Performance of Chaotic Switching Noise Injected to Hopfield NN for Quadratic Assignment Problem

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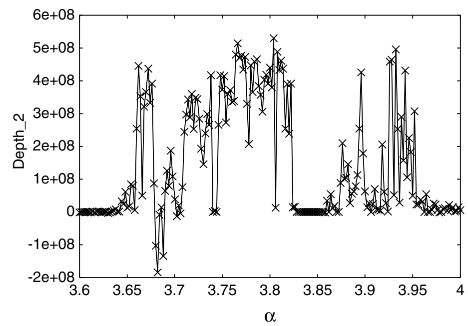
***Abstract*— Solving combinatorial optimization problems is one of the important applications of the neural network. Many researchers have reported that exploiting chaos achieves good solving ability. However, the reason of the good effect of chaos has not been clarified yet. In this study, we investigate a performance of chaotic switching noise injected to the Hopfield neural network for quadratic assignment problems. By computer simulation we confirm that the chaotic switching noise is effective for solving quadratic assignment problems as well as intermittent chaos near three-periodic window.**

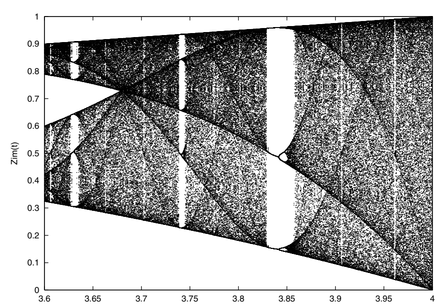
I. Introduction

Combinatorial optimization problems can be solved with the Hopfield neural networks (abbr. NN). If we choose connection weights between neurons appropriately according to given problems, we can obtain a good solution by the energy minimization principle. However, the solutions are often trapped into a local minimum and do not reach the global minimum. In order to avoid this critical problem, several people proposed the method adding some kinds of noise for solving traveling salesman problems (TSP) with the Hopfield NN [1]. Hayakawa and Sawada pointed out the chaos near the three-periodic window of the logistic map gains the best performance [2]. They concluded that the good result might be obtained by a property of the chaos noise; short time correlations of the time-sequence. Hasegawa et al. investigated solving abilities of the Hopfield NN with various surrogate noise, and they concluded that the effects of the chaotic sequence for solving optimization problems can be replaced by stochastic noise with similar autocorrelation [3]. We have also studied the reason of the good performance of the Hopfield NN with chaotic noise. We imitated the intermittency chaos noise by the burst noise generated by the Gilbert model [4] with 2 states; a laminar state and a burst state. We concluded that the irregular switching of laminar part and burst part is one of the reasons of the good performance of the chaotic noise [5][6].

Further, we have investigated the performance of chaos noises when the control parameter of the logistic map is changed, and the intermittency chaos near the three-periodic window of the logistic map is not only special chaos noise gaining a good performance as shown in Fig. 1 [7]. For example, the chaos noise achieves a good performance around $\alpha = 3.67$. At this point, the attractor of the logistic map bifurcates from two-band chaos to one-band chaos via an interior crisis. In the two-band chaos, the solution moves between the upper band and the lower band alternately. Namely, the solution has a kind of regularity even if it is chaotic in each band. However, the crisis makes the two bands to merge into one band and the switching between the upper band and the lower band becomes irregular. We consider that the edge behavior between regular and irregular gains a good performance.



(a) Performance of chaos noises for QAP.



(b) Bifurcation diagram of logistic map.

Fig. 1. Performance of chaos noises when the control parameter is changed.

In this study, we investigate a performance of chaotic switching noise, which is generated by a cubic map, injected to the Hopfield NN for quadratic assignment problems (abbr. QAP). By computer simulation we confirm that the chaotic switching noise is effective for solving QAP as well as the intermittent chaos noise near the three-periodic window.

II. Solving QAP with Hopfield NN

Various methods are proposed for solving the QAP which is one of the NP-hard combinatorial optimization problems. The QAP is expressed as follow: given two matrices, distance matrix $\mathbf{C}$ and flow matrix $\mathbf{D}$, and find the permutation $\mathbf{p}$ which corresponds to the minimum value of the objective function $f(\mathbf{p})$ in Eq. (1).

$$f(\mathbf{p}) = \sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij} D_{p(i)p(j)}, \quad (1)$$

where C_{ij} and D_{ij} are the (i,j)-th elements of $\mathbf{C}$ and $\mathbf{D}$, respectively, $p(i)$ is the i th element of vector $\mathbf{p}$, and N is the size of the problem. There are many real applications which are formulated by Eq. (1). One example of QAP is to find an arrangement of the factories to make a cost the minimum. The cost is given by the distance between the factories and flow of the products between the factories. Other examples are the placement of logical modules in a IC chip, the distribution of medical services in large hospital, and so on.

Because the QAP is very difficult, it is almost impossible to solve the optimum solutions in large problems. The largest problem which is solved by deterministic methods may be only N = 36 in recent study [8]. Further, computation time is very long to obtain the exact optimum solution. Therefore, it is usual to develop heuristic methods which search nearly optimal solutions in reasonable time.

For solving N-element QAP by the Hopfield NN, $N \times N$ neurons are required and the following energy function is defined to fire (i,j)-th neuron at the optimal position:

$$E = \sum_{i,m=1}^{N}\sum_{j,n=1}^{N} w_{im;jn} x_{im} x_{jn} + \sum_{i,m=1}^{N} \theta_{im} x_{im}, \quad (2)$$

The neurons are coupled each other with weight between (i,m)-th neuron and (j,n)-th neuron and the threshold of the (i,m)-th neuron are described by:

$$w_{im;jn} = -2\Big\{ A(1-\delta_{mn})\delta_{ij} + \beta\delta_{mn}(1-\delta_{ij}) + \frac{C_{ij}D_{mn}}{q} \Big\} \quad (3)$$

$$\theta_{im} = A + B \quad (4)$$

where A and B are positive constants, and δ_{ij} is Kroneker's delta. The state of $N \times N$ neurons are asynchronously update due to the following difference equation:

$$x_{im}(t+1) = f\left(\sum_{j,n=1}^{N} w_{im;jn} x_{im}(t) x_{jn}(t) - \theta_{im} + \beta u_{im}(t) \right) \quad (5)$$

where f is sigmoidal function defined as follows:

$$f(x) = \frac{1}{1+\exp\left(-\frac{x}{\varepsilon}\right)} \quad (6)$$

u_{im} is additional noise, and β limits amplitude of the noise.

III. Noise Generation

A. Chaotic Switching Noise

In this section, we describe chaotic switching noise injected to the Hopfield NN.

The following cubic map is used to generate the chaotic switching noise.

$$\hat{y}_{im}(t+1) = -\hat{y}_{im}(t)(\alpha_c \hat{y}_{im}^2(t) + 1 - \alpha_c) \quad (7)$$

Figure 2 shows the shape of the cubic map. The one-parameter bifurcation diagram of this map is shown in Fig. 3. The attractor becomes symmetric at around $\alpha_c = 3.600$ via an interior crisis. Because the transition of the solution from the positive/negative part to the other part is seldom just after the crisis, the behavior looks like an irregular switching as Fig. 4(a). As the parameter increases, the transition becomes more frequent as Figs. 4(b) and (c). For $\alpha_c = 4.000$, the time series become similar to a uniform random noise as Fig. 4(d).

We use the time series in Figs. 4(a), (b) and (c) as chaotic switching noises after the following normalization.

$$y_{im}(t) = \frac{\hat{y}_{im}(t) - \bar{y}}{\sigma_y} \quad (8)$$

where $\bar{y}$ and σ_y are the average and the standard deviation of $\hat{y}(t)$, respectively.

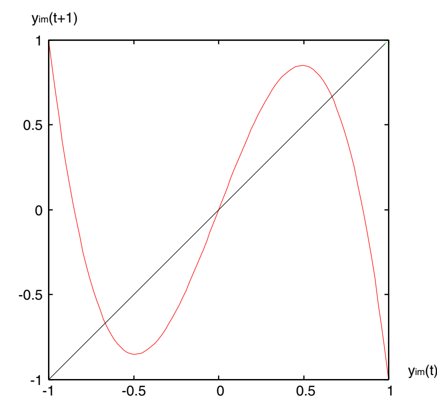


Fig. 2. Cubic map ($\alpha_c = 3.600$).

B. Intermittency Chaos Noise

We have confirmed that the intermittency chaos near the three periodic window has the good performance to the Hopfield NN for combinatorial optimization problems. In order to evaluate the performance of the chaotic switching noise in the last subsection, we carry out the same simulation when the intermittency chaos is injected to the Hopfield NN as a noise.

The logistic map is used to generate the intermittency chaos noise.

$$\hat{z}_{im}(t+1) = \alpha_l \hat{z}_{im}(t)(1 - \hat{z}_{im}(t)). \quad (9)$$

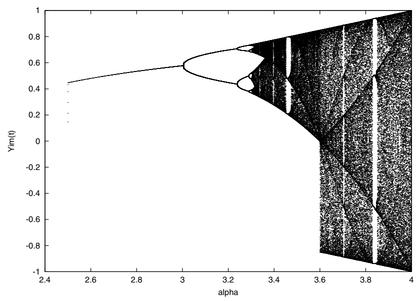


Fig. 3. Bifurcation of cubic map.

Varying parameter α_l, Eq. (9) behaves chaotically via a periodic-doubling cascade. Further, it is well known that the map produces intermittent bursts just before periodic-windows appear. Figure 5(a) shows an example of the intermittency chaos near the three-periodic window obtained from Eq. (9) for $\alpha_l = 3.82676$. As we can see from the figure, the chaotic time series could be divided into two phases; laminar parts of periodic behavior with period three and burst parts of spread points over the invariant interval. As increasing α_l, the ratio of the laminar parts becomes larger and finally the three-periodic window appears. Just for the comparison, we also carry out computer simulations for the case of fully developed chaos in Fig. 5(b) which is obtained from Eq. (9) for $\alpha_l = 4.0000$. This time series is similar to a uniform random noise as well as the cubic map for $\alpha_c = 4.0$.

When we inject the intermittency chaos noise to the Hopfield NN, we normalize $\hat{z}_{im}$ by Eq. (10).

$$z_{im}(t) = \frac{\hat{z}_{im}(t) - \bar{z}}{\sigma_z} \tag{10}$$

where $\bar{z}$ and σ_z are the average and the standard deviation of $\hat{z}(t)$, respectively.

C. Regular Switching Noise

For comparison, we compose a regular switching noise.

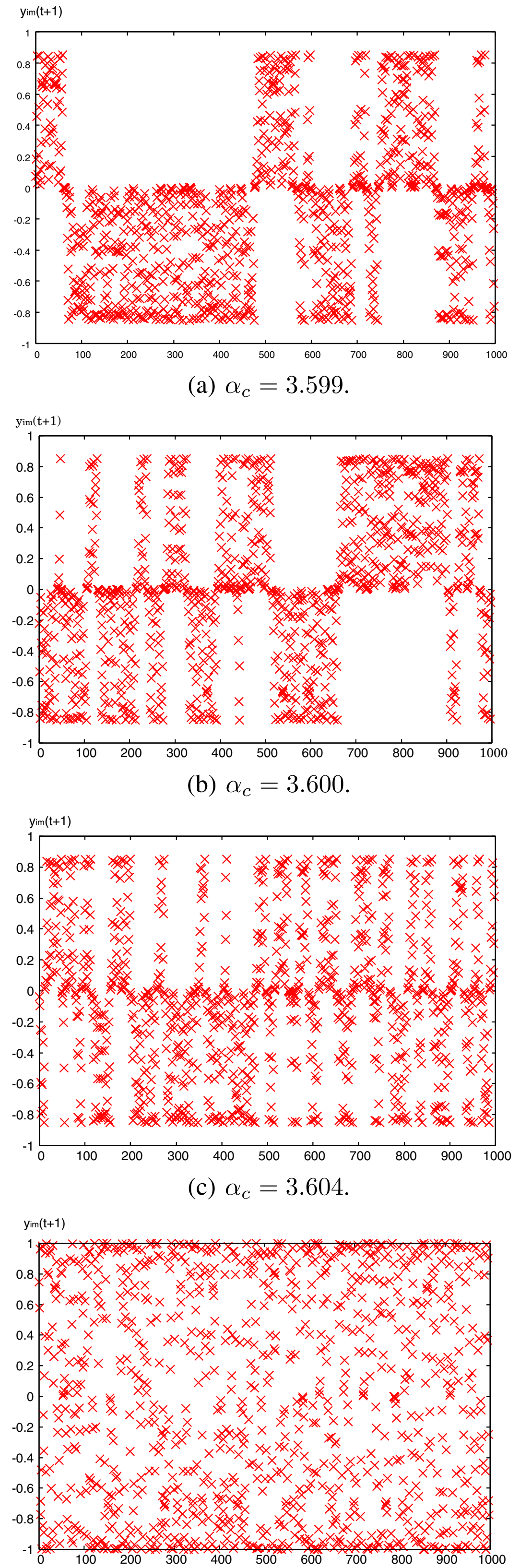
We use the time series of the cubic map (7) just before the interior crisis ($\alpha_c = 3.598$). Because the generated time series do not transit to the negative part before the crisis, we change the sign of the sequence in every L iterations. An example of the regular switching noise is shown in Fig. 6.

This noise is also normalized in a similar way before the injection.

IV. Simulated Results

In this section, the simulated results of the Hopfield NN with the chaotic switching noise, the intermittency chaos noise and the regular switching noise for 12-element QAP are shown. The problem used here was chosen from the site QAPLIB [8] named "Nug12." The global minimum of this target problem is known as 578. The parameters of the Hopfield NN are as $A = 0.9$, $B = 0.9$, $q = 140$, $\varepsilon = 0.02$ and the amplitude of the injected noise is fixed as $\beta = 0.55$.

Table 1 shows simulation results. We carried out each simulation 10 times with different initial conditions and calculated



(a) $\alpha_c = 3.599$.

(b) $\alpha_c = 3.600$.

(c) $\alpha_c = 3.604$.

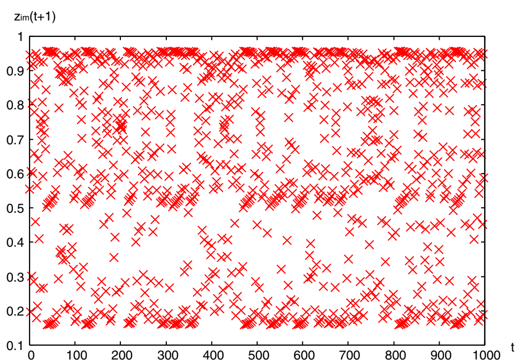
(d) $\alpha_c = 4.000$.

Fig. 4. Time series obtained from cubic map.

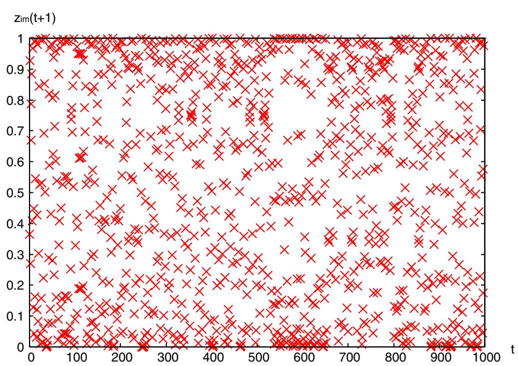
TABLE I

SOLVING ABILITIES FOR 12-ELEMENT QAP.

Iteration	Cubic map				Logistic map		Regular switching			
	Chaotic switching			full	Intermittency chaos	full				
	α_c=3.599	α_c=3.600	α_c=3.604	α_c=4.000	α_l=3.82676	α_l=4.00000	L=50	L=100	L=200	L=500
1000	635.2	632.4	630.0	642.6	623.4	630.2	651.8	654.4	684.2	691.8
2000	627.0	625.4	626.8	642.6	613.4	622.8	634.6	641.6	662.8	669.0
3000	627.0	623.0	625.0	642.6	613.4	622.8	634.2	637.4	651.6	663.8
4000	626.0	615.8	625.0	642.6	607.8	622.8	629.6	631.2	645.8	658.6
5000	621.2	615.6	625.0	642.6	607.8	622.8	624.6	629.6	633.8	652.4
6000	619.6	614.6	624.8	642.6	607.8	622.8	624.6	626.4	633.8	646.0
7000	616.8	614.0	622.6	642.6	607.0	622.8	622.8	625.8	631.4	641.4
8000	616.6	608.8	621.0	642.6	604.0	622.8	622.8	625.8	630.6	640.0
9000	616.6	608.8	621.0	642.6	604.0	622.8	622.8	625.0	630.6	640.0
10000	615.6	**606.6**	621.0	642.6	**604.0**	622.8	621.6	624.6	625.6	622.0



(a). $\alpha_l = 3.82676$.



(b). $\alpha_l = 4.00000$.

Fig. 5. Time series obtained from logistic map.

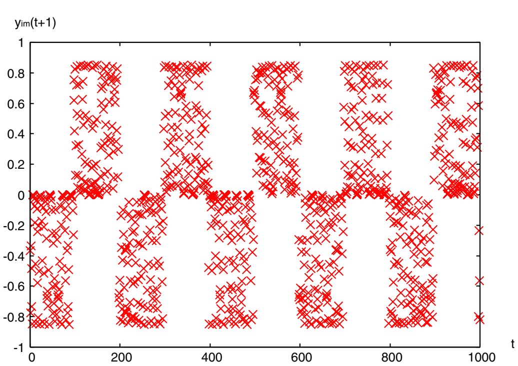


Fig. 6. Regular switching noise ($L = 50$).

the average value. The results show that the chaotic switching noise for $\alpha_c = 3.600$ gains a good performance similar to the intermittency chaos noise for $\alpha_l = 3.82676$. We can also see that the regular switching noise and fully-developed chaos cannot achieve a good performance.

V. CONCLUSIONS

In this study, we have investigated a performance of chaotic switching noise injected to the Hopfield NN for QAP. By computer simulation we confirmed that the chaotic switching noise is effective for solving QAP similar to the intermittency chaos near the three-periodic window.

A positive part and a negative part repeat alternately in the chaotic switching noise. Hence, this noise can be said to be less chaotic than fully-developed chaos. By combining the result in this study with our past study explaining the good performance of the intermittency chaos, we conclude that the edge behavior between regular and irregular can play an important role in information processing.

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