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Synchronization of Chaotic Circuits with Transmission Line

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Abstract

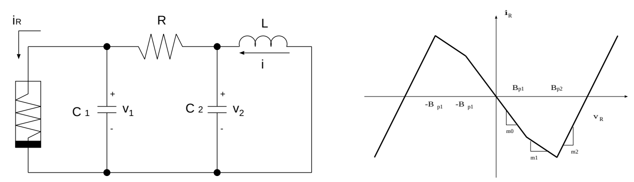
In this study, two Chua's circuits with lossless transmission lines placed in parallel are investigated. The effect of the crosstalk is modeled by the connections via coupling capacitors or mutual inductors. By computer simulations, we confirm that two Chua's circuits are synchronized in in-pahse or anti-phase by the crosstalk effect of the transmission lines.

1. Introduction

Recently several researchers have investigated synchronization of chaotic circuits. Chua's circuit is well known as a simple chaotic circuit. Chua's circuit consists of one resistor, one capacitor, one nonlinear resistor called Chua's diode, and an LC resonator. The $v-i$ characteristics of the nonlinear resistor in Chua's circuit are piecewise-linear as shown in Figure **??** and are described by the following equation.

$$i_R = m_2 v_1 + \frac{1}{2}(m_0 - m_1)\{|v_1 + B_{p1}| - |v_1 - B_{p1}|\} + \frac{1}{2}(m_1 - m_2)\{|v_1 + B_{p2}| - |v_1 - B_{p2}|\} \quad (1)$$

where m_0, m_1 and m_2 are the slopes in the segments of the peacewise-linear function, and B_{p1} and B_{p2} denote the break-points.



(a) (b)

Figure 1: (a) Chua's circuit. (b) $v-i$ characteristics of non-linear resistor.

There are many studies about coupled Chua's circuits, for example, master-slave coupling of Chua's circuits [1], mutual coupling of Chua's circuits [2], a ladder of Chua's circuits, a ring of Chua's circuits [3], two-dimensional array of Chua's circuits [4] [5], etc. However, almost researches consider the couplings by lumped elements. We have investigated chaotic phenomena observed from Chua's circuit when the LC resonator is replaced by a transmission line [6]. Further, we have also investigated synchronization phenomena of two Chua's circuits coupled by a transmission line [7] [8].

In this study, we investigate synchronization of two Chua's circuits with lossless transmission lines coupled by the crosstalk of the transmission lines.

2. Circuit Model

In this study we consider two Chua's circuits with lossless transmission lines placed in parallel as shown in Figure **??**. We consider that the two circuits influence each other by the

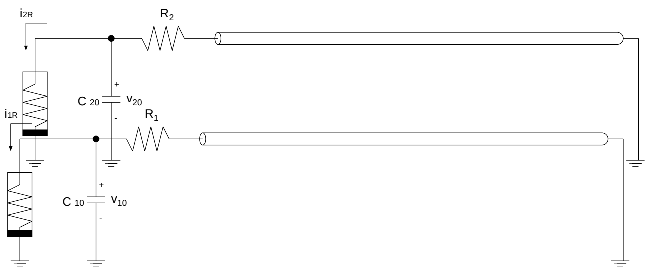


Figure 2: Two Chua's circuits with transmission lines.

effect of the cross talk of transmission lines.

In order to investigate the influence, we model the transmission lines placed in parallel by LC ladder circuits with finite numbers of lumped elements.

2.1. Circuit Model 1 (Crosstalk via Capacitors)

First, we model the transmission lines placed in parallel by the circuit shown in Fig. **??**. In this model, the effect of the crosstalk is given by the connections via coupling capacitors.

The circuit equations of this model can be derived as fol-

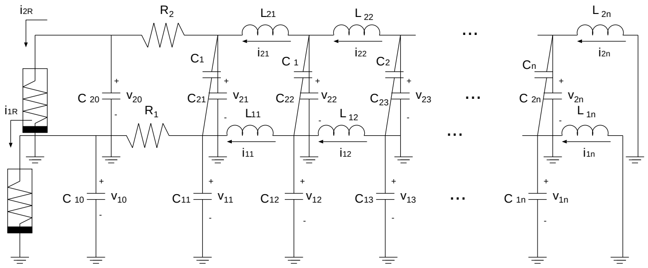


Figure 3: Circuit Model 1.

lows:

$$
\begin{aligned}
C_{j0}\frac{dv_{j0}}{dt} &= \frac{v_{j1}-v_{j0}}{R_j} - i_{Rj}(v_{j0}) \\
C_{j1}\frac{dv_{j1}}{dt} &= i_{j1} - \frac{v_{j1}-v_{j0}}{R_j} + C_1\frac{d(v_{j1}-v_{(j+1)1})}{dt} \\
C_{jk}\frac{dv_{jk}}{dt} &= i_{jk} - i_{j(k-1)} + C_k\frac{d(v_{jk}-v_{(j+1)k})}{dt} \\
L_{jl}\frac{di_{jl}}{dt} &= v_{j(l+1)} - v_{jl}
\end{aligned} \tag{2}
$$

where $v_{31} = v_{11}$, $v_{j(n+1)} = 0$, (k=2, 3, ..., n), (l=1, 2, ..., n) and (j=1, 2).

From (1) and (2), we can obtain the normalized circuit equations as follows:

$$
\begin{aligned}
\dot{x_{10}} &= x_{11} - x_{10} - f(x_{10}) \\
\dot{x_{11}} &= \alpha_{11}(y_{11} - x_{11} + x_{10}) - \beta_{11}(y_{21} - x_{21} + x_{20}) \\
\dot{x_{1k}} &= \alpha_{1k}(y_{1k} - y_{1(k-1)}) - \beta_{1k}(y_{2k} - y_{2(k-1)}) \\
\dot{x_{20}} &= \zeta(x_{21} - x_{20} - f(x_{20})) \\
\dot{x_{21}} &= \alpha_{21}(y_{21} - \zeta(x_{21} - x_{10}) - \beta_{21}(y_{11} - x_{11} + x_{10}) \\
\dot{x_{2k}} &= \alpha_{2k}(y_{2k} - y_{2(k-1)}) - \beta_{2k}(y_{1k} - y_{1(k-1)}) \\
\dot{y_{jl}} &= \gamma_{jl}(x_{j(l+1)} - x_{jl})
\end{aligned} \tag{3}
$$

where $x_{j(n+1)} = 0$,

$$
\begin{aligned}
f(x_{j0}) &= c_j x_{j0} + \frac{1}{2}(a_j - b_j)(|x_{j0}+1| - |x_{j0}-1|) \\
&\quad + \frac{1}{2}(b_j - c_j)(|x_{j0}+d_j| - |x_{j0}-d_j|),
\end{aligned} \tag{4}
$$

$$t = R_1\tau, \quad \text{“}\cdot\text{”} = \frac{d}{d\tau}, \quad v_{jk} = B_{p1}x_{jk}, \quad i_{jl} = \frac{B_{p1}}{R_1}y_{jl},$$

$$\alpha_{jk} = \frac{C_{jk}C_{10}(C_{(j+1)k} - C_k)}{-C_k^2 - (C_{jk}^2+1)C_k + C_{jk}^2 C_{(j+1)k}},$$

$$\beta_{jk} = \frac{C_{jk}C_{10}C_k}{-C_k^2 - (C_{jk}^2+1)C_k + C_{jk}^2 C_{(j+1)k}},$$

$$\gamma_{jl} = \frac{R_1^2 C_{j0}}{L_{jl}},$$

$$\zeta = \frac{C_{10}}{C_{20}}, \quad a_j = R_j m_{j0}, \quad b_j = R_j m_{j1},$$

$$c_j = R_j m_{j2}, \quad d_j = \frac{B_{pj2}}{B_{pj1}},$$

$C_{3k} = C_{1k}$, (k=2, 3, ..., n), (l=1, 2, ..., n) and (j=1, 2).

2.2. Circuit Model 2 (Crosstalk via Mutual Inductors)

Next, we model the transmission lines placed in parallel by the circuit shown in Figure **??**. In this model, the effect of the crosstalk is given by the connections via mutual inductors. The normalized circuit equations of this model can be derived

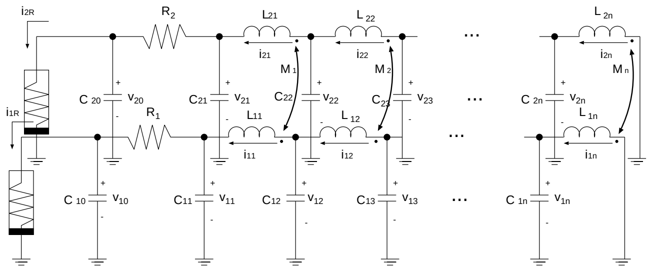


Figure 4: Circuit Model 2.

as follows:

$$
\begin{aligned}
\dot{x_{10}} &= x_{11} - x_{10} - f(x_{10}) \\
\dot{x_{11}} &= \alpha_{11}(y_{11} - (x_{11} - x_{10})) \\
\dot{x_{1k}} &= \alpha_{1k}(y_{1k} - y_{1(k-1)}) \\
\dot{x_{20}} &= \zeta(x_{21} - x_{20} - f(x_{20})) \\
\dot{x_{21}} &= \alpha_{21}(y_{21} - \zeta(x_{21} - x_{10})) \\
\dot{x_{2k}} &= \alpha_{2k}(y_{2k} - y_{2(k-1)}) \\
\dot{y_{jl}} &= \gamma_{jl}(x_{j(l+1)} - x_{jl}) + \beta_l(x_{(j+1)(l+1)} - x_{jl})
\end{aligned} \tag{5}
$$

where $x_{j(n+1)} = 0$, $x_{3(l+1)} = x_{1(l+1)}$, and

$$\alpha_{jk} = \frac{C_{10}}{C_{jk}},$$

$$\beta_l = \frac{R_1^2 C_{10} M_l}{(L_{2l} - M_l)(L_{1l} - M_l) - M_l^2},$$

$$\gamma_{jl} = \frac{R_1^2 C_{10}(L_{(j+1)l} - M_l)}{(L_{2l} - M_l)(L_{1l} - M_l) - M_l^2},$$

where $L_{3l} = L_{1l}$, (k=2, 3, ..., n), (l=1, 2, ..., n) and (j=1, 2).

3. Simulation Results

We carry out computer simulation for the above-mentioned two models of the circuit by using the 4th order Runge-Kutta method. In the simulations, we consider only the case that the two Chua's circuits are identical. Hence, we rewrite the parameters as

$$
\begin{aligned}
&\alpha_1 = \alpha_2 = \alpha, \quad \beta_1 = \beta_2 = \beta, \quad \gamma_1 = \gamma_2 = \gamma, \\
&a_1 = a_2 = a, \quad b_1 = b_2 = b, \\
&c_1 = c_2 = c, \quad d_1 = d_2 = d.
\end{aligned} \tag{6}
$$

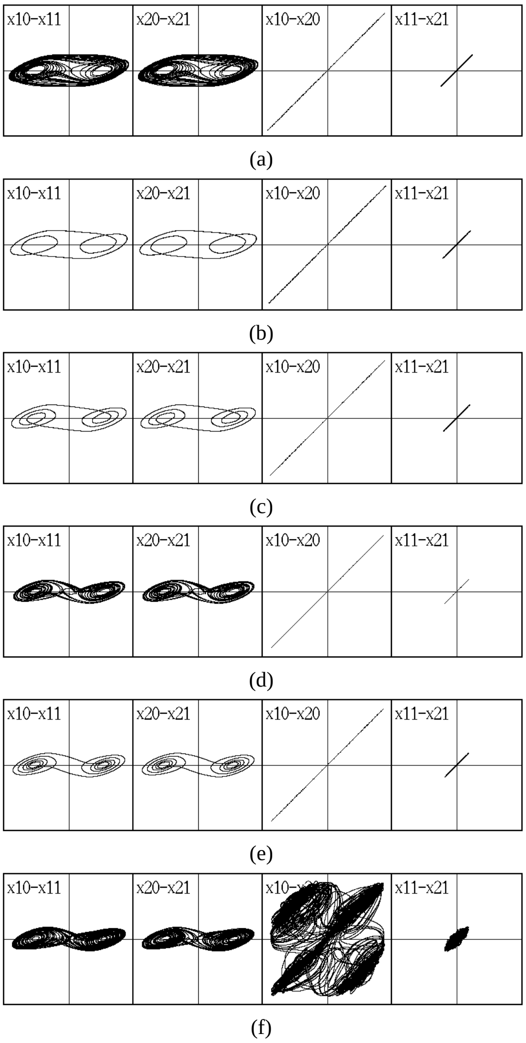


Figure 5: Synchronization phenomena of Circuit Model 1 (γ=1.0). (a) β=0.150, (b) β=0.128, (c) β=0.115, (d) β=0.092, (e) β=0.0905, (f) β=0.0900.

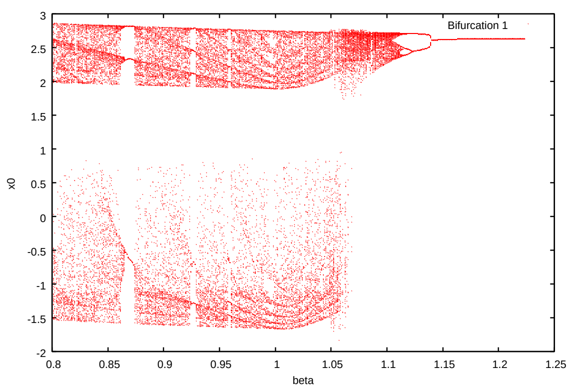


Figure 6: Bifurcation diagram of Circuit Model 1 (β=0.1).

In the following simulations, we fix the parameters as follows:

$$a = -1.2,\ \ b = -0.75,\ \ c = 10,\ \ d = 8,\ \ \alpha = 16. \qquad (7)$$

3.1. Results of Circuit Model 1

Figure **??** shows the simulation results of the Circuit Model 1. In this simulation, we vary β, namely, the coupling strength.

We can confirm that the two circuits synchronize in in-phase even when each circuit produce the double scroll attractor. However, for a relatively small coupling parameter value, the synchronization breaks down as shown in Fig. **??**(f). Figure **??** shows the one-parameter bifurcation diagram when γ is chosen as a control parameter. Namely, in this figure, the coupling strength is fixed and nonlinear strength of each circuit is changed. The horizontality axis is γ and vertical axis is x_{10} when the solution hits the hyperplane X_{11}=0. From Fig. **??**, we can see that the bifurcation is similar to that of the original Chua's circuit.

3.2. Results of Circuit Model 2

Figure **??** shows the simulation results of the Circuit Model 2. In this simulation, we vary β, namely, the coupling strength.

Interestingly, in this case, we can confirm that the two circuits synchronize in anti-phase. Figure **??** shows the one-parameter bifurcation diagram when γ is chosen as a control parameter. From Figure **??**, we can say that the bifurcation is similar to the case of the capacitor coupling.

4. Conclusions

In this study, we have investigated two Chua's circuits with lossless transmission lines placed in parallel. We have modeled the effect of the crosstalk by the connections via coupling capacitors or mutual inductors. By computer simulations, we have confirmed that two Chua's circuits are synchronized in in-pahse or anti-phase by the crosstalk effect of the transmission lines.

In our future work, we investigate Chua's circuit with lossy transmission line, because real transmission lines should have loss actually. Furthermore, we investigate crosstalk phenomena between conductor boards placed in parallel and apply the result to chaotic circuits.

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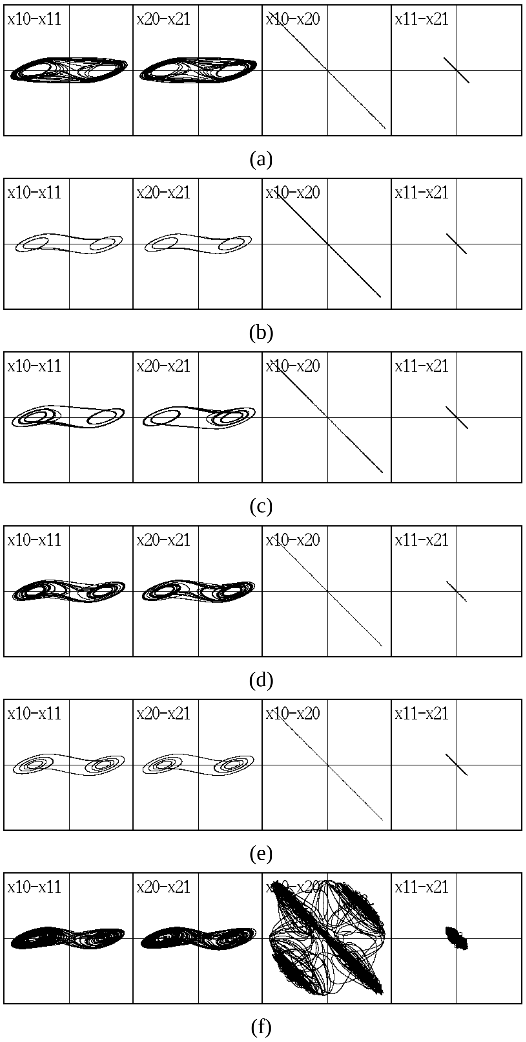

Figure 7: Synchronization phenomena of Circuit Model 2 (γ=1.0). (a) β=4.9, (b) β=4.7, (c) β=4.3, (d) β=4.0, (e) β=3.8, (f) β=3.5.

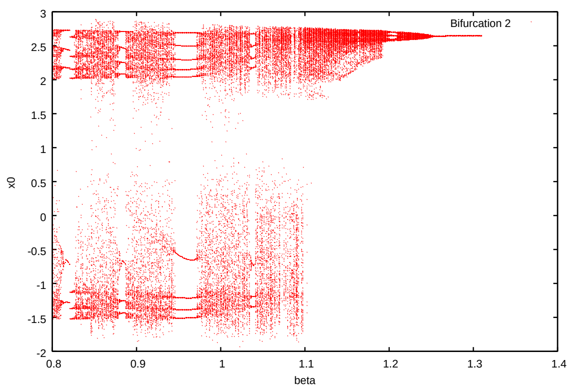

Figure 8: Bifurcation diagram of Circuit Model 2 (β=3.5).

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