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Synchronization of Chaotic Circuits Linked by Cross Talk

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Abstract—In this study, we investigate synchronization of two Chua's circuits with a lossless transmission lines. Two circuits with transmission lines placed in parallel cause the cross talk phenomena. The effect of the cross talk is modeled by the connections via coupling capacitors or mutual inductors.

By computer simulations, we investigate the synchronization of the two chaotic circuits linked by the cross talk. Especially, in this study, we pay our attentions to the breakdown of chaos synchronization.

1. Introduction

Recently, several researchers have investigated synchronization of chaotic circuits. Chua's circuit is one of the simplest autonomous chaotic circuits. Chua's circuit consists of a resistor, a capacitor, a nonlinear resistor and an LC resonator. The nonlinear resistor in Chua's circuit has a piecewise linear $v - i$ characteristics as shown in Fig. 1 and it is described by the following equation;

$$
\begin{aligned}
i_R = m_2 v_1 &+ 0.5(m_0 - m_1)[|v_1 + B_{p1}| - |v_1 - B_{p1}|] \\
&+ 0.5(m_1 - m_2)[|v_1 + B_{p2}| - |v_1 - B_{p2}|], \qquad (1)
\end{aligned}
$$

where m_0, m_1 and m_2 are the slopes in the segments of the piecewise linear function, and B_{p1} and B_{p2} denote the breakpoints. There are many studies about coupled Chua's

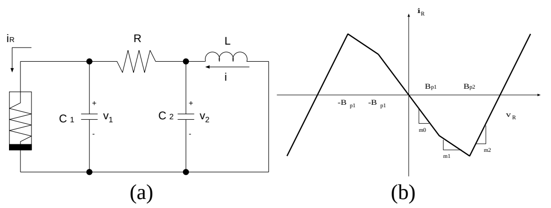


Figure 1: (a) Chua's circuit. (b) $v - i$ characteristics of nonlinear resistor.

circuits, for example, master-slave coupling of Chua's circuits [1], mutual coupling of Chua's circuits [2], a ladder of Chua's circuits, a ring of Chua's circuits [3], two-dimensional array of Chua's circuits [4], etc. However, almost researches consider the couplings by lumped elements. We have investigated chaotic phenomena observed from Chua's circuit when the LC resonator is replaced by a transmission line [5]. Further, we have also investigated synchronization phenomena of two Chua's circuits coupled by a transmission line [6].

We consider two Chua's circuits with lossless transmission lines placed in parallel. Transmission lines placed in parallel cause the cross talk phenomena. Cross talk phenomena appear in long powerlines or very high speed VLSI and its effect is normally not preferable. We have already confirmed that the two Chua's circuit with transmission lines could be synchronized by the effect of the cross talk [7]. In this study, we investigate the synchronization of the two Chua's circuits linked by the cross talk in detail, especially, paying our attentions to the breakdown of chaos synchronization.

2. Basic Circuit Model [7]

In our previous research, we reported that two Chua's circuits with lossless transmission lines could be synchronized. The circuit model is shown in Fig. 2.

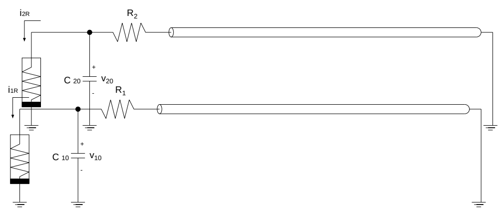


Figure 2: Two Chua's circuits with transmission lines.

We modeled the transmission lines by LC ladder circuits with finite numbers of lumped elements. Further, we modeled the cross talk effect by coupling capacitors or mutual inductors. The circuit equations could be derived as follows;

$$
\begin{array}{rcl}
C_{j0}\frac{dv_{j0}}{dt} & = & \frac{v_{j1}-v_{j0}}{R_j} - i_{Rj}(v_{j0}) \\
C_{j1}\frac{dv_{j1}}{dt} & = & i_{j1} - \frac{v_{j1}-v_{j0}}{R_j} + C_1\frac{d(v_{j1}-v_{(j+1)1})}{dt} \\
C_{jk}\frac{dv_{jk}}{dt} & = & i_{jk} - i_{j(k-1)} + C_k\frac{d(v_{jk}-v_{(j+1)k})}{dt} \\
L_{jl}\frac{di_{jl}}{dt} & = & v_{j(l+1)} - v_{jl} + \frac{M_k}{R^2 C_{10}}\frac{d(i_{jk}-i_{(j+1)k})}{dt}
\end{array} \qquad (2)
$$

where $v_{31} = v_{11}$, $v_{j(n+1)} = 0$, $i_{31} = i_{11}$, (k=2, 3, ..., n), (l=1, 2, ..., n) and (j=1, 2). From Eqs. (1) and (2), we could obtain the normalized circuit equations as follows;

$$
\begin{array}{rcl}
\dot{x_{10}} & = & x_{11} - x_{10} - f(x_{10}) \\
\dot{x_{11}} & = & \alpha_{11}(y_{11} - x_{11} + x_{10}) \\
 & & -\beta_{11}(y_{21} - x_{21} + x_{20}) \\
\dot{x_{1k}} & = & \alpha_{1k}(y_{1k} - y_{1(k-1)}) - \beta_{1k}(y_{2k} - y_{2(k-1)}) \\
\dot{x_{20}} & = & \zeta(x_{21} - x_{20} - f(x_{20})) \\
\dot{x_{21}} & = & \alpha_{21}(y_{21} - \zeta(x_{21} - x_{10})) \\
 & & -\beta_{21}(y_{11} - x_{11} + x_{10}) \\
\dot{x_{2k}} & = & \alpha_{2k}(y_{2k} - y_{2(k-1)}) - \beta_{jk}(y_{1k} - y_{1(k-1)}) \\
\dot{y_{jl}} & = & \gamma_{jl}(x_{j(l+1)} - x_{jl}) + \beta_l(x_{(j+1)(l+1)} - x_{jl}
\end{array} \qquad (3)
$$

where $x_{j(n+1)} = 0$,

$$
\begin{aligned}
f(x_{j0}) = c_j x_{j0} &+ 0.5(a_j - b_j)(|x_{j0} + 1| - |x_{j0} - 1|) \\
&+ 0.5(b_j - c_j)(|x_{j0} + d_j| - |x_{j0} - d_j|), \qquad (4)
\end{aligned}
$$

$$t = R_1\tau, \ \ \text{“}\cdot\text{”} = \frac{d}{d\tau}, \ \ v_{jk} = B_{p1}x_{jk}, \ \ i_{jl} = \frac{B_{p1}}{R_1}y_{jl},$$

$$\alpha_{jk} = \frac{C_{jk}C_{10}(C_{(j+1)k} - C_k)}{-C_k^2 - (C_{jk}^2 + 1)C_k + C_{jk}^2 C_{(j+1)k}},$$

$$\beta_{jk} = \frac{C_{jk}C_{10}C_k}{-C_k^2 - (C_{jk}^2 + 1)C_k + C_{jk}^2 C_{(j+1)k}},$$

$$\beta_l = \frac{R_1^2 C_{10} M_l}{(L_{2l} - M_l)(L_{1l} - M_l) - M_l^2},$$

$$\gamma_{jl} = \frac{R_1^2 C_{j0}}{L_{jl}}, \ \ \zeta = \frac{C_{10}}{C_{20}}, \ \ a_j = R_j m_0,$$

$$b_j = R_j m_1, \ \ c_j = R_j m_2, \ \ d = \frac{B_{p2}}{B_{p1}},$$

$C_{3k} = C_{1k}$, (k=2, 3, ..., n), (l=1, 2, ..., n) and (j=1, 2).

Some examples of chaos synchronization obtained by computer simulations are shown in Figs. 3 and 4. We confirmed that the circuits were synchronized in anti-phase in the case of mutual inductors and in in-phase in the case of coupling capacitors.

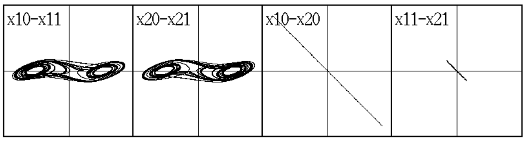


Figure 3: Anti-phase synchronization via mutual inductors (γ=1.0 and β=4.0).

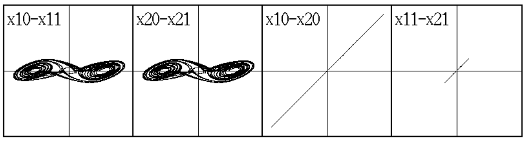


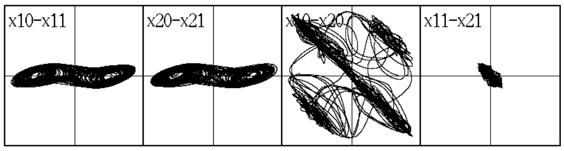
Figure 4: In-Phase synchronization via coupling capacitors (γ=1.0 and β=0.092).

3. Breakdown of Synchronization

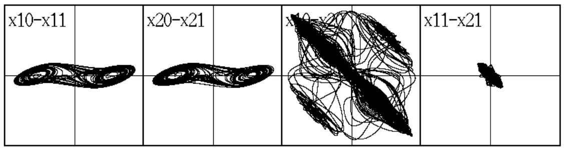
In order to understand the synchronization phenomena via cross talk effects of the transmission lines, we investigate the breakdown of the synchronization when the circuit parameters are gradually changed.

We fix the coupling parameter β and increase the error between the two Chua's circuits. Figure 5 shows the computer simulated results when the cross talk effect is modeled by mutual inductors. As the error between the Chua's circuits $\alpha_{1k} - \alpha_{2k}$ increases, the solution escapes from the synchronized plane more often. Figure 5(c) shows the time waveform of $x_{10} + x_{20}$. We can see from this figure that the solution behaves under the on-off intermittency.

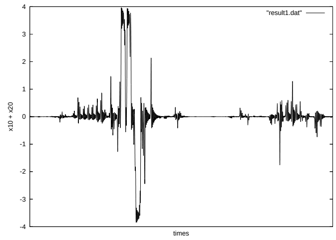
Figure 6 shows the computer simulated results when the cross talk effect is modeled by coupling capacitors. We can observe the similar phenomena to the case of mutual inductors. However, the breakdown seems to be less sensitive to the error of the parameters than the case of mutual inductors.



(a) α_{1k}=16.000000005, α_{2k}=16.0, and β=4.0.



(b) α_{1k}=16.000000010, α_{2k}=16.0, and β=4.0.



(c) Time waveform of $x_{10} + x_{20}$. Parameters are the same as (b).

Figure 5: Breakdown of synchronization. Cross talk effect is modeled by mutual inductors.

4. Cross Talk from Opposite Direction

Now, we consider the two Chua's circuits with lossless transmission lines placed in parallel but from the opposite direction as shown in Fig. 7.

Similar to the circuit model in Fig. 2, we model the lossless transmission lines by LC ladder circuits with finite numbers of lumped elements as shown in Fig. 8.

Further, we consider that the effect of the cross talk is modeled by mutual inductors as shown in Fig. 9.

We carry out computer simulation for this models by using the 4th order Runge-Kutta method. In this simulation, we consider only the case that the two Chua's circuits are identical. So, we can rewrite the parameters as follows;

$$\begin{array}{lll} \alpha_1 = \alpha_2 = \alpha, & \gamma_1 = \gamma_2 = \gamma, & a_1 = a_2 = a, \\ b_1 = b_2 = b, & c_1 = c_2 = c, & d_1 = d_2 = d. \end{array} \quad (5)$$

In the following simulations, we fix the parameters as follows;

$$\begin{array}{llll} a = -1.2, & b = -0.75, & c = 10, & d = 8, \\ \alpha = 18, & \gamma = 1, & n = 10. & \end{array} \quad (6)$$

4.1. Cross Talk via Mutual Inductor M

Figure 10 shows the synchronization of the two circuits. It is interesting to observe that the two circuits can be synchronized even if they are placed from the opposite direction. The parameter m corresponds to the overlap of the two transmission lines. Namely, $m = 10 (= n)$ means that all of inductors are coupled to one of the other inductors via mutual inductors. While $m = 5$ means that only the half of the transmission lines is influenced each other. Further, m=1 means that only the last inductors in the two transmission lines are coupled each other.

4.2. Cross Talk via Mutual Inductor $-M$

Next, we consider the case that the cross talk effect is modeled by negative mutual inductors. Figure 11 shows

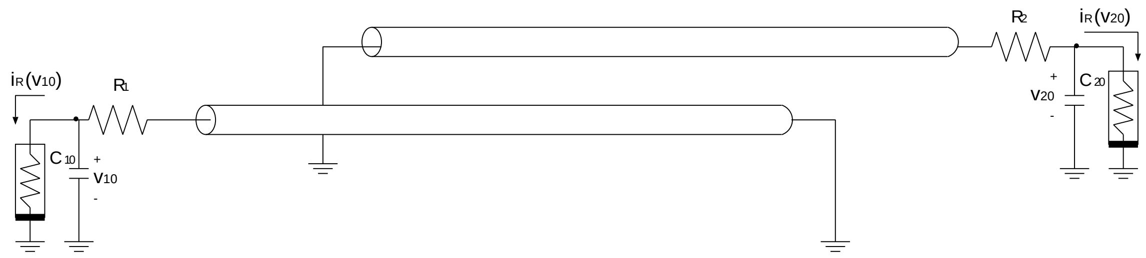


Figure 7: Two Chua's circuits with transmission lines placed from the opposite direction.

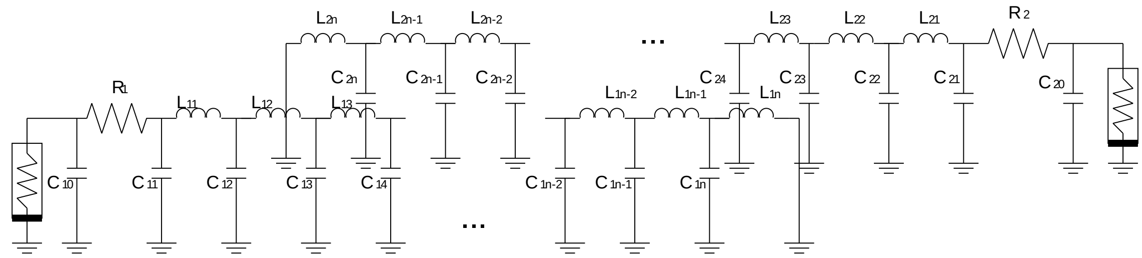
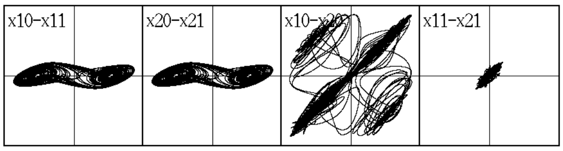
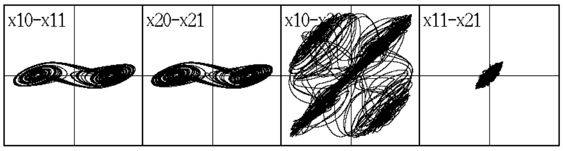


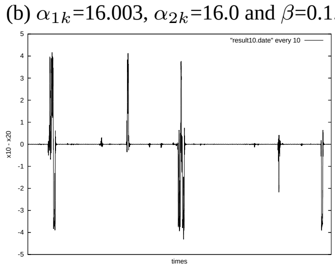
Figure 8: Discrete model by LC ladder circuits with finite number of lumped elements.



(a) α_{1k}=16.001, α_{2k}=16.0 and β=0.1.



(b) α_{1k}=16.003, α_{2k}=16.0 and β=0.1.



(c) Time wave form of $x_{10} - x_{20}$. Parameters are the same as (b).

Figure 6: Breakdown of synchronization. Cross talk effect is modeled by coupling capacitors.

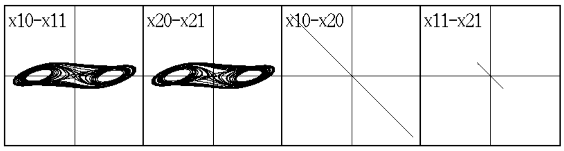
the simulation results for the case that M is negative. In this case, we confirm that the two circuits synchronize in in-phase.

4.3. Effect of Coupling Strength

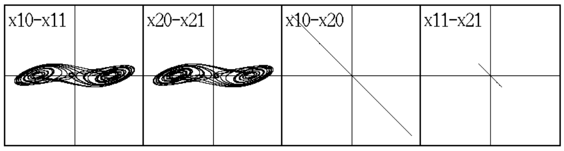
Finally, we carry out computer simulation as varying β, namely, the coupling strength. Figure 12 shows the simulation result when $m = 3$. From Fig. 12, we can say that the synchronization is robust against the coupling strength.

While Fig. 13 shows the simulation result when $m = 5$. This results show that the synchronization is sensitive to the change of the coupling strength.

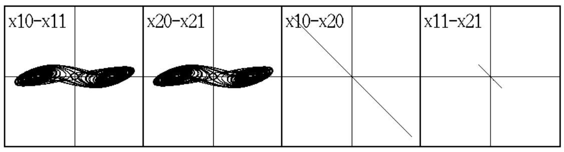
At the moment, we can not explain why the robustness of the synchronization against the coupling strength is changed according to the position of the coupling of the transmission line



(a) $m = 10$ and β = 6.5.



(b) $m = 5$ and β = 9.



(c) $m = 1$ and β = 14.

Figure 10: Simulation results of cross talk via mutual inductor M.

5. Conclusions

In this study, we have considered the two Chua's circuits with lossless transmission lines placed in parallel. We have modeled the effect of the cross talk by mutual inductors. By computer simulations, we have investigated various interesting phenomena related with chaos synchronization.

In our future work, we investigate Chua's circuit with lossy transmission line, because real transmission lines should have loss actually. Furthermore, we investigate crosstalk phenomena between conductor boards placed in parallel and apply the result to chaotic circuits.

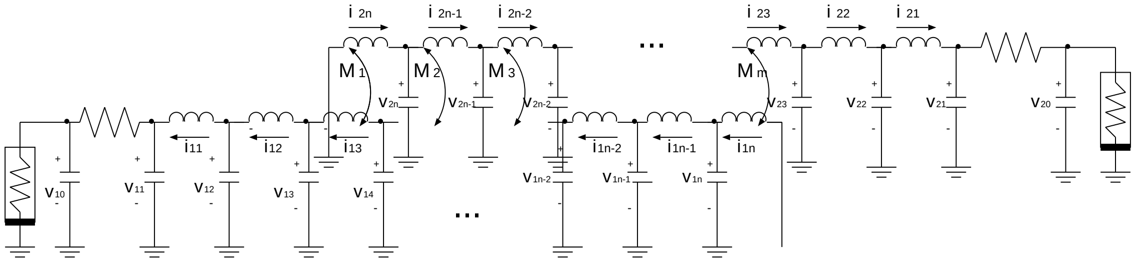
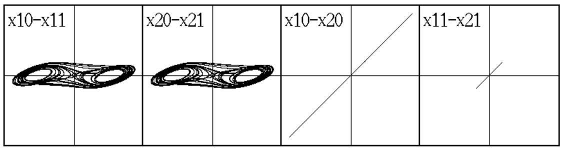
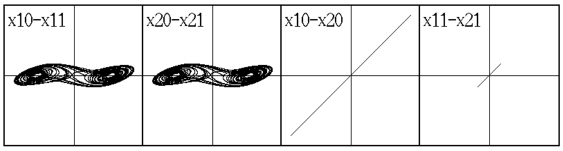


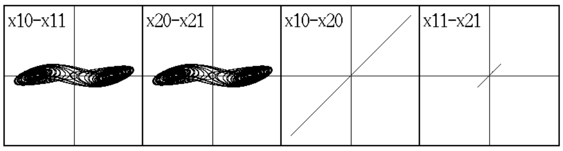
Figure 9: Model of cross talk effect by mutual inductors.



(a) $m = 10$ and β=$-$6.5.

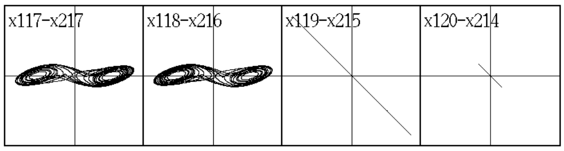


(b) $m = 5$ and $\beta = -9$.

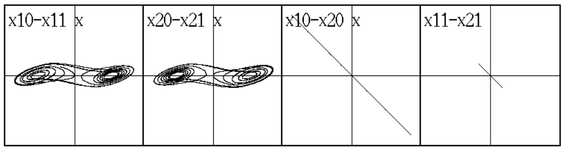


(c) $m = 1$ and $\beta = -14$.

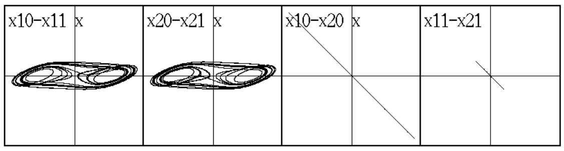
Figure 11: Simulation results of cross talk via mutual inductor $-M$.



(a) $m = 3$ and β = 6.3.

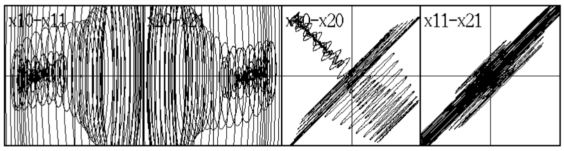


(b) $m = 3$ and β = 7.0.

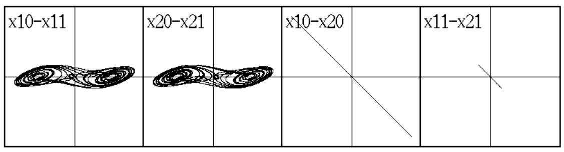


(c) $m = 3$ and β = 8.0.

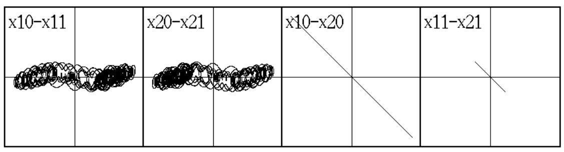
Figure 12: Effect of coupling strength when m = 3.



(a) $m = 5$ and β = 8.7.



(b) $m = 5$ and β = 9.5.



(c) $m = 5$ and β = 9.9.

Figure 13: Effect of coupling strength when m = 5.

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