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Solving Ability of Hopfield Neural Network with Mix Noise for Quadratic Assignment Problem

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Abstract

Solving combinatorial optimization problem is one of the important applications of neural network (abbr. NN). However, the solutions are often trapped into a local minimum and do not reach the global minimum. In order to avoid this critical problem, several people proposed the method adding some kinds of noise.

In this study, we consider torus noise generated by the sine circle map for the Hopfield NN. By computer simulations, solving abilities of Hopfield NN for quadratic assignment problem (QAP) with various kinds of noises based on the torus noise are investigated.

1. Introduction

The use of Hopfield NN in broad fields is expected, such as combinatorial optimization problem, associative memory, and pattern recognition. If we choose connection weights between neurons appropriately according to given problems, we can obtain a good solution by the energy minimization principle. However, the solution are often trapped into a local minimum and do not reach the global minimum. In order to avoid this critical problem, several people proposed the method adding some kind of noise for solving traveling salesman problem (TSP) with the Hopfield NN [1]. Hayakawa and Sawada pointed out the chaos near the three-periodic window of the logistic map gains the best performance [2]. They concluded and that the good result might be obtained by a property of the chaos noise; short time correlations of the time-sequence. Hasegawa et al. investigated solving abilities of the Hopfield NN with various surrogate noise, and they conclude that the effects of the chaotic sequence for solving optimization problems can be replaced by stochastic noise with autocorrelation [3]. We have also studies the reason of the good performance with Hopfield NN with chaotic noise. We imitated the intermittency chaos noise by the burst noise generated by Gilbert model [4] with 2 states; a laminar state and a burst state. We conclude that the irregular switching of laminar part and burst part is one of the reasons of the good performances of the chaotic noise [5][6].

In this study, we consider torus noise generated by the sine circle map. We tune the parameters of the sine circle map to generate torus including intermittent feature. Further, we propose a mix noise, which is a mixture of the intermittent torus and chaos/random noise, in order to investigate the performance of the noise including intermittet feature. By computer simulations, solving abilities of Hopfield NN for QAP with various kinds of noises based on the torus noise are investigated.

2. Solving QAP with Hopfield NN

Various methods are proposed for solving QAP which is one of the NP-hard combinatorial optimization problems. QAP is expressed as follows: given two matrices, distance matrix **C** and flow matrix **D**, and find the permutation **P** which corresponds to the minimum value of the objective function $f(\mathbf{P})$ in (1).

$$f(\mathbf{P}) = \sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij} D_{P(i)P(j)}, \tag{1}$$

where C_{ij} and D_{ij} are the (i,j)-th elements of **C** and **D**, respectively, $P(i)$ is the i-th element of the vector **P**, and N is the size of the problem. There are many real applications which are formulated by (1). One example of QAP is to find an arrangement of the factories to make a cost the minimum. The cost is given by the distance between the factories and flow of the products between the factories. Other examples are the placement of logical modules in an IC chip, the distribution of medical services in large hospital, and so on.

Because QAP is very difficult, it is almost impossible to solve the optimum solutions in large problems. The largest problem which is solved by deterministic methods may be only 24 in recent study. Further, computation time is very long to obtain the exact optimum solution. Therefore, it is usual to develop heuristic methods which search nearly optimal solutions in reasonable time.

For solving N-element QAP by Hopfield NN, $N \times N$ neurons are required and the following energy function is defined to fire (i, j)-th neuron at the optimal position:

$$E = \sum_{i,m=1}^{N} \sum_{j,n=1}^{N} w_{im;jn} x_{im} x_{jn} + \sum_{i,m=1}^{N} \theta_{im} x_{im}. \tag{2}$$

The weight between (i, m)-th neuron and (j, n)-th neuron and the threshold of the (i, m)-th neuron are described by:

$$w_{im;jn} = -2\left\{ A(1-\delta_{mn})\delta_{ij} + \beta\delta_{mn}(1-\delta_{ij}) + \frac{C_{ij}D_{mn}}{q} \right\} \tag{3}$$

$$\theta_{im} = A + B \tag{4}$$

where A and B are positive constants, and δ_{ij} is Kroneker's delta. The state of $N \times N$ neurons are asynchronously updated due to the following difference equation:

$$x_{im}(t+1) = f\left(\sum_{j,n=1}^{N} w_{im;jn} x_{im}(t) x_{jn}(t) - \theta_{im} + \beta z_{im}(t) \right) \tag{5}$$

where $f(\cdot)$ is the sigmoidal function defined as follows:

$$f(x) = \frac{1}{1 + \exp\left(-\dfrac{x}{\varepsilon}\right)}, \tag{6}$$

z_{im} is additional noise, and β limits the amplitude of the noise.

3. Intermittency noise

3.1. Chaos noise

The logistic map is used to generate the intermittency chaos noise.

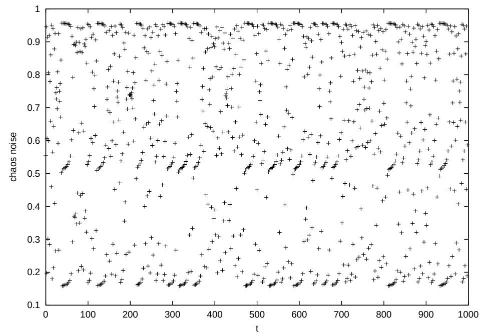
$$\hat{z}_{im}(t+1) = \alpha_z \hat{z}_{im}(t)(1 - \hat{z}_{im}(t)). \tag{7}$$

Increasing the parameter α_z, the logistic map behaves chaotically via a periodic-doubling cascade. Further, it is well known that the map produces intermittent burst just before periodic-windows appear. Figure 1(a) shows an example of the intermittency chaos near the three-periodic window obtained from (7) for α_z = 3.82676. As we can see from Fig. 1(a), the chaotic time series could be divided into two phases; laminar part of periodic behavior with period three and burst parts of spread points over the invariant interval. We obtain fully-developed chaos shown in Fig. 1(b) for α_z = 3.999. This time series are more similar to random uniform noise.

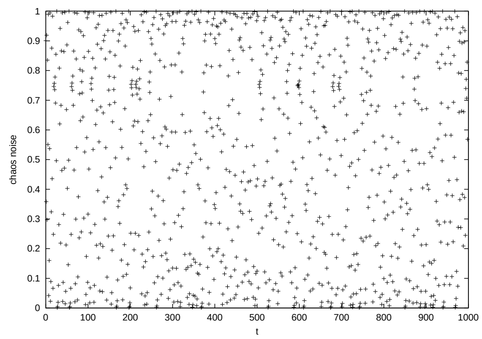
When we inject the intermittency chaos noise to the Hopfield NN, we normalize $\hat{z}_{im}$ by the following equation.

$$z_{im}(t) = \frac{\hat{z}_{im}(t) - \bar{z}}{\sigma_z} \tag{8}$$

where $\bar{z}$ and σ_z are the average and the standard deviation of $\hat{z}(t)$, respectively.



(a) Intermittency chaos (α_z=3.82676).



(b) Fully-developed chaos (α_z=3.999).

Figure 1: Chaotic time series.

3.2. Torus noise

The following sine circle map is used to generate the torus noise.

$$\hat{y}_{im}(t+1) = \hat{y}_{im}(t) + \alpha_y \sin\{6\,\hat{y}_{im}(t)\} + D. \tag{9}$$

Figure 2 shows the shape of the sine circle map for α_y=0.04 and D=0.05. We tune the parameters of the sine circle map to generate torus including intermittent feature. Figure 3 shows an example of intermittency torus.

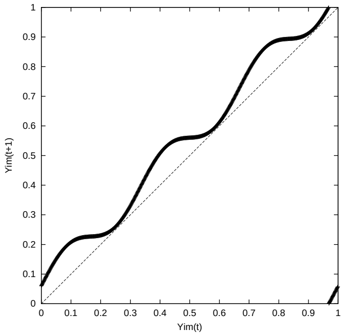
When we inject the intermittency torus noise to the Hopfield NN, we normalize $\hat{y}_{im}$ by the following equation.

$$y_{im}(t) = \frac{\hat{y}_{im}(t) - \bar{y}}{\sigma_y} \tag{10}$$

where $\bar{y}$ and σ_y are the average and the standard deviation of $\hat{y}(t)$, respectively.

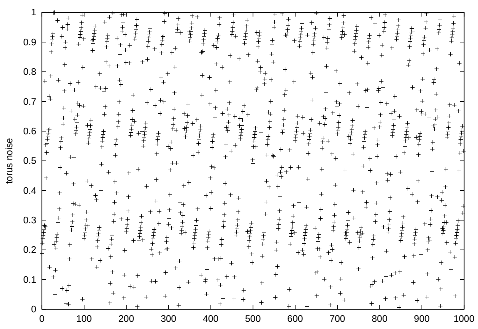
3.3. Simulated results

We carry out computer simulations of the Hopfield NN with the intermittency chaos noise and the intermittency torus noise for 12-element QAP. The problem used here was chosen from the site QAPLIB [7] named "Nug12," The global minimum of this target problem is known as 578. The parameters of the Hopfield NN are as $A = 0.9$, $B = 0.9$, $q = 140$,



α_y=0.04 and D=0.05.

Figure 2: Sine circle map.



α_y=0.04 and D=0.05.

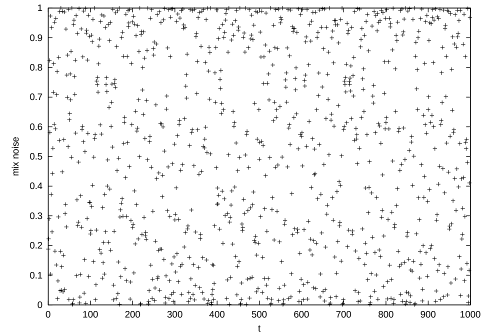
Figure 3: Intermittency torus time series.

ε=0.02, and the amplitude of the injected noise is fixed as $\beta = 0.55$.

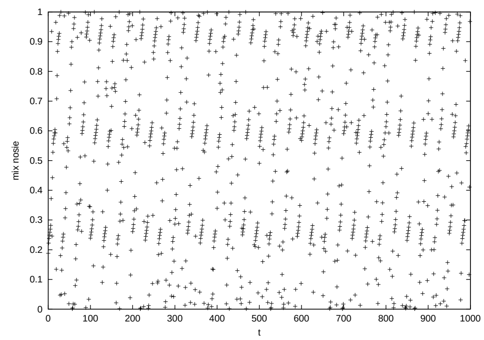
The average solution and the best minimum solution are summarized in Table 1. For comparison, the results for the cases of the fully-developed chaos noise and random noise are shown together. The results show that the intermittency chaos noise gains the best performance. The intermittency torus noise is better than the random noise, but still far from the chaotic noises.

4. Mix noise

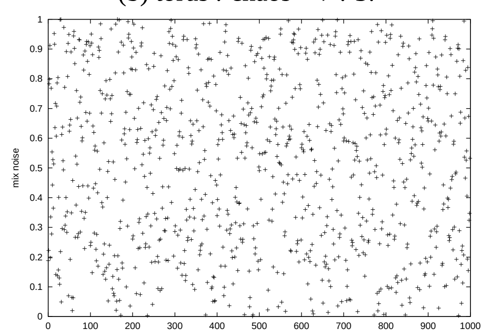
Since the intermittency torus noise does not have a better performance, we try to add a burst feature of the intermittency chaos. Mix noise is made by inserting fully-developed chaos or random noise into the intermittency torus noise. The obtained mix noise is shown in Fig. 4. Figures 4(a) and (b) are the mix noise obtained by inserting the fully-developed chaos with different rates. While Figs. 4(d) and (e) are the mix noise obtained by inserting random noise with different rates. The simulated results of the Hopfield NN with the mix noise are summarized in Table 2. The problem and the parameters of the Hopfield NN are the same as the case of the intermittency



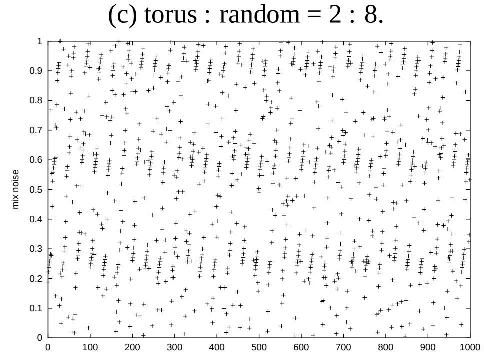
(a) torus : chaos = 2 : 8.



(b) torus : chaos = 7 : 3.



(c) torus : random = 2 : 8.



(d) torus : random = 7 : 3.

Figure 4: Mix noise.

Table 1: Solving abilities of intermittency noise.

Iteration	Chaos Noise				Intermittency Torus Noise		Random Noise	
	Intermittency		Fully-developed					
	average	best	average	best	average	best	average	best
2000	613.4	586	636.0	606	667.0	638	724.2	680
4000	607.8	586	636.0	606	667.0	638	724.2	680
6000	607.8	586	636.0	606	667.0	638	724.2	680
8000	604.0	586	636.0	606	667.0	638	724.2	680
10000	604.0	586	636.0	606	667.0	638	724.2	680

Table 2: Solving abilities for of mix noise.

Iteration	Mix Noise							
	torus noise : chaos noise				torus noise : random noise			
	2 : 8		7 : 3		2 : 8		7 : 3	
	average	best	average	best	average	best	average	best
2000	604.4	578	617.4	586	612.0	582	614.6	590
4000	595.2	578	608.6	578	604.8	582	604.2	590
6000	591.6	578	601.4	578	601.6	582	599.8	586
8000	590.4	578	595.6	578	601.6	582	598.8	586
10000	587.8	578	595.6	578	596.6	582	598.0	586

noises in Table 1.

We can see that the mix noise with chaos achieves the best performance and that the mix noise with random noiose also achieves a good performance.

We can conclude that the burst feature of the noise is important to gain a good performance of the Hopfield NN for QAP.

5. Conclusions

In this study, we have considered the torus noise generated by the sine circle map for the Hopfield NN. We have investigated the performance of various noises based on the torus noise injected to Hopfield NN for QAP. By computer simulations, we confirmed that the mix noise of the intermittency torus noise with chaos achieved the best performance.

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