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CHAOSOM: Collaboration between Chaos and Self-Organizing Map

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Abstract

In this study, we try to implant chaotic features into the learning algorithm of self-organizing map. We call this concept as *Chaotic SOM (CHAOSOM)*. As a first step to realize CHAOSOM, we consider the case that learning rate and neighboring coefficient of SOM are refreshed by chaotic pulses generated by the Hodgkin-Huxley equation. We apply the CHAOSOM to solve a traveling salesman problem and confirm that the chaotic feature improves the performance.

keywords: self-organizing map, chaos, Hodgkin-Huxley equation, traveling salesman problem

1. Introduction

The Self-Organizing Map (SOM) is a one of the neural network methods for unsupervised learning, introduced by Kohonen in 1982 [1]. Self-Organization is to change an internal structure to adjust to the signal from the outside. SOM is a model simplifying self-organization process of brain. SOM obtains a statistical feature of input data and is applied to a wide field of data classifications. SOM can be also applied to the traveling salesman problem (TSP) [1]. TSP is one of combinatorial optimization problems and a prominent illustration of a class of problems in computational complexity theory which are hard to solve. This problem is to search the shortest round-trip route of visiting each city once and then returns to the starting city, when given a number of cities and the distance of traveling from any city to any other city.

On the other hand, chaos is said to exist in the brain and to play an important role to realize higher functions of information processing. Several researchers have tried to exploit the features of chaos to solve combinatorial optimization problems and some good results have been obtained [2]-[5]. Hence, it is important to investigate the possibility of adding chaotic features to SOM. Actually, chaotic self-organization map has been proposed in [6]. In [6], chaos is used to choose the winner neuron in a probabilistic manner. However, the effect is not examined for any difficult problems.

In this study, we try to implant chaotic features into the learning algorithm of SOM. We call this concept as *Chaotic SOM (CHAOSOM)*. As a first step to realize CHAOSOM, we consider the case that learning rate and neighboring coefficient of SOM are refreshed by chaotic pulses generated by the Hodgkin-Huxley equation [7]. We apply the CHAOSOM to solve TSP and confirm that the chaotic feature improves the performance.

2. Self-Organizing Map (SOM) Algorithm

We explain the learning algorithm of the conventional SOM for TSP. SOM consists of m neurons located on a one-dimensional line like a circle. The basic SOM algorithm is iterative. Each neuron i has a d-dimensional weight vector $\boldsymbol{w}_i = (w_{i1}, w_{i2}, \cdots, w_{id})$ $(i = 1, 2, \cdots, m)$. The initial values of all the weight vectors $\boldsymbol{w}$ are given over the input space at random.

(SOM1) An input vector $\boldsymbol{x}_j = (x_{j1}, x_{j2}, \cdots, x_{jd})(j = 1, 2, \cdots, N)$ is inputted to all the neurons at the same time in parallel.

(SOM2) Distances between $\boldsymbol{x}_j$ and all the weight vectors are calculated. The winner neuron, denoted by c, is the neuron with the weight vector closest to the input vector $\boldsymbol{x}_j$,

$$c = \arg\min_i \{\|\boldsymbol{w}_i - \boldsymbol{x}_j\|\}. \quad (1)$$

In this study, Euclidean distance is used for (1).

(SOM3) A set of neighboring neurons of the winner neuron is denoted as $Nc(t)$. The weight vector of the winner neuron c and the neighboring neuron $Nc(t)$ are updated as;

$$\boldsymbol{w}_i(t+1) = \boldsymbol{w}_i(t) + \alpha(t)(\boldsymbol{x}_j - \boldsymbol{w}_i(t)), i \in Nc(t) \quad (2)$$

Both the learning rate $\alpha(t)$ and the neighboring neuron $Nc(t)$ decrease with time, in this study, according to the following equations;

$$\begin{aligned} \alpha(t) &= \frac{1}{\ln(t+2)}, \\ Nc(t) &= \left\lfloor -\frac{1}{n_t}t + \frac{m}{6} + 1 \right\rfloor, \end{aligned} \quad (3)$$

where $\lfloor \ \rfloor$ denotes the floor function.

(SOM4) The steps from (SOM1) to (SOM3) are repeated for all the input data.
(SOM5) Each input data $\boldsymbol{x}_j$ $(j = 1, 2, \cdots, N)$ is assigned to the neuron with the weight vector closest to $\boldsymbol{x}_j$ in order. However, it is only one input vector which is assigned to one neuron, so, if another input data is assigned to the closest neuron already, this input data is assigned to the second-closest neuron. We can route the tour of the input data set by sorting input data in numerical order of neuron assigned to each input data.

3. Chaotic Self-Organizing Maps (CHAOSOM)

3.1. Hodgkin-Huxley Equations

The Hodgkin-Huxley equation is the mathematical model which simulate the action potential in a squid giant axon, proposed by Hodgkin and Huxley [7]. The Hodgkin-Huxley equation is expressed as;

$$\left\{\begin{aligned}
\frac{dV}{dt} = \;& -120.0m^3h(V-55.0) \\
& -36.0n4(C+72.0) \\
& -0.24(V+49.387), \\
\frac{dm}{dt} = \;& \frac{0.1(-35-V)}{\exp\left(\frac{-35-V}{10}\right)-1}(1-m) \\
& -4\exp\left(\frac{-60-V}{18}\right)m, \\
\frac{dh}{dt} = \;& 0.07\exp\left(\frac{-60-V}{20}\right)(1-h) \\
& -\frac{1}{\exp\left(\frac{-30-V}{10}\right)+1}h, \\
\frac{dn}{dt} = \;& \frac{0.01(-50-V)}{\exp\left(\frac{-50-V}{10}\right)-1}(1-n) \\
& -0.125\exp\left(\frac{-60-V}{80}\right)n,
\end{aligned}\right. \tag{4}$$

where V is a membrane potential, m and h denote a sodium activation coefficient and a sodium inactivation coefficient, respectively, and n is a potassium activation coefficient. I denotes the current stimulus.

The Hodgkin-Huxley equation is known to generate chaotic oscillation for periodic external force [8][9]. Figure 1 shows the time series of the first variable $V(t)$, when (4) is stimulated by the following external force.

$$I = I_0 + A\sin 2\pi ft. \tag{5}$$

where I_0=20, A=40 and f=300.2. We can see that the time interval between spikes is not completely periodic. By changing the parameter values, the interval becomes more irregular.

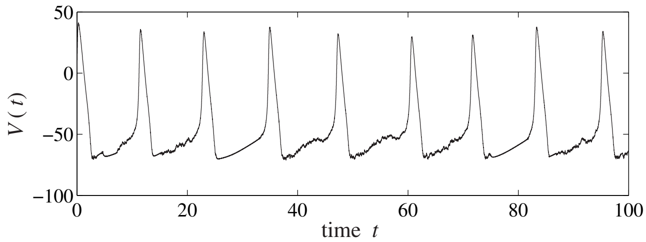

Figure 1: Chaotic response of Hodgkin-Huxley equation.

3.2. Learning Algorithm

The basic algorithm of CHAOSOM is the same as SOM, however, the important feature of CHAOSOM is to refresh the learning rate and neighboring coefficient at the timing of the spikes generated chaotically by the Hodgkin-Huxley equation.

We explain the learning algorithm of CHAOSOM in detail. The learning rate $\alpha_s(s)$ and the neighboring neuron $Nc_s(s)$ of CHAOSOM are defined as;

$$\begin{aligned}
\alpha_s(s) &= \frac{1}{\ln(s+2)}, \\
Nc_s(s) &= \left\lfloor -\frac{1}{n_s}s + \frac{m}{6(p+1)} + 1 \right\rfloor,
\end{aligned} \tag{6}$$

where p represents the number of the spikes of $V(t)$ in (4) and increases by one. In order to adjust the time constant of (4) to the CHAOSOM algorithm, we use the value of $V_k = V(k \times \Delta t)$ for Δt=1/150. The parameter step s increases monotonically with time t, however, if the chaotic signal V_k generates a spike, the parameter step s is refreshed to a smaller value by the following equation;

$$s = 500p. \tag{7}$$

Furthermore, at the same timing of the spikes, we route the tour of the input data set according to (SOM5), Finally, the minimum tour among the obtained tours during the learning is defined as the result of CHAOSOM.

4. Simulation Results

We carry out the computer simulations for SOM and CHAOSOM. The target problem of TSP, namely the input data set, is *att48* from TSPLIB. The number of the input data is $N = 48$ and the optimal solution of this problem is known as 33524. We iterated the simulations 100 times with different initial conditions and the average values are evaluated. Each SOM has $m = 96$ neurons, namely twice of the number of input data. The parameters of the learning are chosen as follows,

$$n_t = 600, \quad n_s = 100.$$

The maximum number of the learning of each SOM is 15,000 times.

The example of the learning process of CHAOSOM are shown in Figs. 2. We can see that CHAOSOM obtains a feature of input data as learning progressed.

The results are summarized in Table. 1. In the table, Error means the difference between the obtained distances and the optimal distance and Error (%) means the percentages when the optimal distance is regarded as 100%. We can see that the CHAOSOM improve the results.

Table 1: Results of solving TSP using SOM and CHAOSOM.

	SOM	CHAOSOM
Distance	34850	34202
Error	1326	678
Error (%)	3.96	2.02

5. Conclusions

In this study, we have proposed a concept of CHAOSOM, which is a collaboration between chaos and SOM. We considered the case that learning rate and neighboring coefficient of SOM were refreshed by chaotic pulses generated by the Hodgkin-Huxley equation. We applied the CHAOSOM to solve a traveling salesman problem and confirmed that the chaotic feature improved the performance.

Actually, this result is just a first step to develop the CHAOSOM concept. More effective implantation of chaotic features into SOM is our important future subject.

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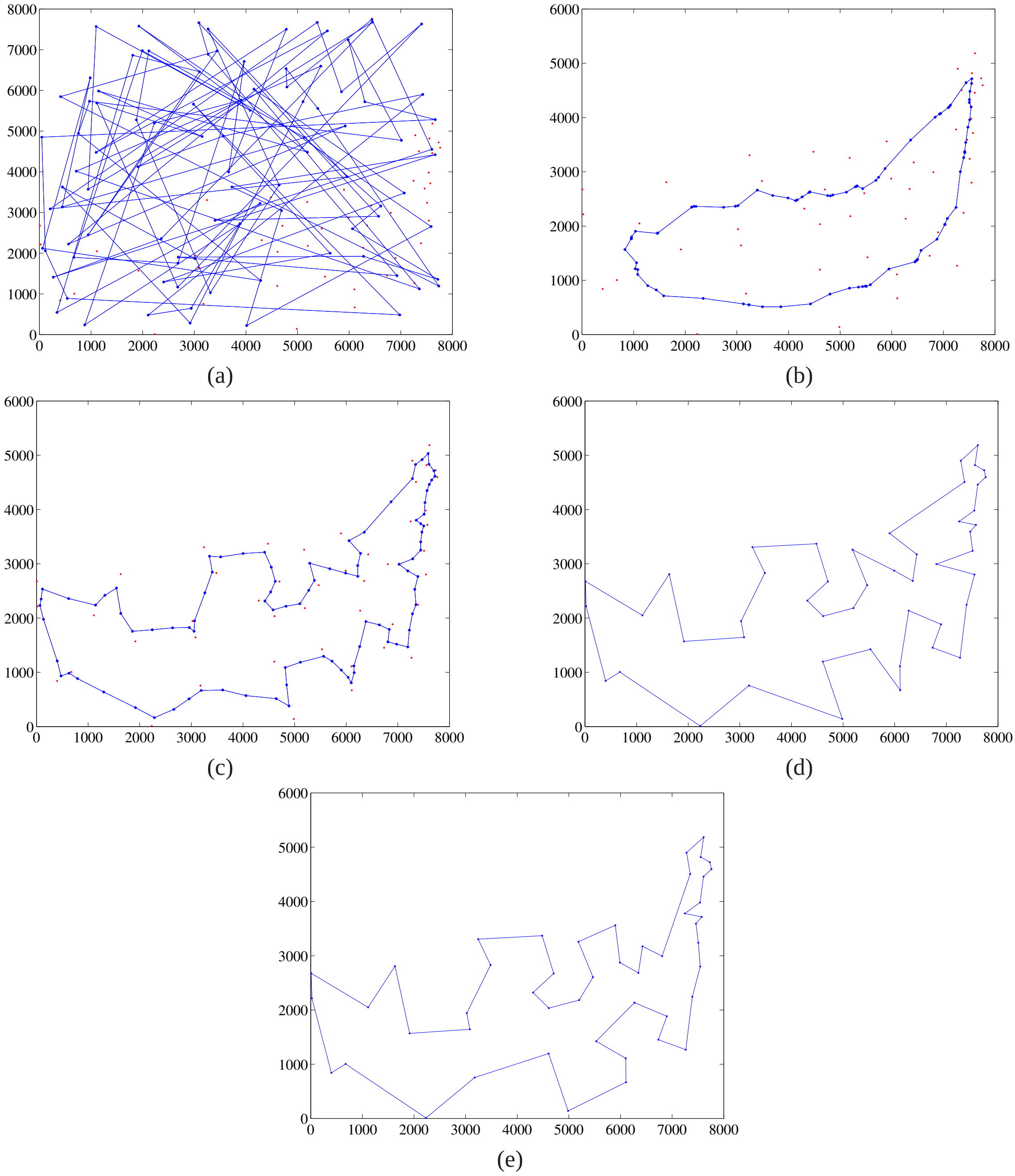

Figure 2: Example of learning process of CHAOSOM. (a) $t = 0$ (Initial state). (b) $t = 1000$. (c) $t = 1791$ (First learning result). (d) First result of solving TSP. (e) Best result of solving TSP in this learning.