NOLTA 2006
11-14 september
Bologna - ITALY

Pattern Dynamics of Phase Synchronization in a Family of Coupled Several One-Dimensional Chaotic Maps

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Abstract— In this study, phase synchronization behavior and control of its patterns in coupled chaotic maps are investigated. There are many types of chaotic map, then coupled chaotic systems yield wide variety of complex phenomena and further it is shown possibility to several engineering applications. The chaotic maps which have been governed by *n*-th power polynomial or sinusoidal functions is properly selected as a chaotic cell, then each chaotic map is connected to neighbors as a ring array or network structure. Several phase synchronization patterns and its control method are shown.

1. Introduction

We have now interests how to various patterns in nature were cleated. Coupled chaotic systems attract many researchers' attention as a good model which can realize the complicated phenomena in the natural world, and further its dynamics can yield a wide variety of complex and strange phenomena. The coupled systems existing in nature exhibit great variety of phenomena such as complex mechanisms for all of the systems in the universe. These phenomena can be found in a metabolic network, a human society, the process of a life, self organization of neuron, a biological system, an ecological system and so many nonlinear systems. Among the studies on such coupled systems, many interesting researches relevant to the spatio-temporal chaos phenomena on the coupled chaotic systems have been studied until now, e.g. mathematical model in one- or two-dimensional network investigated earnestly by Kaneko [1]-[4], and found in physical circuit model [5]. The construction of multi–agent system on the coupled cubic map system has been reported [6]. Moreover, research of complicated phenomena and emergent property in the coupled cubic maps on 2-dimensional network system has been also reported [7]. The studies of coupled map lattice(CML), globally coupled maps(GCM) and so many studies concerned with such complex systems provided us tremendous interesting phenomena. This is an interesting report that phase dynamics are controlled due to change its parameter themselves. We had also reported the research on spatio-temporal phase patterns in coupled maps using a fifth-power function [8][9], in which it has been carried out in the unique case. However many coupled chaotic systems have wide variety of features and moreover its dynamics is also expected to be applied much engineering applications, there are many problems which should be solved in large scale coupled network systems by their complexity.

In this study, spatio-temporal chaotic behavior in coupled chaotic maps is investigated from the point of view in more faithful natural world. The chaotic map which has been governed by *n*-th power polynomial or sinusoidal functions is properly selected as a chaotic cell. We consider the model which chaotic cells are mutually connected to neighbors as a ring structure (i.e. CML type) by arbitrary coupling strength. Then, we show some phenomena which spatio-temporal chaos, complex behavior and several phase patterns can be found in the proposed coupled systems. Furthermore, its control dynamics are realized by changing a perturbation parameter.

2. Model Description

Chaotic maps are generally used for several approaches to investigate chaotic phenomena on coupled chaotic systems. Especially, the logistic map and the other types of chaotic maps such as a cut map, a circle map, a tent map, a cubic map are well known and popular. Obviously, it is necessary to have a lot of equilibrium points with the complex phenomena that corresponds to the natural world. Let us consider two types of chaotic map. Firstly, the chaotic map from an *n*-th power polynomial function written as follows.

$$f(x) = \sum_{i=1}^{n} a_i x^i + \varphi \tag{1}$$

where a_i is a parameter which can determine for their chaotic feature, further φ is a new parameter for perturbation as a small variable value. The parameter φ should be normally set as zero. If it is needed to adopt the map with respect to the origin, odd-numbered coefficients a_i are only set suitable values in (1). In other words, even-numbered coefficients are set as all zero. Then, we can easily confirm that it generates chaos in this function. The some diagrams of the function (1) are shown in Fig. 1 with some equilibrium points.

Secondary, the chaotic map from a sinusoidal function written as follows.

$$f(x) = e^{c|x|}\sin(ax) + bx + \varphi \tag{2}$$

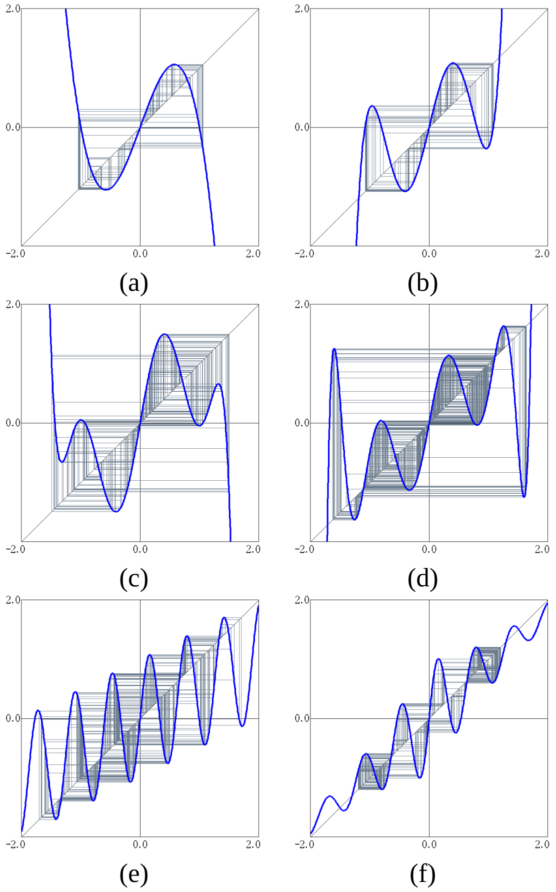


Figure 1: Several chaotic maps by n-th power polynomial and sinusoidal functions for $\varphi = 0$. Setting parameters: (a) $n = 3$, $a_3 = -2.75$, $a_1 = 2.75$, (b) $n = 5$, $a_5 = 5.50$, $a_3 = -10.0$, $a_1 = 4.17$, (c) $n = 7$, $a_7 = -2.85$, $a_5 = 11.6$, $a_3 = -14.6$, $a_1 = 5.80$, (d) $n = 9$, $a_9 = 2.08$, $a_7 = -13.15$, $a_5 = 27.4$, $a_3 = -21.3$, $a_1 = 5.46$, (e) $\sin(10x) + 0.5x$, (f) $e^{-0.9|x|}\sin(10x) + 0.9x$, and the others are all 0.

where a, b and c are parameters which can determine for their chaotic feature. Especially the parameter b is an important factor in order to suppress its divergence. The some diagrams of the function (2) are also shown in Fig. 1. From (1) and (2), it can be calculated rigorously several bifurcation conditions and boundary region.

In order to evaluate the function (1), Lyapunov exponent can be calculated as follows.

$$\lambda = \lim_{N\to\infty} \sum_{k=1}^{N} \log \left| \frac{df(x_k)}{dx} \right| \tag{3}$$

Lyapunov exponent is a very important measurement often used to show the existence of chaos. Some Lyapunov exponents with bifurcation diagram by changing one parameter are shown in Fig. 2. These are typical results which can be obtained from computer calculation. In case of using polynomial functions, period doubling and tangent bifurcation can be confirmed. On the other hand, in case of using sinusoidal functions, complicated bifurcation property can be confirmed even if the result of Lyapunov exponent is seen. Therefore chaotic maps possessing several equilibrium points can yield various wide interesting behavior.

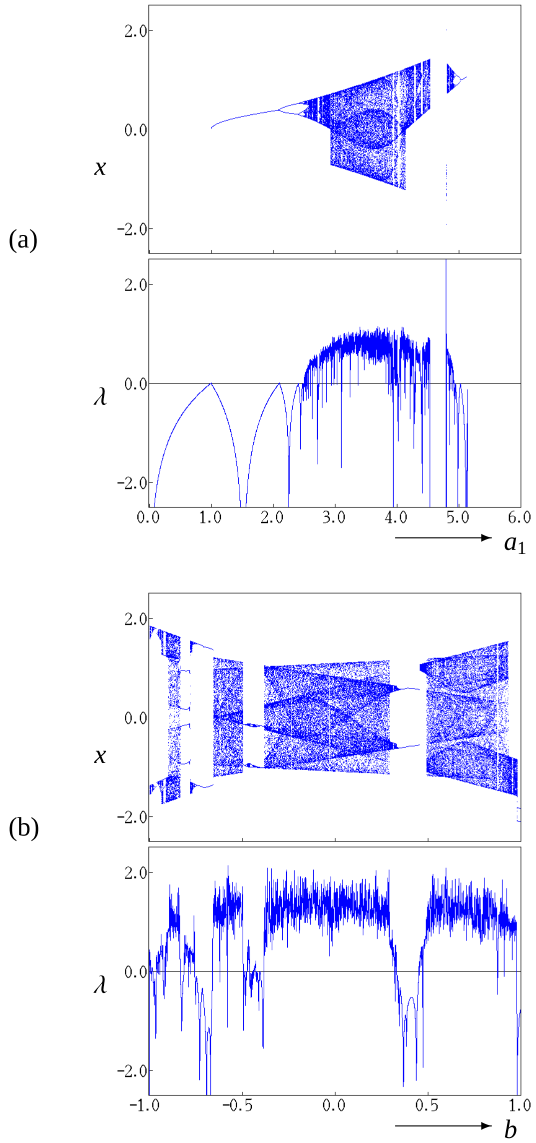


Figure 2: Bifurcation diagram and Lyapunov exponent: (a) changing a_1 for $a_7 = -3.10$, $a_5 = 11.5$ and $a_3 = -12.4$ in case of $n = 7$, (b) changing b for $a = 10.0$ and $c = -0.90$ in case of using sinusoidal function (2).

3. Phase Patterns in Coupled Chaotic Maps

In this section, we consider a coupled chaotic system that one of these maps as a chaotic subsystem in each cell. It can be considered easily that coupled chaotic systems have wide variety of phase patterns. The term "spatio-temporal" is extensively used for irregular dynamical behavior observed from large scale complex systems of the relevant to both time and space. In this study, in order to confirm spatio-temporal chaos or phase patterns in the faithful natural world, consider a coupled model of the chaotic maps which are connected to neighbors on a ring array structure as shown in Fig. 3. Each chaotic cell is connected to neigh-

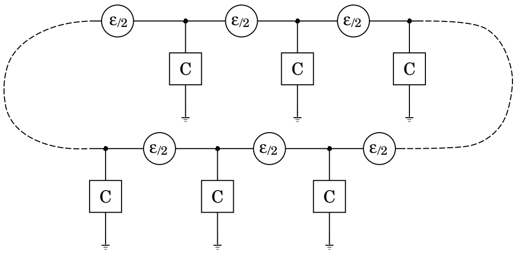


Figure 3: Coupled chaotic system as a ring array or coupled lattice structure.

bors by arbitrary coupling strength ε. The total system by CML is represented as

$$x_k(t+1) = (1-\varepsilon) f\big(x_k(t)\big) + \frac{\varepsilon}{2}\Big(f\big(x_{k-1}(t)\big) + f\big(x_{k+1}(t)\big)\Big), \quad (k = 1, 2, \cdots, N) \tag{4}$$

where t is an iteration, k is an index number of the cell which follows the cyclic rule, and N is a size of coupled cell number, respectively.

Some numerical simulation results of model (4) for $N = 50$ are shown in Fig. 4 with coupling strength $\varepsilon = 0.30$. The initial condition for each cell is given as $x_k(0) \in [0.49, 0.51]$ uniformly. The figure indicates a grade of synchronization state for phase difference, with gray scale colors between white □ and black ■ which correspond to synchronous and asynchronous state, respectively. Hereby the synchronous state with gray scale colors in 100 steps is displayed. A lot of interesting phenomena were confirmed though all the results can not be represented more here.

4. Pattern Dynamics and Control

Further, we attempt to control phase patterns of entire coupled system to become synchronous state with additional swing of function or changing the coupling strength. As shown in Fig. 4, it is confirmed that some parts are asynchronous state. Although all subsystem is the same, it is difficult to perform to control entire system synchronously. However control method should be simple as possible. Therefore, we propose two simple methods below.

(method 1)

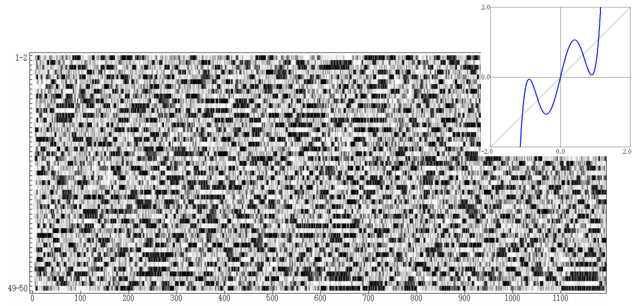
$$\varepsilon_k = \varepsilon^* \qquad \text{if } |x_k - x_{k+1}| < 0.5$$

(method 2)

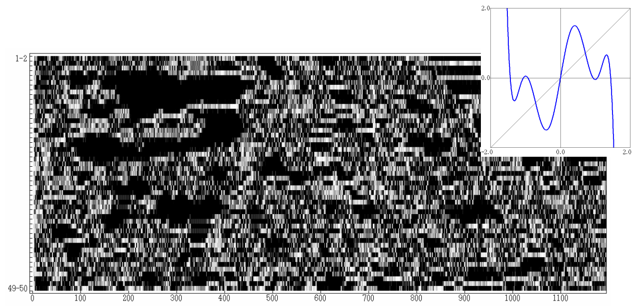
$$\varepsilon_k = \varepsilon^* \qquad \text{if } |x_k - x_{k+1}| < 0.5$$

$$\varphi = \begin{cases} -0.1 & \text{if } x_k \geq 0 \\ +0.1 & \text{if } x_k < 0 \end{cases}$$

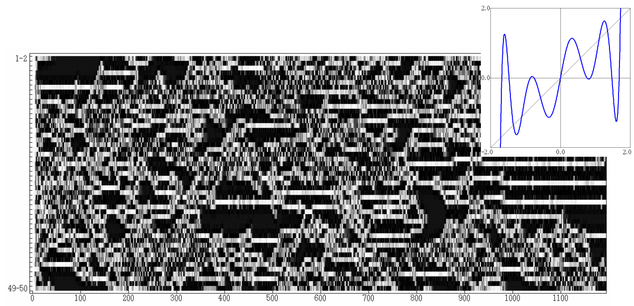
In case of method 1, it is changed only coupling parameter ε_k in each cell. When the value of difference between two neighbors is larger than 0.5, the coupling strength ε_k of the target cell changes to ε^*. In case of method 2, we use the condition of swing parameter φ in addition to the method 1. Figure 5 shows a pattern obtained by method



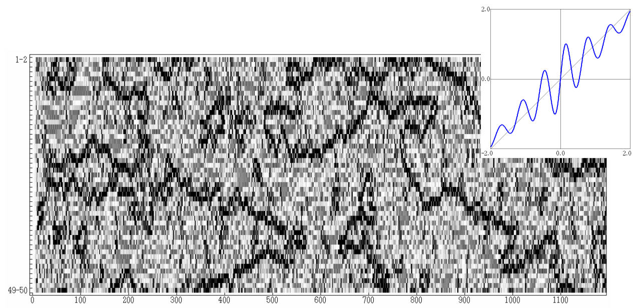
(a)



(b)

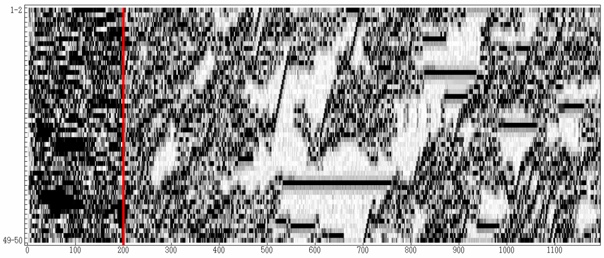


(c)

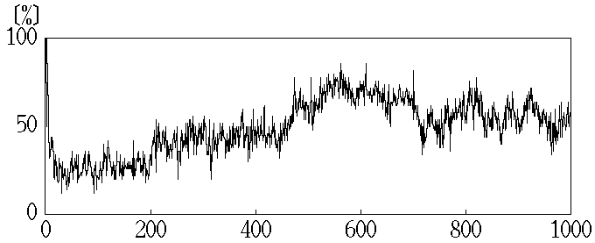


(d)

Figure 4: Simulation results of phase synchronization state in coupled chaotic maps as a ring array for $N = 50$: (a) $n = 5$, $a_5 = 6.20$, $a_3 = -10.0$, $a_1 = 4.10$, (b) $n = 7$, $a_7 = -2.85$, $a_5 = 11.6$, $a_3 = -14.6$, $a_1 = 5.80$, (c) $n = 9$, $a_9 = 2.08$, $a_7 = -13.15$, $a_5 = 27.4$, $a_3 = -21.3$, $a_1 = 5.46$, and (d) $e^{-0.9|x|}\sin(10x) + 0.9x$. Each result is performed in the condition $\varepsilon = 0.30$.



(a)



(b)

Figure 5: Simulation results of pattern dynamics with control method 2: (a) $n = 7$, $a_7 = -2.85$, $a_5 = 11.6$, $a_3 = -14.6$, $a_1 = 5.80$, (b) transition of synchronous ratio. Each result is performed in the condition of change parameters $\varepsilon = 0.30 \leftrightarrow \varepsilon^* = 0.50$ and $\varphi = \pm 0.1$.

2 when the control is executed at $t = 200$. A lot of parts are made synchronous can be confirmed. To clarify a synchronous ratio, the difference of the adjoined cells counted the number of 0.2 or less and it was calculated. The mean values of the increase ratio before and after the control when the initial condition is changed and tries it 1000 times are shown in Table 1 and 2. The result that either was excellent was not obtained because two methods were carried out by a specific parameter, and the control method should be changed according to the characteristic of the chaotic map. Therefore, it is necessary to investigate whether other parameters are changed, and which control method is excellent though only control by the change in the coupling strength and the swing parameter were used in this experiment.

5. Conclusions

In this paper, some chaotic maps by n-th power polynomial or sinusoidal functions for using as a chaotic cell of coupled network have been proposed. Some illustrated computer simulation results of spatio-temporal chaotic behavior and several phase patterns in coupled chaotic maps have been shown. The simple method for controlling the patterns has been proposed, and the effectiveness has been investigated. We conclude that the supposed or similar coupled chaotic systems can be regarded as a good model for realizing complex phenomena in the universe concerned with self organization, mechanisms of pattern formation and so on. However some studies of pattern dynamics and the mechanism of clustering phenomena in such complex phenomena and many works have been left.

Table 1: Increase ratio of synchronous state when the setting parameters are the same as figure 4.

	ε^*					
	0.4	0.5	0.6	0.7	0.8	
n = 5	11.7	4.4	6.7	9.0	11.7	
n = 7	9.1	13.4	14.5	15.0	13.4	
n = 9	21.8	23.9	25.3	28.9	25.8	
e^x sin	6.4	6.0	2.5	-6.1	-12.1	[%]

Table 2: Increase ratio of synchronous state when the setting parameters are the same as figure 4 with swing parameter φ.

	ε^*						
	0.3	0.4	0.5	0.6	0.7	0.8	
n = 5	-8.1	-0.7	-0.9	0.8	4.3	2.7	
n = 7	17.1	20.8	20.8	34.1	22.8	15.3	
n = 9	8.0	16.3	17.3	17.6	18.7	15.6	
e^x sin	-0.1	6.6	6.0	2.4	-6.1	-12.2	[%]

Acknowledgment

This work was partly supported by GRANT-IN-AID for "Open Research Center" project from MEXT, Japan.

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