2006 RISP International Workshop
on Nonlinear Circuits and Signal Processing (NCSP'06)
Waikiki Beach Marriott, Honolulu,
Hawaii, USA, March 3-5, 2006

# Investigation of Strange Synchronization Phenomena in Coupled Wien-Bridge Oscillators

Koichi Matsumoto† , Yoko Uwate† , Yoshifumi Nishio† and Seiichiro Moro‡

†Department of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima 770-8506, Japan

‡Dept. of Electrical and Electronics Engineering, University of Fukui
3-9-1 Bunkyo, Fukui 910-8507, Japan

E-mail: †{koichi, uwate, nishio}@ee.tokushima-u.ac.jp, ‡moro@ppc8100.fuee.fukui-u.ac.jp

**Abstract –** The synchronization phenomena are the representative phenomena in the nonlinear science and can be often observed in nature world. Coupled oscillatory systems have attracted a great deal of attentions in various fields. In particular, synchronization in such systems is very important phenomenon and many researches have been reported. In this study, we investigate the interesting synchronization phenomena with 143 degree phase difference in detail, as varying for example the nonlinearity of the oscillator. The results would be a first step to make clear the mechanism of the generation of the 143 degrees synchronization.

## 1. Introduction

Coupled oscillatory systems have attracted a great deal of attentions in various fields. In particular, synchronization in such systems is very important phenomenon and many researches have been reported. In our past studies [1][2], we have reported that a certain class of coupled systems of the Wien-Bridge oscillators synchronizes with 0 or 120 degrees.

In [3], we have investigated synchronization phenomena in coupled Wien-Bridge oscillators by both circuit experiments and computer simulations using SPICE. We observed the synchronization state with 143 degrees phase difference. The phase difference of 143 degrees is very strange, because in-phase, anti-phase, or N-phase synchronization is typical.

In this study, we investigate the strange but interesting synchronization phenomenon with 143 degrees phase difference in detail, as varying for example the nonlinearity of the oscillator. The results would be a first step to make clear the mechanism of the generation of the 143 degrees synchronization. In Sec.2, some results in our past studies are introduced. In Sec.3, the circuit model in this study and the circuit equations are given. In Sec.4, simulation results are shown. In Sec.5, we conclude our study.

## 2. Coupled Wien-Bridge Oscillators

The Wien-Bridge oscillator is shown in Fig.1. This circuit consists of two capacitors, four resistors and one OP amp.

The parameters for the circuit experiments are chosen as follows. C = 15nF, R=10kΩ, $R_i$ = 4.7kΩ, $R_f$ = 14.7kΩ.

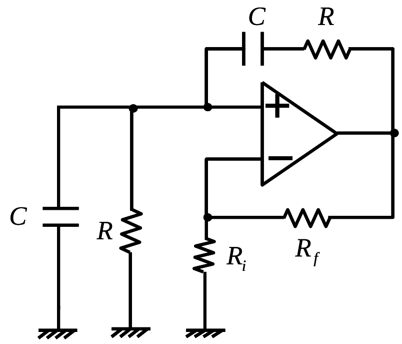



Fig.1 Wien-Bridge oscillator.

Figure 2 shows one of the coupled systems of the Wien-Bridge oscillators reported in our past studies. In this system, N Wien-Bridge oscillators are coupled as a ring. We observed the synchronization state with 143 degrees phase difference for the ring by both circuit experiments and SPICE simulations. Typical examples of the results for the case of N=2 are shown in Figs. 3 and 4. This strange synchronization coexists with the in-phase synchronization.

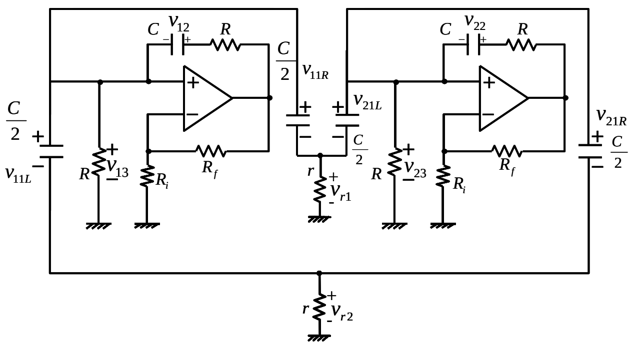
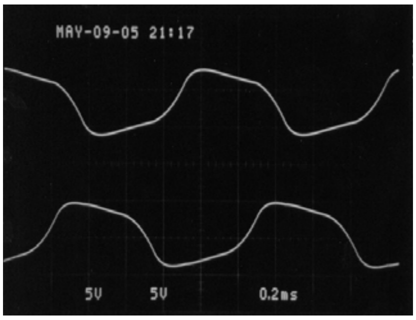
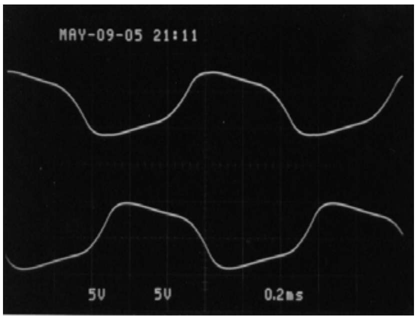



Fig.2 Ring of coupled Wien-Bridge oscillators (N=2).

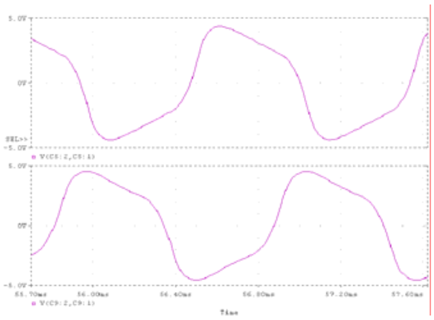



(a)

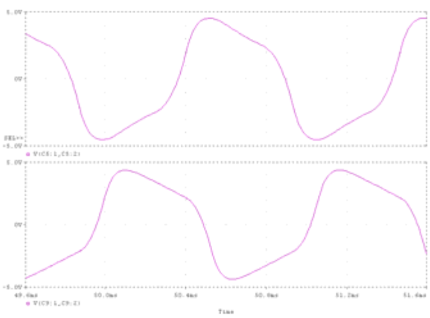



(b)

Fig.3 Strange synchronization with 143 degrees phase difference (circuit experiment) . Two phase states coexist.



(a)



(b)

Fig.4 Strange synchronization with 143 degrees phase difference (SPICE simulation) . Two phase states coexist.

## 3. Circuit Model

In this study, we consider a simple coupled system of two Wien-Bridge oscillators to investigate the generation of the 143 degrees synchronization. The circuit model is shown in Fig. 5. In this model, two Wien-Bridge oscillators are coupled by one resistor r.

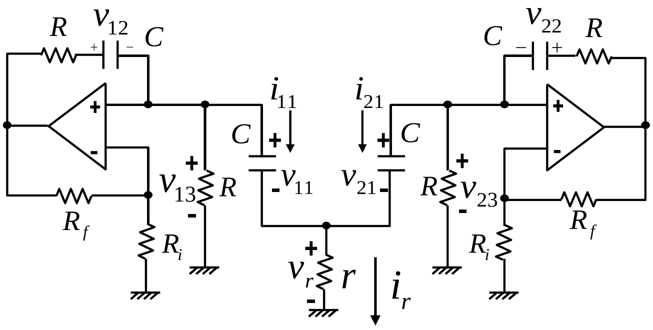


Fig.5 Coupled Wien-Bridge oscillators.

The circuit equations are given as follows.

$$\begin{cases} C\dfrac{dv_{k1}}{dt} = C\dfrac{dv_{k2}}{dt} - \dfrac{1}{R}v_{k3} \\ C\dfrac{dv_{k2}}{dt} = \dfrac{1}{R}\left(g_1 v_{k3} - g_3 v_{k3}^3 - v_{k2} - v_{k3}\right) \\ C\left(\dfrac{dv_{11}}{dt} + \dfrac{dv_{21}}{dt}\right) = \dfrac{1}{r}\left(v_{k3} - v_{k1}\right) \end{cases} \quad (1)$$

$$(k = 1,2).$$

By changing variables and parameters,

$$\begin{aligned} & t = RC\tau, \\ & x_k = \sqrt{\frac{g_1 - 3}{3g3}}v_{k1},\ y_k = \sqrt{\frac{g_1 - 3}{3g3}}v_{k2}, \\ & z_k = \sqrt{\frac{g_1 - 3}{3g3}}v_{k3},\ \alpha = \frac{r}{R},\ \varepsilon = g_1 - 3, \end{aligned} \quad (2)$$

the circuit equations are normalized as follows.

$$\begin{cases} \dot{x}_k = \dot{y}_k - z_k \\ \dot{y}_k = \varepsilon\left(z_k - \dfrac{z_k^3}{3}\right) - y_k + 2z_k \\ z_k = x_k + \alpha\left(\dot{x}_1 + \dot{x}_2\right) \end{cases} \quad (3)$$

$$(k = 1,2).$$

In equation (3), α is the coupling factor and ε is the strength of nonlinearity.

## 4. Simulation Results

We investigate synchronization phenomena when ε is changed by the computer simulation using the Runge-Kutta method. The relation between ε and φ is shown in Fig. 6. φ is the phase difference between the two oscillators. We set the coupling parameter as α = 0.01.

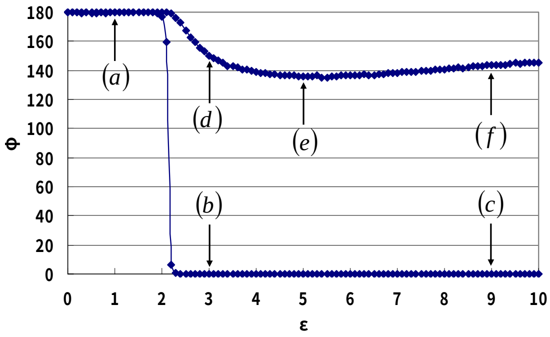


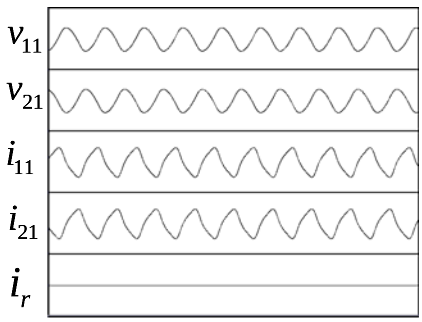

Fig. 6 The relation between nonlinearity and phase difference.

This figure shows that only the anti-phase synchronization state (φ=180 ) can be observed for ε < 2.3. Namely, all of the initial states converge to the anti-phase synchronization. The voltage and the current waveforms for this case are shown in Fig. 7(a). Because the nonlinearity of the oscillators is weak, the voltage waveforms are similar to sine waves.

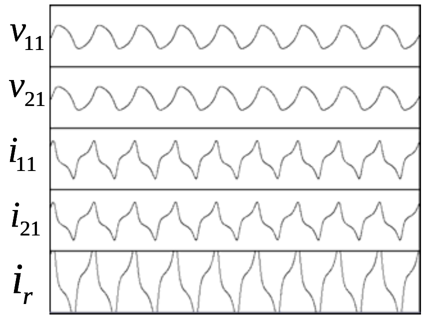
For ε > 2.3, we can observe the coexistence of the in-phase synchronization and the other strange synchronization whose phase difference varies with ε . From Fig. 6, we can see that the phase difference of the strange synchronization state decreases from 180 to around 135 as ε increases. However, for ε >5.0, the phase difference increases toward 180 again.

The voltage and the current waveforms for the in-phase synchronization are shown in Figs. 7(b) and (c). We can observe that the shapes of the waveforms are different because of the difference of the nonlinearity.

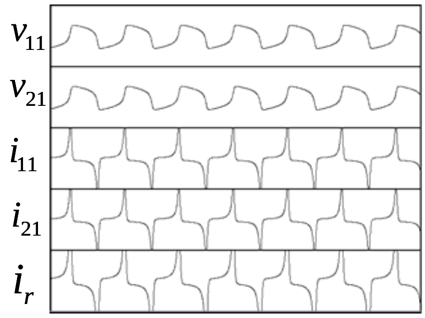
The voltage and the current waveforms for the strange synchronization are shown in Figs. 7(d)-(f). We consider that the shape of the current through the coupling resistor $i_r$ plays an important role to decide the phase difference of this strange synchronization.



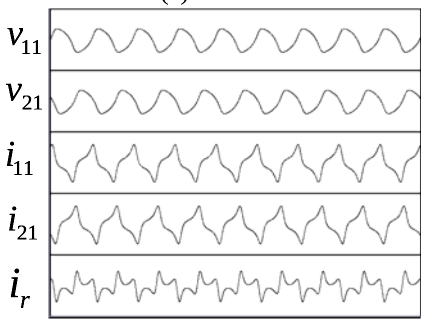

(a) ε =1.0.



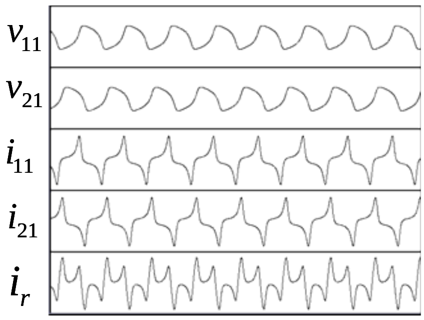

(b) ε =3.0.



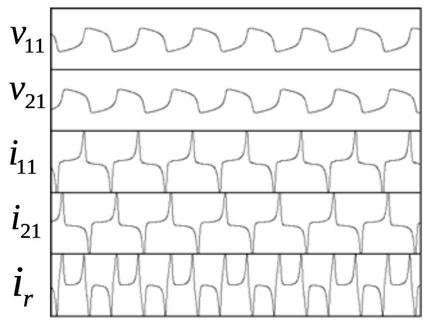

(c) ε =9.0.




(d) ε =3.0.




(e) ε =5.0.



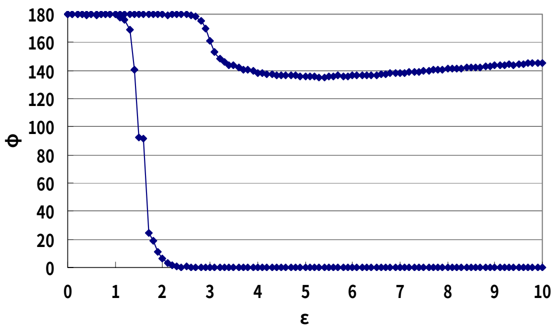

(f) ε =9.0.

Fig.7 The voltage and the current waveforms.

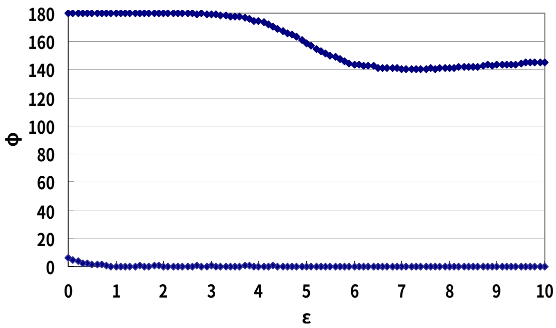
## 5. Effect of Coupling Strength

Next, we investigate how the graph of the phase difference of the strange synchronization changes according to the change of the coupling strength $\alpha$. The relation between $\varepsilon$ and $\phi$ for different coupling strengths ($\alpha$ = 0.005 and 0.001) are shown in Figs. 8 (a) and (b).

We can obtain an interesting change of the graph. However, at the moment we cannot explain the reason of the change.




(a) $\alpha$ =0.005.




(b) $\alpha$ =0.001.

Fig. 8 Effect of Coupling Strength.

## 6. Conclusions

In this study, we investigate the strange synchronization phenomena with 143 degrees phase difference in detail, as varying for example the nonlinearity of the oscillator. The results would be a first step to make clear of the mechanism of the generation of the 143 degrees synchronization.

## Reference


[1] S. Moro, Y. Nishio and S. Mori, "Synchronization Phenomena in RC Oscillators Coupled by One Resistor," IEICE Transactions on Fundamentals, vol. E78-A, no. 10, pp. 1435-1439, Oct. 1995.

[2] S. Moro and T. Matsumoto, "Various Kinds of Coupled Networks with Wien-Bridge Oscillators," Proc. of NOLTA'00, pp. 547-550, Sep. 2000.

[3] K. Matsumoto, Y. Uwate, Y. Nishio and S. Moro, "Synchronization Phenomena in a Ring of Coupled Wien-Bridge Oscillators," Proc. of NDES'05, p. O28, Sep. 2005.