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Solving Ability of Hopfield Neural Network for QAP by Changing Chaotic Behavior of Switching Noise

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Abstract—In our previous research, we confirmed that the chaotic switching noise generated by the cubic map gained a good performance for solving combinatorial optimization problems when the noise was injected to the Hopfield neural network. However, the reason of the good effect of chaotic switching noise has not been clarified completely. In this study, we investigate the solving ability of Hopfield neural network for QAP when the chaotic behavior of the switching noise is changed.

1. Introduction

Combinatorial optimization problems can be solved with the Hopfield neural network (abbr. NN). If we choose connection weights between neurons appropriately according to given problems, we can obtain a good solution by the energy minimization principle. However, the solutions are often trapped into a local minimum and do not reach the global minimum. In order to avoid this critical problem, several people proposed the method adding some kinds of noise for solving traveling salesman problems (TSP) with the Hopfield NN [1]. Hayakawa and Sawada pointed out the chaos near the three-periodic window of the logistic map gains the best performance [2]. They concluded that the good result might be obtained by a property of the chaos noise; short time correlations of the time-sequence. Hasegawa et al. investigated solving abilities of the Hopfield NN with various surrogate noise, and they concluded that the effects of the chaotic sequence for solving optimization problems can be replaced by stochastic noise with similar autocorrelation [3]. We have also studied the reason of the good performance of the Hopfield NN with chaotic noise. We imitated the intermittency chaos by the burst noise generated by the Gilbert [4] model with 2 states; a laminar state and a burst state. We concluded that the irregular switching of laminar part and burst part is one of the reasons of the good performance of the chaotic noise [5] [6]. Further, we have investigated a performance of chaotic switching noise generated by the cubic map when the noise is injected to the Hopfield NN for quadratic assignment problem (abbr. QAP). We have confirmed that the chaotic switching noise was effective for solving QAP similar to the intermittency chaos noise near the three-periodic window [9]. However, the reason of the good effect of chaotic switching noise has not been clarified completely.

In this study, we investigate solving ability of Hopfield NN for QAP when the chaotic behavior of the switching noise is changed. By computer simulation, we confirm that the network can find good solutions, even when the chaotic behavior is partly replaced by random time series.

2. Solving QAP with Hopfield NN

Various methods are proposed for solving the QAP which is one of the NP-hard combinatorial optimization problems.The QAP is expressed as follow: given two matrices, distance matrix C and flow matrix D, and find the permutation P which corresponds to the minimum value of the objective function f(p) in Eq. (1).

$$f(P) = \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} D_{p(i)p(j)}, \quad (1)$$

where C_{ij} and D_{ij} are the (i, j)-th elements of C and D, respectively, p(i) is the i th element of vector P, and N is the size of the problem. There are many real applications which are formulated by Eq. (1). One example of QAP is find an arrangement of the factories to make a cost the minimum. The cost is given by the distance between the factories and flow of the products between the factories. Other examples are the placement of logical modules in a IC chip, the distribution of medical services in large hospital.

Because the QAP is very difficult, it is almost impossible to solve the optimum solutions in large problems. The largest problem which is solved by deterministic methods may be only 24 in recent study. Further, computation times is very long to obtain the exact optimum solution. Therefore, it is usual to develop heuristic methods which search near optimal solutions in reasonable time.

For solving N-element QAP by Hopfield NN, $N \times N$ neurons are required and the following energy function is defined to fire (i, j)-th neuron at the optimal position:

$$E = \sum_{i,m=1}^{N} \sum_{j,n=1}^{N} w_{im;jn} x_{im} x_{jn} + \sum_{i,m=1}^{N} \theta_{im} x_{im}. \quad (2)$$

The neurons are coupled each other with the following weight between (i,m)-th neuron and (j,n)-th neuron and the threshold of the (i,m)-th neuron is described by:

$$w_{im;jn} = -2\left\{A(1-\delta_{mn})\delta_{ij}+B\delta_{mn}(1-\delta_{ij})+\frac{C_{ij}D_{mn}}{q}\right\} \quad (3)$$

$$\theta_{im} = A + B \quad (4)$$

where A and B are positive constant, and δ_{ij} is Kroneker's delta. The state of $N\times N$ neurons are asynchronously update due to following difference equation:

$$x_{im}(t+1) = f\left(\sum_{j,n=1}^{N} w_{im;jn}x_{im}(t)x_{jn}(t)-\theta_{im}+\beta z_{im}(t)\right) \quad (5)$$

where f is sigmoidal function defined as follows:

$$f(x) = \frac{1}{1+\exp\left(-\frac{x}{\varepsilon}\right)}, \quad (6)$$

z_{im} is additional noise, and β limits the amplitude of the noise.

3. Chaotic Switching Noise

In this section, we describe chaotic switching noise injected to the Hopfield NN.

The following cubic map is used to generate the chaotic switching noise.

$$\hat{y}_{im}(t+1) = -\hat{y}_{im}(t)(\alpha_c\hat{y}_{im}^2(t)+1-\alpha_c). \quad (7)$$

Figure 1 shows the shape of the cubic map Eq. (7). The one-parameter bifurcation diagram of this map is shown in Fig. 2. The attractor becomes symmetric at around $\alpha_c = 3.600$ via an interior crisis. Because the transition of the solution from the positive/negative part to the other part is seldom just after the crisis, the behavior looks like an irregular switching as shown in Fig. 3. We use these time series after the following normalization.

$$y_{im}(t+1) = \frac{\hat{y}_{im}(t)-\bar{y}}{\sigma_y} \quad (8)$$

where $\bar{y}$ and σ_y are the average and the standard deviation of $\hat{y}(t)$, respectively.

In our previous research [9], we have confirmed that this chaotic switching noise was effective for solving QAP.

4. Changing Chaotic Behavior of Switching Noise

We consider that the chaotic switching noise (Figs. 3) contains two kinds of chaotic features; one is the switching timing of the upper part and the lower part, the other is the behavior inside the each part.

In the previous research [9], we have confirmed that regular switching between the two parts makes the performance worse. Hence, the former chaotic feature can be

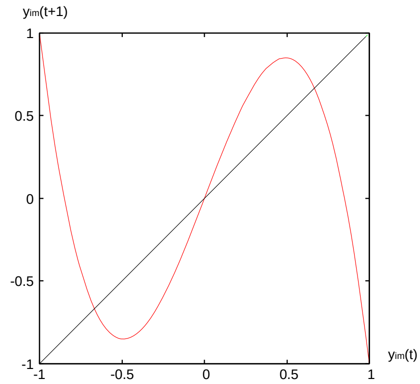


Figure 1: Cubic map ($\alpha_c = 3.600$).

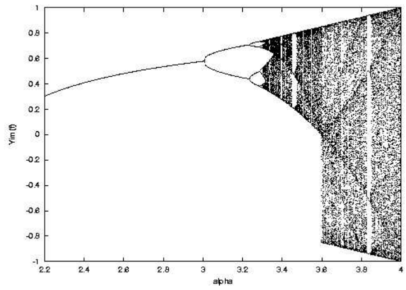
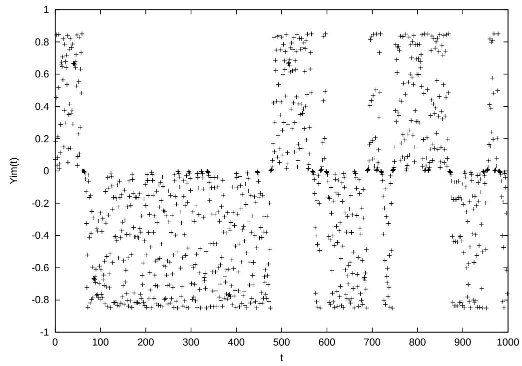
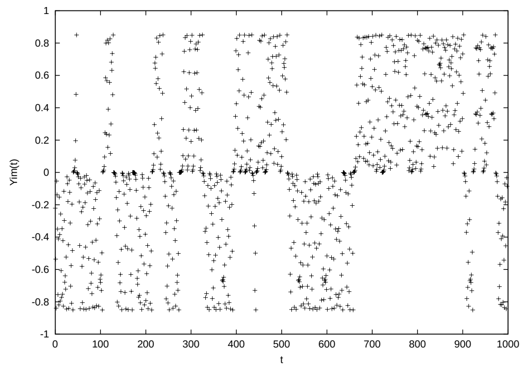


Figure 2: Bifurcation diagram of cubic map.



(a) $\alpha_c = 3.599$.

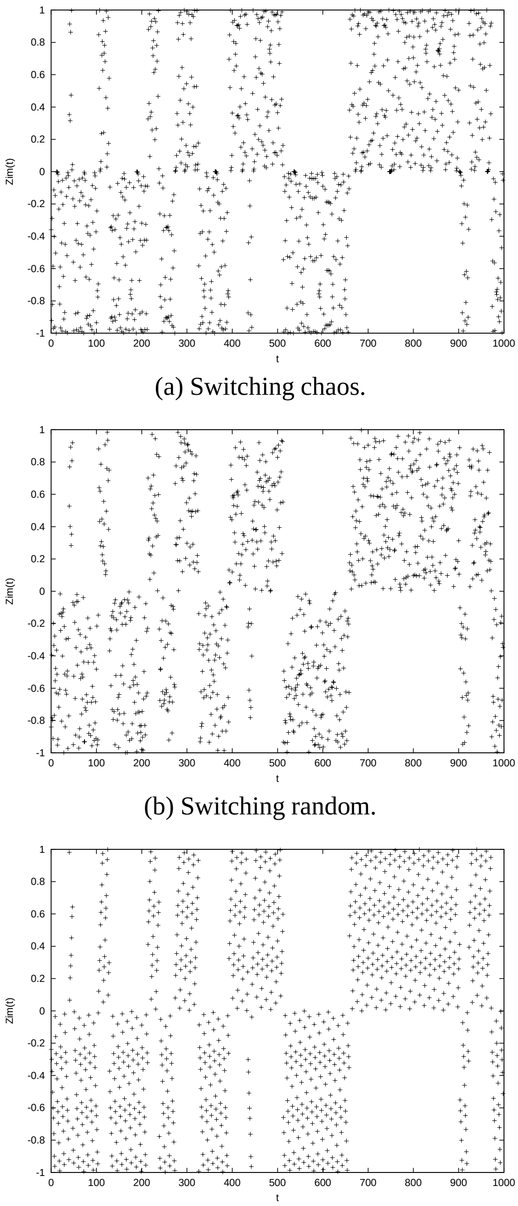


(b) $\alpha_c = 3.600$.

Figure 3: Time series obtained from cubic map.

said to be important for solving QAP. How about the latter chaotic feature? In order to answer this question, we replace the chaotic time series inside the each interval of the chaotic switching noise by other time series.

Figure 4 shows three kinds of time series made by replacing the original chaotic time series by chaos, random, and torus. The switch was to multiply the sequance number either by 1 or by -1. Please note that we keep the switching timing between the upper part and the lower part.



(a) Switching chaos.

(b) Switching random.

(c) Switching torus.

Figure 4: Changing chaotic behavior of switching noise.

4.1. Switching Chaos

The logistic map is used to generate the chaotic time series inside the each interval.

$$\hat{l}_{im}(t+1) = \alpha_l \hat{l}_{im}(t)(1 - \hat{l}_{im}(t)). \quad (9)$$

Varying parameter α_l, Eq. (9) behaves chaotically via a periodic-doubling cascade. In this study, we carry out computer simulations using the fully-developed chaos, which is obtained from Eq. (9) for $\alpha_l = 4.0$.

4.2. Switching Random

Random time series are generated by using random function of C compiler. The range of random noise is set to 0 $\sim$ 1.

4.3. Switching Torus

The sine circle map is used to generate the torus time series inside the each interval.

$$\hat{x}_{im}(t+1) = \hat{x}_{im}(t) + \alpha_x \sin\{6\pi \hat{x}_{im}(t)\} + D. \quad (10)$$

The parameters are fixed as α_x=0.04 and D=0.1.

5. Simulation Results

In this section, the simulation results of Hopfield NN with three kinds of switching noise for 12-elements QAP are summarized in Tables 1 and 2. The problem used here was chosen from QAPLIB, whose name is "Nug12." The global minimum of this target problem is known as 578. The parameters of Hopfield NN are fixed as A = 0.9, B = 0.9, q = 140, ε = 0.02 and β= 0.55.

The tables show the average of the obtained solutions in 10 trials with different initial conditions, the minimum solution in 10 trials, and the error between the average and the optimal solution calculated by the following equation.

$$Error = \frac{Ave - Opt}{Opt} \times 100, \quad (11)$$

where Opt denotes the optimal solution of the target problem.

From these tables, we can confirm that the switching chaos and the switching random can gain similar performance to the cubic map (chaotic switching noise). This means that the chaotic feature inside the each interval is not very important for solving QAP.

However, we can also notice that the switching torus can not find a good solution. We can conclude that some kinds of irregularity are important inside the each interval of the switching noise.

Table 1: Solving abilities for 12-elements QAP. (α_c=3.599)

Iteration	switching chaos			switching random			switching torus			Cubic Map		
	Ave	Min	error	Ave	Min	error	Ave	Min	error	Ave	Min	error
1000	624.8	586	8.097	645.4	626	11.661	657.6	632	13.772	635.2	616	9.896
2000	620.2	586	7.301	631.2	606	9.204	654.8	632	13.287	627.0	616	8.478
3000	619.0	586	7.903	627.6	606	8.581	646.4	606	11.834	627.0	616	8.478
4000	617.8	586	6.886	623.4	594	7.855	646.4	606	11.834	626.0	616	8.304
5000	615.8	586	6.540	619.8	594	7.232	646.4	606	11.834	621.2	608	7.470
6000	615.8	586	6.540	617.2	594	6.782	646.4	606	11.834	619.6	608	7.197
7000	614.0	586	6.228	616.8	594	6.713	646.4	606	11.834	616.8	606	6.713
8000	613.4	586	6.125	614.6	594	6.332	646.4	606	11.834	616.6	606	6.678
9000	611.4	586	5.779	614.0	594	6.228	646.4	606	11.834	616.6	606	6.678
10000	611.4	586	5.779	612.6	594	5.986	646.4	606	11.834	615.6	606	6.505

Table 2: Solving abilities for 12-elements QAP. (α_c=3.6)

Iteration	switching chaos			swithing random			swithing torus			Cubic Map		
	Ave	Min	error	Ave	Min	error	Ave	Min	error	Ave	Min	error
1000	635.8	612	10.000	637.0	606	10.207	662.4	664	14.602	632.4	582	9.412
2000	633.6	612	9.619	627.4	606	8.547	656.6	622	13.599	625.4	582	8.201
3000	623.8	598	7.924	624.8	606	8.097	654.8	622	13.287	623.0	582	7.785
4000	621.4	598	7.509	617.8	606	6.886	654.8	622	13.287	615.8	582	6.540
5000	619.2	598	7.128	613.6	586	6.159	654.8	622	13.287	615.6	582	6.505
6000	614.6	598	6.332	613.6	586	6.159	654.4	622	13.218	614.6	582	6.332
7000	610.6	598	5.640	613.4	586	6.125	654.4	622	13.218	614.0	582	6.228
8000	609.4	598	5.433	613.4	586	6.125	654.4	622	13.218	608.8	582	5.329
9000	609.4	598	5.433	611.8	586	5.848	654.4	622	13.218	608.8	582	5.329
10000	608.4	598	5.259	611.8	586	5.848	654.4	622	13.218	606.6	582	4.948

6. Conclusions

In this study, we have investigated the solving ability of Hopfield NN when the chaotic behavior of the switching noise was replaced by another chaos, random and torus. By computer simulation, we confirmed that the network can find good solution, even when the chaotic behavior was partly replaced by random time series. We conclude that the reason of the good performance of the chaotic switching noise generated from the cubic map is the chaotic switching between the upper part and the lower part, and that chaotic behavior inside the each interval is not important but some kinds of irregularity are necessary.

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