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Synchronization in Chaotic Circuits Coupled by Mutual Inductors

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Abstract

In this research, synchronization phenomena observed from simple chaotic circuits coupled by mutual inductors are investigated. A simple three-dimensional autonomous circuit is considered as a chaotic subcircuit. By carrying out circuit experiments and computer calculations for two or three subcircuits cases, various kinds of synchronization phenomena of chaos are observed.

1. Introduction

Many nonlinear dynamical systems in various fields have been confirmed to exhibit chaotic oscillations. Recently applications of chaos to engineering systems are expected such as chaos noise generators, control of chaos, synchronization of chaos, and so on. In those applications, we are especially interested in synchronization of chaos. Synchronization and the related bifurcation in chaotic systems are good models to describe various high-dimensional nonlinear phenomena in the field of natural science and many excellent studies on synchronization of chaos have been reported. Now mechanisms of chaotic phenomena generated in low-dimensional systems have been elucidated theoretically, and complex phenomena observed from higher dimensional circuits represented by coupled plural chaotic circuits attract attentions [1]-[3].

In this research, synchronization phenomena observed from simple chaotic circuits coupled by mutual inductors are investigated. A simple three-dimensional autonomous circuit is considered as a chaotic subcircuit. This subcircuit is a symmetric version of the chaotic circuit proposed by Inaba *et al.* [4]. They used ideal piecewise linear model of diodes in [4], but in this research the $i - v$ characteristics of the nonlinear resistor consisting of diodes are approximated by a smooth function. This is more real than piecewise linear approximation in the sense that every real elements in the natural field are not piecewise linear. By carrying out computer calculations for two or three subcircuits cases, various kinds of synchronization phenomena of chaos are observed. In the two subcircuits case, in-phase and anti-phase synchronization are observed. Moreover in-phase and three-phase synchronization are observed in the three subcircuit case.

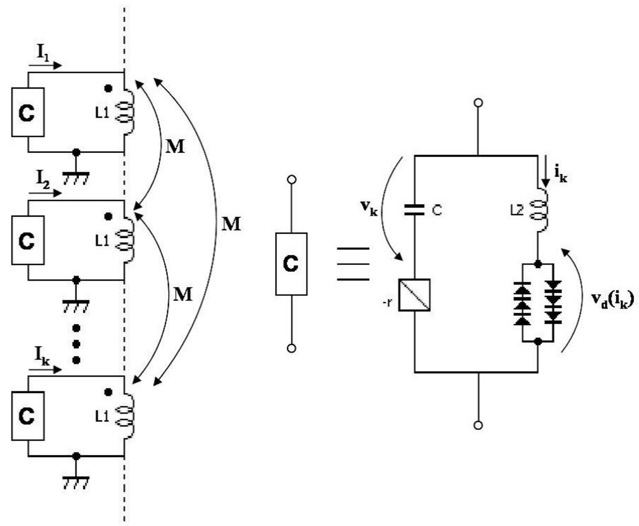

Figure 1: Circuit model.

2. Circuit Model

Fig. 1 shows the circuit model. In the circuit, N identical chaotic circuits are coupled by mutual inductors. Each chaotic subcircuit is a symmetric version of the circuit model proposed by Inaba *et al.* [4]. It consists of three memory elements, one linear negative resistor and one nonlinear resistor, which is realized by connecting some diodes, and is one of the simplest autonomous chaotic circuits. First, we approximate the $i - v$ characteristics of the nonlinear resistor consisting of diodes by the following function.

$$v_d(i_k) = \sqrt[9]{r_d\, i_k}. \tag{1}$$

By changing the variables and parameters,

$$\begin{aligned} &t = \sqrt{(L_1 - M)\,C}\,\tau, \quad a = \sqrt{\frac{C}{L_1 - M}}, \\ &b = \sqrt[8]{r_d\, a}, \quad I_k = a\,b\,x_k, \quad i_k = a\,b\,y_k, \quad v_k = b\,z_k, \\ &\text{“ . ”} = \frac{d}{d\tau}, \quad \alpha = \frac{L_1 - M}{L_2}, \quad \beta = r\,a, \quad \gamma = \frac{M}{L_1}, \end{aligned} \tag{2}$$

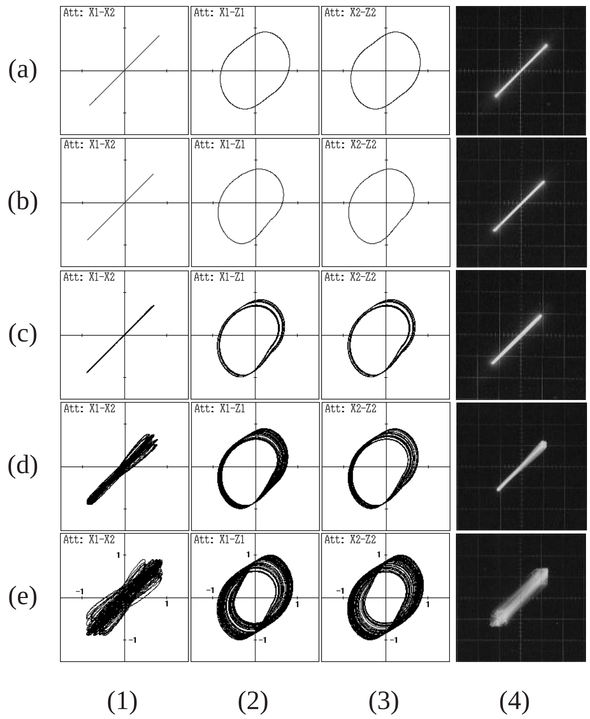

Figure 2: In-phase synchronization of two chaotic circuits. γ = 0.01 (M = 2.63mH). (a) β = 0.24 (r = 585Ω). (b) β = 0.27 (r = 708Ω). (c) β = 0.29 (r = 753Ω). (d) β = 0.293 (r = 758Ω). (e) β = 0.3 (r = 771Ω). (1) x_1 vs. x_2. (2) x_1 vs. z_1. (3) x_2 vs. z_2. (4) Circuit experimental results. I_1 vs. I_2. 10 V/div.

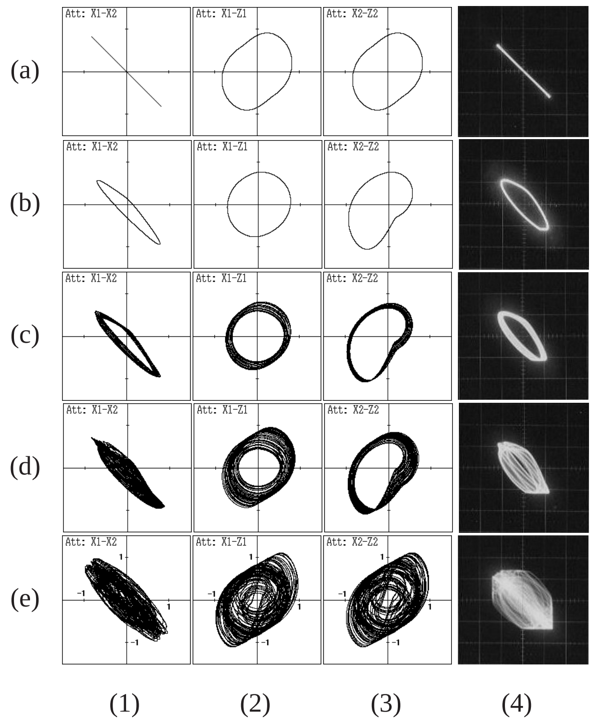

Figure 3: Anti-phase synchronization of two chaotic circuits. γ = 0.4 (M = 102mH). (a) β = 0.24 (r = 571Ω). (b) β = 0.25 (r = 646Ω). (c) β = 0.265 (r = 655Ω). (d) β = 0.273 (r = 689Ω). (e) β = 0.28 (r = 706Ω). (1) x_1 vs. x_2. (2) x_1 vs. z_1. (3) x_2 vs. z_2. (4) Circuit experimental results. I_1 vs. I_2. 10 V/div.

the circuit equations are normalized and described as

$$
\begin{aligned}
\dot{x}_k &= \beta(x_k + y_k) - z_k \\
&\quad - \frac{\gamma}{1+(N-1)\gamma}\sum_{j=1}^{N}\{\beta(x_j + y_j) - z_j\} \\
\dot{y}_k &= \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\} \\
\dot{z}_k &= x_k + y_k \qquad\qquad (k = 1, 2, \cdots, N)
\end{aligned}
\tag{3}
$$

where

$$
f(y_k) = \sqrt[9]{y_k}. \tag{4}
$$

In the following circuit experiments, the values of the inductors and the capacitor in each chaotic subcircuit are fixed and those values are measured as L_1 = 208 mH ± 0.5%, L_2 = 10.22 mH ± 1.6% and C = 33.58 nF ± 1.4%. While in the following computer calculations, the parameter value corresponding to the inductors is fixed as α = 18.0 and (3) is calculated by using the Runge-Kutta method with step size Δt = 0.001.

3. Two Subcircuits Case

In this section, we consider the case of $N = 2$, namely two chaotic subcircuits are coupled by a mutual inductor.

Computer calculated results and the corresponding circuit experimental results are shown in Figs. 2 and 3. Please note that in-phase and anti-phase synchronizations coexist, but subcircuits are easy to be synchronized at in-phase for lower γ and at anti-phase for higher γ. Fig. 2(a) shows in-phase synchronization of one-periodic attractors. As the parameter β increases, one-periodic attractor bifurcates to chaotic attractor via period-doubling route keeping in-phase synchronization. When the attractor is chaotic, two subcircuits are not synchronized completely, but are almost synchronized as shown in Figs. 2(d) and (e). We call the situation as quasi-synchronization of chaos.

While, Fig. 3 shows anti-phase synchronization. The anti-phase synchronization undergoes complicated bifurcation route explained as follows. One-periodic attractor with symmetry on the $x_1 - x_2$ plane in Fig. 3(a) bifurcates to two one-periodic attractors with asymmetry as Fig. 3(b). Each asymmetric attractor bifurcates to torus via Hopf bifurcation as Fig. 3(c) and to chaos via torus breakdown as Fig. 3(d). Two asymmetric chaos collide each other and one chaotic attractor with symmetry is generated via symmetry-recovering crisis as Fig. 3(e). Namely, the anti-phase synchronization exhibits symmetry breaking and recovering and torus via Hopf bifurcation.

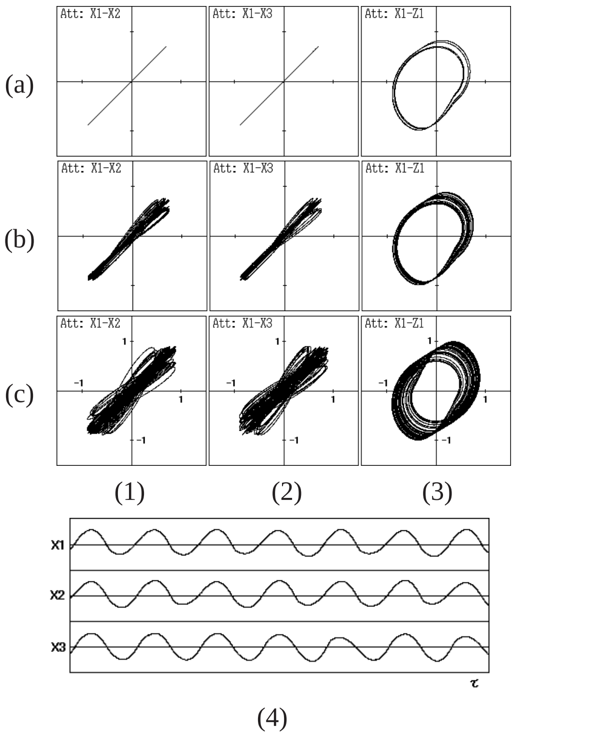

Figure 4: In-phase synchronization of three chaotic circuits (computer calculated results). γ = 0.01. (a) β = 0.29. (b) β = 0.293. (c)β = 0.3. (1) x_1 vs. x_2. (2) x_1 vs. x_3. (3) x_1 vs. z_1. (4) Time waveform for β = 0.3.

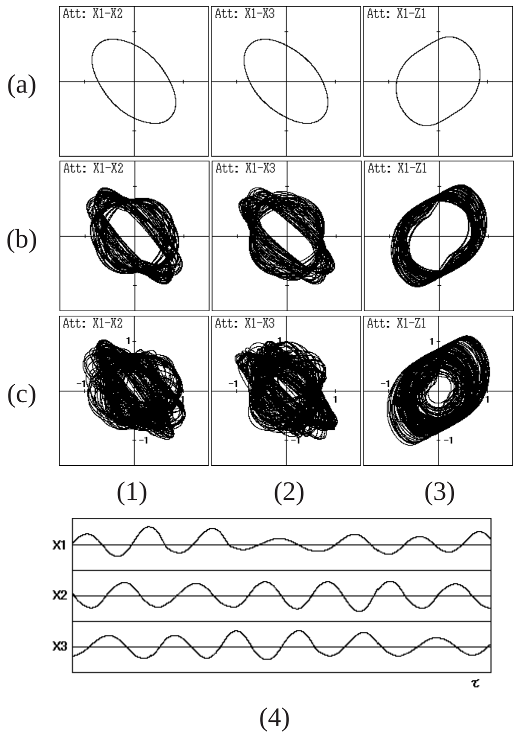

Figure 5: Three-phase synchronization of three chaotic circuits (computer calculated results). γ = 0.4. (a) β = 0.2. (b) β = 0.25. (c) β = 0.262. (1) x_1 vs. x_2. (2) x_1 vs. x_3. (3) x_1 vs. z_1. (4) Time waveform for β = 0.262.

4. Three Subcircuits Case

In this section, we consider the case of $N = 3$, namely three chaotic subcircuits are coupled by three mutual inductors.

Computer calculated results are shown in Figs. 4 and 5. In figures we omit attractors on the $x_i - z_i$ plane for $i = 2, 3$, because the shape is almost same as the attractors on the $x_1 - z_1$ plane. As well as two subcircuits case, in-phase synchronization in Fig. 4 is confirmed to undergo period-doubling route to chaos. While, three-phase synchronization in Fig. 5 undergoes torus breakdown.

In order to investigate the bifurcation route in detail, we consider the Poincare map of each synchronization mode. The poincare section is defined as $z_1 = 0$, $x_1 < 0$. The projections of the Poincare map onto $x_1 - x_2$ plane of in-phase and three-phase synchronization are shown in Figs. 6 and 7, respectively. From Fig. 6 we can see that one-periodic attractor (a) bifurcates to two-periodic (b), four-periodic (c), two-band chaos (d) and one-band chaos (e). This is well-known period doubling route to chaos. As β increase further, chaos grows as (f). From Fig. 7 we can see the bifurcation route of three-phase synchronization via torus breakdown. One-periodic attractor (a) bifurcates torus (b) via Hopf bifurcation. As β increases, torus grows as (c)(d). At $\beta \cong 0.258$, chaos (e) appears. As β increases further, Poincare map has thickness (f) and area-expanding chaos is considered to be generated.

Moreover, we made one-parameter bifurcation diagrams of the Poincare map as shown in Figs. 8 and 9. Fig. 8 shows that in-phase synchronization exhibits logistic chaos. We can also observe the generation of five-periodic window around β = 0.302. It is clear that the synchronization becomes weak for larger β value. From Fig. 9 we can confirm the bifurcation route of three-phase synchronization, namely bifurcation of the one-periodic solution to torus around β = 0.239, the generation of periodic solution around β = 0.241 and the generation of chaotic solution for β values more than about 0.258.

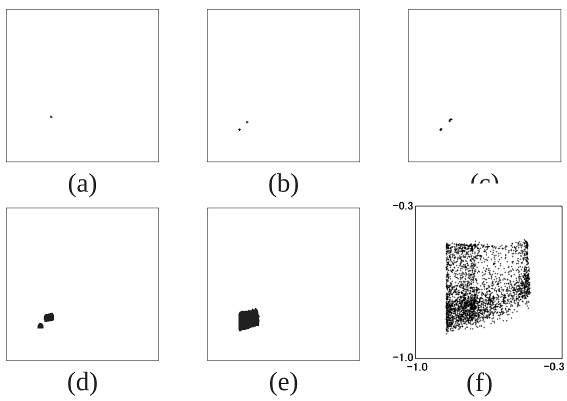

Figure 6: Poincare map of the in-phase synchronization. Horizontal: x_1. Vertical: x_2. γ = 0.01. (a) β = 0.27. (b) β = 0.285. (c) β = 0.29. (d) β = 0.291. (e) β = 0.293. (f) β = 0.3.

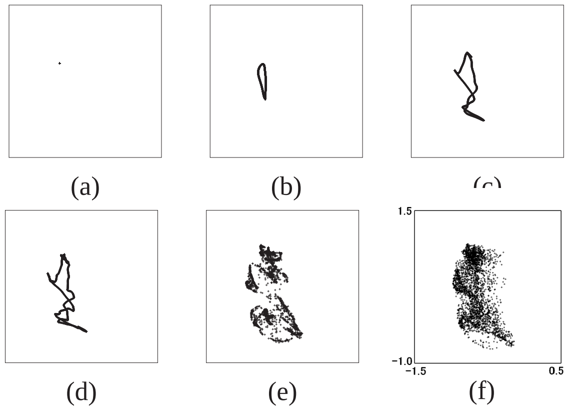

Figure 7: Poincare map of the three-phase synchronization. Horizontal: x_1. Vertical: x_2. γ = 0.4. (a) β = 0.2. (b) β = 0.24. (c) β = 0.25. (d) β = 0.255. (e) β = 0.258. (f) β = 0.262.

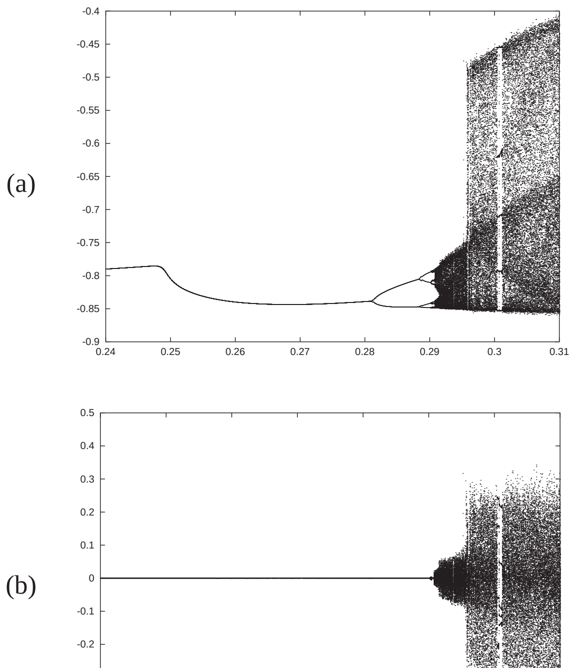

Figure 8: Bifurcation diagram of the Poincare map for the in-phase synchronization. γ = 0.01. (a) Horizontal: β. Vertical: x_1. (b) Horizontal: β. Vertical: $x_1 - x_2$.

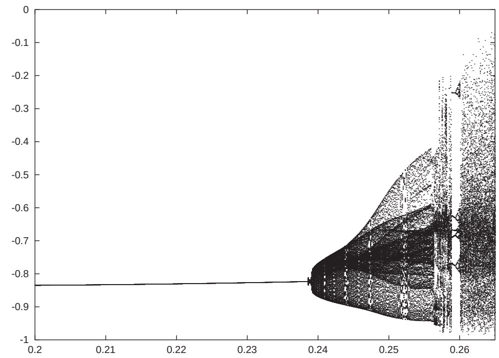

Figure 9: Bifurcation diagram of the Poincare map for the three-phase synchronization. γ = 0.4. Horizontal: β. Vertical: x_1.

5. Conclusions

In this study, we investigated quasi-synchronization phenomena observed from simple chaotic circuits coupled by mutual inductors. By carrying out circuit experiments and computer calculations for two or three subcircuits case, we confirmed that various quasi-synchronization phenomena of chaos were stably observed.

In the future, we investigate phenomena observed from the case N $\geq$ 4. Moreover we investigate synchronization phenomena observed from two identical chaotic circuits are coupled by a nonlinear mutual inductor.

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