the case of thirty subcircuits. Therefore, we consider that these phenomena are not the phenomena generated on a special system.

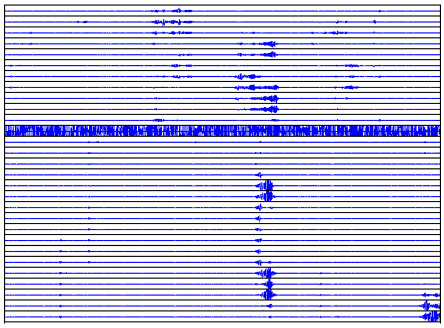


Figure 10: Voltage difference between two subcircuits in the case of thirty subcircuits. $m = 12$, $n = 18$, $\alpha = 0.600$, $\beta = 0.500$, $\gamma = 20.0$, $\delta = 0.013$, $\varepsilon = 0.6$, $\zeta = 1.5$ and $\eta = 0.5$.

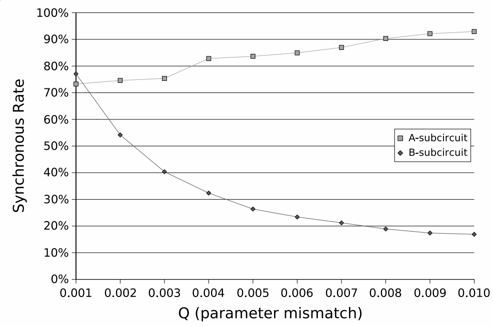


Figure 11: Synchronous rate of chaotic circuits to the parameter mismatch of B-subcircuit in the case of thirty subcircuits. $m = 12$, $n = 18$, $\alpha = 0.600$, $\beta = 0.500$, $\gamma = 20.0$, $\delta = 0.013$, $\varepsilon = 0.6$, $\zeta = 1.5$ and $\eta = 0.5$.

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