Two van der Pol Oscillators Coupled by Chaotically Varying Resistor

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Abstract—*In this study, two van der Pol oscillators coupled by a time-varying resistor are investigated. We assume that the time-varying resistor is realized by switching a positive and a negative resistors alternately with a fixed frequency and chaotically varying duty ratio. By carrying out computer simulations, we confirm that interesting change of phase difference between the two oscillators is observed.*

I. Introduction

Synchronization phenomena in complex systems are very interesting to describe various higher-dimensional nonlinear phenomena in the field of natural science. Studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields, physics [1]-[3], biology [4], engineering [5]-[12] and so on. Because many researchers suggest that synchronization phenomena of coupled oscillators have some relations to information processing in the brain. We consider that it is very important to investigate the synchronization phenomena of coupled oscillators to realize a brain computer for the future engineering application.

On the other hand, there are some systems whose dissipation factors vary with time, for example, under the time-variation of the ambient temperature, an equation describing an object moving in a space with some friction and an equation governing a circuit with a resistor whose temperature coefficient is sensitive such as thermistor. However, there are few discussion about coupled oscillators coupling by a time-varying resistor.

In our previous research, we have investigated synchronization phenomena in van der Pol oscillators coupled by a time-varying resistor [13]. We realized the time-varying resistor by switching a positive and a negative resistors periodically [14]. By changing the duty ratio p, we can confirmed that the characteristics of the synchronization phenomena as follows. First, for smaller p, the two coupled oscillators are synchronized only in anti-phase. Second, for intermediate p, the coexistence of the in-phase and the anti-phase synchronizations can be observed. Finally, for larger p, only the in-phase synchronization can be confirmed. What happen in the two coupled oscillators? if the duty ratio changes to between smaller and larger values. This question is very interesting to make clear the mechanism of coupled oscillators with time-varying resistor.

In this study, we investigate two van der Pol oscillators coupled by a time-varying resistor when the switching duty ratio of the time-varying resistor is determined by the value of the logistic map as an external signal. First, we investigate the phenomena when the switching duty ratio is changed as one-period, two-period, four-period, and chaos. Next, we investigate the phenomena when the switching duty ratio is around the intermittency chaos near the three-periodic window. By carrying out computer calculations, we can confirm that the phase difference between the two coupled oscillators changes according to the switching duty ratio.

II. Circuit Model

Figure 1 shows the circuit model. In this circuit, two identical van der Pol oscillators are coupled by a Time-Varying Resistor (TVR). We have known the synchronization phenomena for the case that the coupling resistor is a simple time-invariant resistor [5][6]. Namely, the in-phase synchronization is stable for a positive coupling resistor, while the anti-phase synchronization is stable for a negative coupling resistor. In this study, we consider the case that the coupling resistance $R(t)$ of the TVR varies with time. The characteristics of the TVR is shown in Fig. 2. In this study, we consider the case that the function representing the variation of the TVR is the square wave with the angular frequency ω_t and the duty ratio p.

Firstly, the $v_k - i_{Rk}$ characteristics of the nonlinear resistor are defined as follows,

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3. \qquad (1)$$

By changing the variables and the parameters,

$$v_k = \sqrt{\frac{g_1}{g_3}} x_k, \quad i_k = \sqrt{\frac{g_1}{g_3}} \sqrt{\frac{C}{L}} y_k, \quad t = \sqrt{LC}\tau,$$

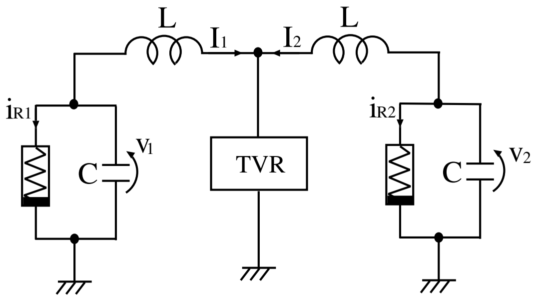


Fig. 1. Circuit model (TVR is a Time-Varying Resistor).

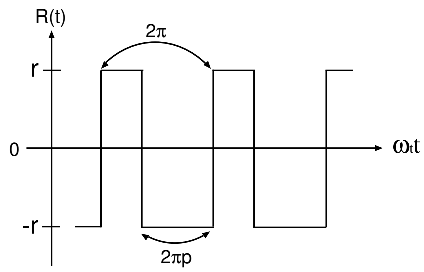


Fig. 2. Characteristics of the TVR.

$$\varepsilon = g_1\sqrt{\frac{L}{C}},\ \gamma = r\sqrt{\frac{C}{L}},\ \omega = \frac{1}{\sqrt{LC}}\omega_t,$$

the normalized circuit equations are given as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon(1 - {x_k}^2) - y_k \\ \dfrac{dy_k}{d\tau} = x_k \pm \gamma(\tau)\displaystyle\sum_{j=1}^{2} y_j \end{cases} \quad (k = 1, 2) \qquad (2)$$

where the sign of the coupling term changes according to the value of the time-varying resistor.

III. Synchronization Phenomena [13]

We observed that the two coupled oscillators are synchronized in in-phase and anti-phase as shown in Fig. 3. These two synchronization states can be obtained by giving different initial conditions.

We have investigated the influence of the duty ratio of the switching, namely we vary the ratio of time intervals connecting to the positive and the negative resistors. Figure 4 shows the phase difference between both oscillators in dependence on the duty ratio p. We set the parameters as $\varepsilon = 2.0$, $\gamma = 0.1$ and $\omega = 2.0$.

From this figure, we can see the hysteresis of the synchronization phenomena. First, for p smaller than 0.46, the two coupled oscillators are synchronized only in anti-phase. Second, when the p is between

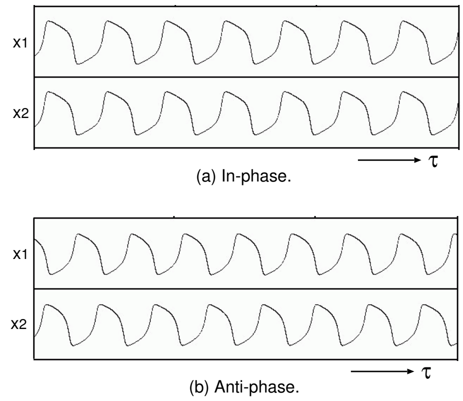


Fig. 3. Time waveform of two synchronization states (computer calculated results). $\varepsilon = 2.0, \gamma = 0.1, p = 0.5$, $\omega = 1.5$. (a) In-phase synchronization. (b) Anti-phase synchronization.

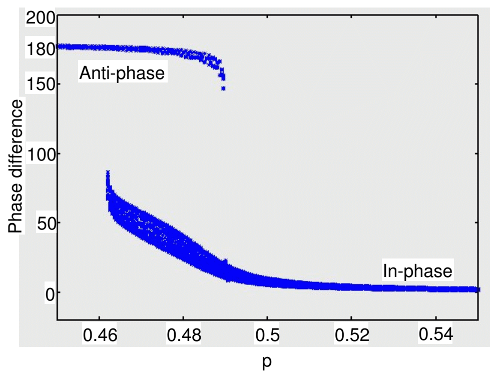


Fig. 4. Synchronization in dependence on p ($\varepsilon = 2.0$, $\gamma = 0.1$, $\omega = 2.0$).

0.46 and 0.492, the coexistence of the in-phase and the anti-phase synchronizations can be observed. Finally, for p larger than 0.492, only the in-phase synchronization can be confirmed.

IV. Switching Duty Ratio by Logistic Map

We investigate the phase difference between the oscillators when the duty ratio of the TVR changes between smaller and larger values. We assume that the switching duty ratio p is determined by the value of the logistic map as an external signal. The logistic map is defined by the following equation.

$$z_{n+1} = \alpha z_n(1.0 - z_n). \qquad (3)$$

Varying parameter α, Eq. (3) behaves chaotically via a periodic-doubling cascade. We transform the value of the logistic map into the switching duty ratio p by the following equation:

$$p_n = \frac{1}{3}(z_n + 0.85). \qquad (4)$$

Further, we assume that the duty ratio p_n is fixed during N periods.

In the following sections, we set the parameters of the circuit as $\varepsilon = 2.0$, $\gamma = 0.1$ and $\omega = 2.0$.

V. Periodic and Chaotic Switching

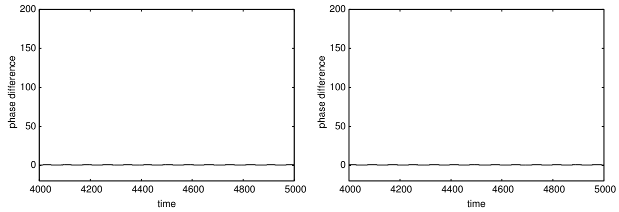
Figure 5 shows the phase differences when the logistic map is one-periodic. The phase difference is constant. This is because the duty ratio, which is determined by the value of the logistic map, is also constant. Figures 6 and 7 show the phase differences when the logistic map is two-periodic and four-periodic, respectively. From these figures, we can confirm that the phase difference oscillates as almost one or two periodic according to the change of the duty ratio. Further, when the period N becomes large, the amplitude of the oscillation of the phase difference becomes large, and the changing speed of the phase difference becomes slow. Further, the phase differences when the logistic map is chaotic are shown in Fig. 8. We can see that the phase difference between the two coupled oscillators behaves chaotically.

VI. Intermittency and Fully-Developed Chaotic Switching

Next, we compare the phenomena for the cases that the logistic map is three-periodic, intermittency chaos near the three periodic window and fully-developed chaos. We consider the three types of intermittency chaos near the three periodic window. Namely, the laminar part of the intermittency chaos is long, middle and short. The simulated results are shown in Fig. 9. From these figures, we can see the different behaviors of the phase differences by changing the switching duty ratio. First, for the case that the laminar part is long (Fig. 9(b)), the behavior is similar to the three periodic (Fig. 9(a)). Secondly, for the case that the laminar part is short (Fig. 9(d)), the behavior is similar to the fully-developed chaos (Fig. 9(e)). Finally, for the case that the laminar part is middle (Fig. 9(c)), the behavior includes both the three periodic and the fully-developed chaos. In that case, the changing of the phase difference is very complicated.

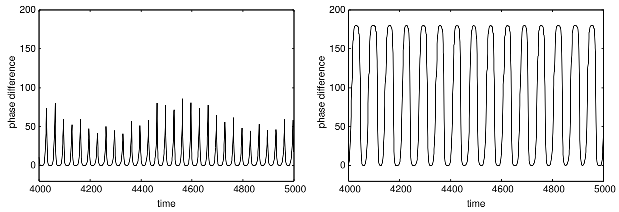
VII. Conclusions

In this study, we have investigated phase differences in two van der Pol oscillators coupled by a time-varying resistor when the duty ratio of the time-varying resistor varies chaotically. We realized the time-varying resistor by switching a positive and a negative resistors using the value of the logistic map. By carrying out computer calculations, we confirmed that the synchronization phenomena of the two coupled oscillators depended on the switching duty ratio. Namely, for the case of the intermittency chaos, the synchronization had coexistence of the three period and the fully-developed chaos. The varying phase difference was very complex.



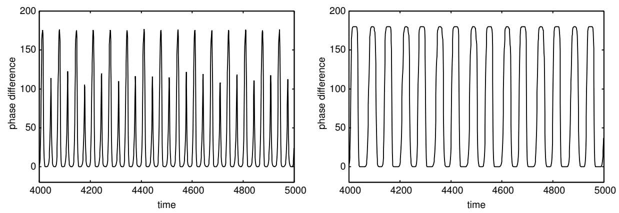
(a) N=200. (b) N=400.

Fig. 5. Phase differences for 1-periodic duty ratio ($\alpha = 2.9$).



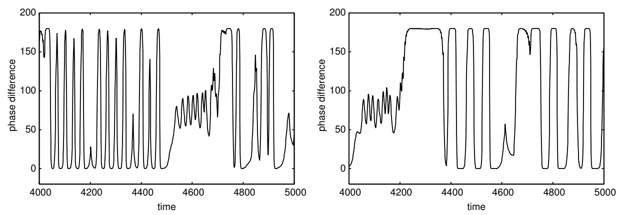
(a) N=200. (b) N=400.

Fig. 6. Phase differences for 2-periodic duty ratio ($\alpha = 3.2$).



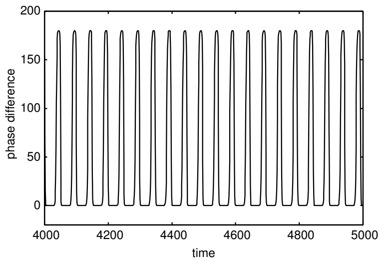
(a) N=200. (b) N=400.

Fig. 7. Phase differences for 4-periodic duty ratio ($\alpha = 3.5$).

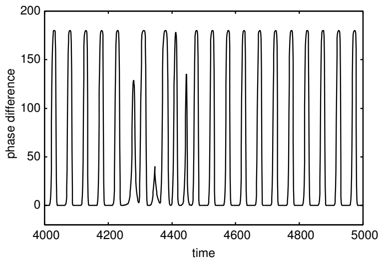


(a) N=200. (b) N=400.

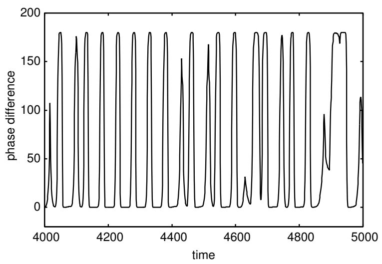
Fig. 8. Phase differences for chaotic duty ratio ($\alpha = 3.68$).



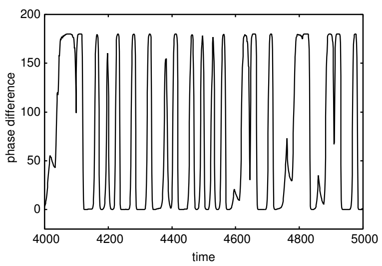
(a) 3 periodic ($\alpha = 3.835$).



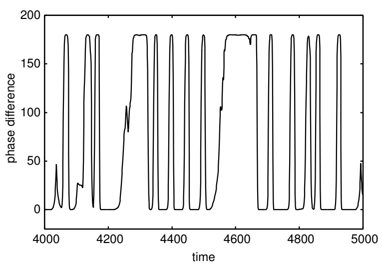
(b) Intermittency chaos: laminar part is long. ($\alpha = 3.8284$).



(c) Intermittency chaos: laminar part is middle. ($\alpha = 3.8260$).



(d) Intermittency chaos: laminar part is short. ($\alpha = 3.8250$).



(e) Fully-developed chaos ($\alpha = 3.99$).

Fig. 9. Comparison of intermittency chaos and fully-developed chaos.

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