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Phase Propagation Phenomena in Two-Layer Cellular Neural Networks

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1. Introduction

Cellular Neural Networks (CNNs) [1] are constructed by cells connected each other. The cell contains linear and nonlinear current sources controlled by voltage. It is known that two-layer CNNs can exhibit oscillation by choosing an appropriate set of the parameters.[2]

In this work, we present that a simplification of CNN constructed by two cells can be described by van der Pol equation. Moreover, we confirm the generation of the phase propagation phenomena, which can be observed in coupled van der Pol oscillators [3], in the two-layer CNNs by computer simulations. We discuss the similarity between the two-layer CNNs and coupled van der Pol oscillators.

2. Oscillation in CNN constructed by two cells

CNN constructed by two cells used in this study is shown in Fig. 1.

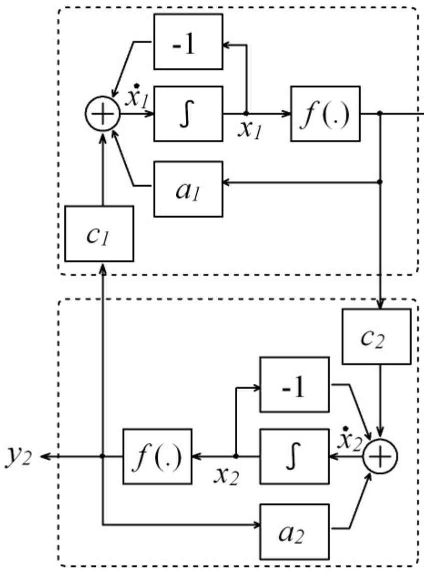

Figure 1: CNN constructed by two cells

The circuit equations governing these cells are as follows:

$$\dot{x_1} = -x_1 + a_1 y_1 + c_1 y_2 \quad (1)$$

$$\dot{x_2} = -x_2 + a_2 y_2 + c_2 y_1 \quad (2)$$

$$y_\ell = f(x_\ell) = 0.5\left(|x_\ell + 1| - |x_\ell - 1|\right) \quad (3)$$

$$(\ell = 1, 2)$$

where x_ℓ is state, y_ℓ is output in cell ℓ.

For simplicity, Equation (1), (2) are rewritten as follows:

$$\dot{x_1} = -x_1 + a_1 y_1 + c_1 x_2 \quad (4)$$

$$\dot{x_2} = -x_2 + a_2 x_2 + c_2 x_1 \quad (5)$$

And we approach piece-wised-linear function $f(\cdot)$ of output feedback from its own cell by 3rd order polynomial expression.

$$-x_1 + a_1 y_1 = -x_1 + 0.5 a_1\left(|x_1 + 1| - |x_1 - 1|\right) \quad (6)$$

$$\sim \epsilon\left(x_1 - x_1^3/3\right) \quad (7)$$

Thus, from (4), (5),

$$\ddot{x_1} - \epsilon(1 - x_1^2)\dot{x_1} - c_1(a_2 - 1)x_2 - c_1 c_2 x_1 = 0 \quad (8)$$

If $a_2 = 1$, Equation (8) is corresponding to van der Pol equation.

$$\ddot{x_1} - \epsilon(1 - x_1^2)\dot{x_1} - c_1 c_2 x_1 = 0, \quad (9)$$

Numerical analysis of (9) with $\epsilon = 0.30, c_1 c_2 = 1$ shown in Fig 2 is oscillated.

Equation (7) with $\epsilon = 0.30$ is shown in a dotted line of Fig.3. Determining a_1 equation (6) is similar to the dotted line. Equation (6) with $a_1 = 1.2$ is shown in a solid line of Fig.3,

We expect that the CNN constructed by 2 cells (1)–(3) given same parameters of oscillated van der Pol equation will be oscillated. Equation (1)–(3) with $a_1 = 1.2$, $a_2 = 1.0$, $c_1 = 1.0$, $c_2 = -1.0$ calculated by Runge-Kutta-Gill method is shown in Fig. 4.

3. Phase propagation phenomena in coupled van der Pol oscillators

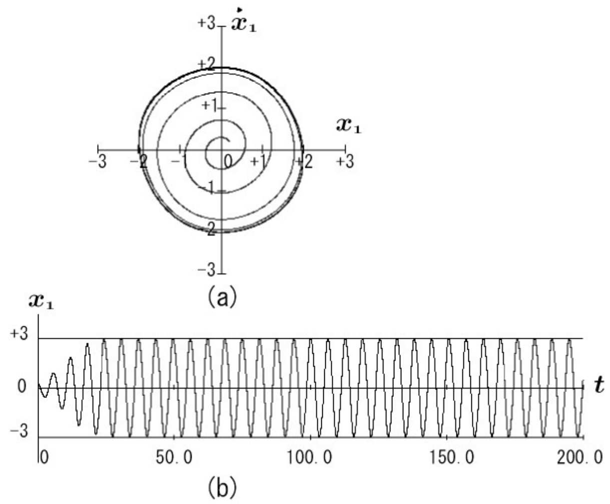

Figure 2: Oscillation in van der Pol Equation

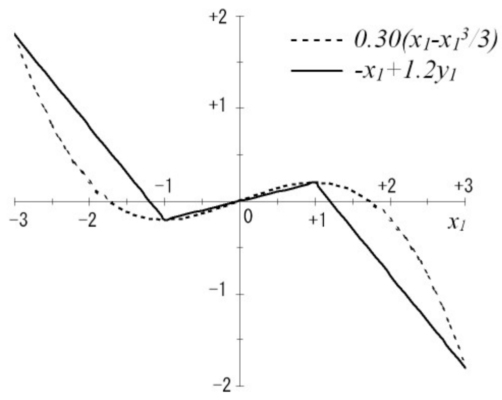

Figure 3: Approach by piece-wised-linear function

3.1. Circuit model

The van der Pol oscillations coupled by inductor L_0 as a comparative object used in this study is shown in Fig. 5.

We assume the v - i characteristics of the nonlinear negative resistors in the each circuit as the following function.

$$i_r(v_k) = -g_1 v_k + g_3 v_k^3 \quad (g_1, g_3 > 0) \tag{10}$$

The circuit equations governing the circuit in Fig.5 are written as

[First Oscillator]

$$\begin{aligned} \frac{du_1}{d\tau} &= w_1 \\ \frac{dw_1}{d\tau} &= -u_1 + \alpha(u_2 - u_1) + \epsilon\left(w_1 - w_1^3/3\right) \end{aligned} \tag{11}$$

[Middle Oscillator]

$$\begin{aligned} \frac{du_k}{d\tau} &= w_k \\ \frac{dw_k}{d\tau} &= -u_k + \alpha(u_{ki+1} - 2u_k + u_{k-1}) \\ &+ \epsilon\left(w_k - w_k^3/3\right) \end{aligned} \tag{12}$$

[Last Oscillator]

$$\begin{aligned} \frac{du_N}{d\tau} &= w_N \\ \frac{dw_N}{d\tau} &= -u_N + \alpha(u_{N-1} - u_1) + \epsilon\left(w_N - w_N^3/3\right) \end{aligned} \tag{13}$$

where

$$t = \sqrt{L_1 C}\tau, i_{L_1 k} = \sqrt{\frac{Cg_1}{3L_1 g_3}} u_k, v_k = \sqrt{\frac{g_1}{3g_3}} w_k,$$

$$\alpha = \frac{L_1}{L_0}, \epsilon = g_1\sqrt{\frac{L_1}{C}} \tag{14}$$

It should be noted that α corresponds to the coupling of the oscillators and ϵ corresponds to the nonlinearity of the oscillators. In this study, we calculate (2)-(4) by using the Runge-Kutta-Gill method.

3.2. Phase propagation phenomena

In this study, we use $N = 8$, $\alpha = 0.050$, $\epsilon = 0.30$ for numerical analysis. And the initial conditions are given as follows:

1. Setting the initial conditions of all oscillators as the same.

2. Putting the arbitrary phase difference of the voltage and the current to one oscillator.

Putting the phase difference +180 [deg] to 1 oscillator, result of simulation is shown in Fig 6. In Fig. 6, the vertical axis is the sum of voltages of adjacent oscillators, and horizontal axis is time. If the sum of voltages of adjacent oscillators is zero, the phase difference between adjacent oscillators have phase difference +180 [deg]. At first, only 1 oscillator has phase difference +180[deg], phase difference propagating to adjacent oscillator is shown though a time.

4. Phase propagation phenomena in 2 layer CNN

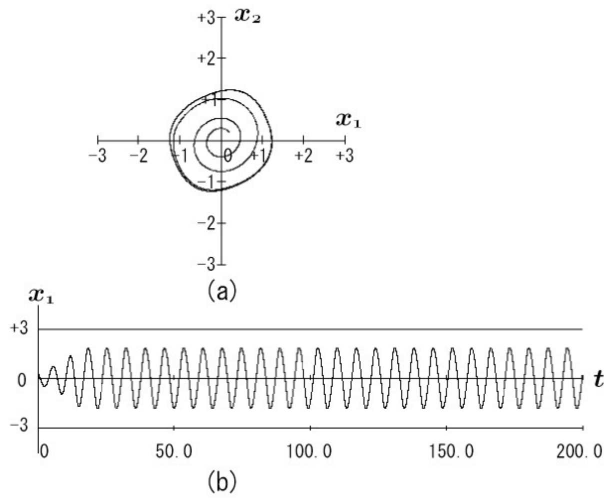

Figure 4: Oscillation in CNN constructed by 2 cells

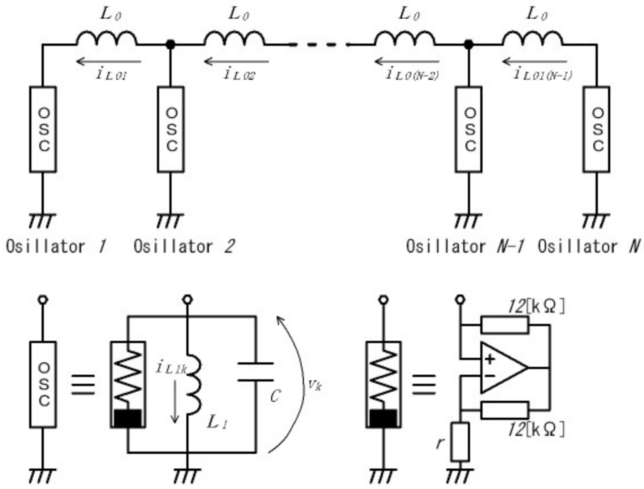

Figure 5: coupled van der Pol oscillators

4.1. Circuit model

In this study, we use 1-dimension 2-layer CNN , shown in Fig. 7, that coupled CNN oscillators shown in Fig.1.

The circuit equations governing the CNN in Fig.7 are written as

$$\dot{x}_{1,k} = -x_{1,k} + a_1 y_{1,k} + c_1 y_{2,k} \tag{15}$$

$$\dot{x}_{2,k} = -x_{2,k} + a_2 y_{2,k} + c_2 y_{1,k} + d_2 y_{1,(k-1)} + d_2 y_{1,(k+1)} \tag{16}$$

$$y_{\ell,k} = f(x_{\ell,k}) = 0.5\left(|x_{\ell,k}+1| - |x_{\ell,k}-1|\right) \tag{17}$$

$$(\ell = 1, 2)$$

4.2. Comparison with coupled van der Pol Oscillators

For simplicity, Equation (15), (16) are rewritten as follows:

$$\dot{x}_{1,k} = -x_{1,k} + a_1 x_{1,k} + c_1 x_{2,k} \tag{18}$$

$$\dot{x}_{2,k} = -x_{2,k} + a_2 y_{2,k} + c_2 x_{1,k} + d_2 x_{1,(k-1)} + d_2 x_{1,(k+1)} \tag{19}$$

And we approach piece-wised-linear function $f(\cdot)$ of output feedback from its own cell by 3rd order polynomial expression like (6), (7).

$$-x_{2,k} + a_2 y_{2,k}$$

$$= -x_{2,k} + 0.5 a_2\left(|x_{2,k}+1| - |x_{2,k}-1|\right) \tag{20}$$

$$\sim \epsilon\left(x_{2,k} - x_{2,k}^3/3\right) \tag{21}$$

Thus, from (18), (19),

$$\dot{x}_{1,k} = (a_1 - 1)x_{1,k} + c_1 x_{2,k} \tag{22}$$

$$\dot{x}_{2,k} = +c_2 x_{1,k} + \epsilon\left(x_{2,k} - x_{2,k}^3/3\right) + d_2 x_{1,(k-1)} + d_2 x_{1,(k+1)}$$

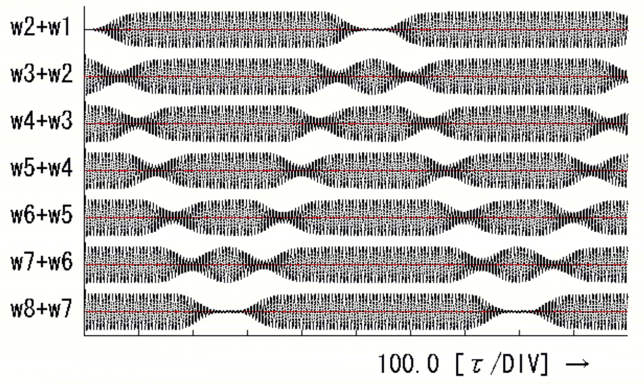

Figure 6: Phase propagation phenomena in coupled van der Pol oscillators

The parameters are determined by comparison with circuit equations governing coupled van der Pol Oscillotors (12), as follow:

$$a_1 = 1,\ \ c_1 = 1,\ \ c_2 = -(1+2\alpha),\ \ d_2 = \alpha$$

Now, we consider boundary condition as follows:

$$x_{1,0} = x_{1,1} \tag{23}$$

$$x_{1,(N+1)} = x_{1,N} \tag{24}$$

4.3. Result of simulation

To calculate the same conditions as 3.2 the parameters in 2-layer CNN are given as follow. Since $\alpha = 0.050$,

$$a_1 = 1,\ \ c_1 = 1,\ \ c_2 = -1.1, d_2 = 0.050$$

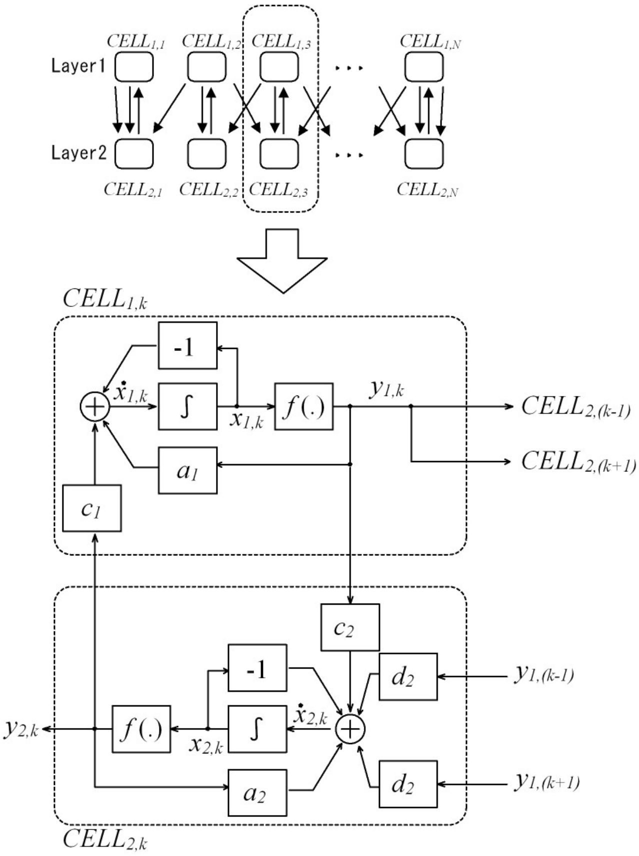

Figure 7: Two-layer CNN

Since $\epsilon = 0.30$ and Fig. 3,

$a_2 = 1.2$

The state initial values are given as follow:

1. Setting the state initial conditions of all cell as the same.
2. Putting the arbitrary phase difference of the state value to one cell.

Putting the phase difference +180 [deg] to state of $\mathrm{Cell}_{1,1}$ result of simulation is not observed phase propagation phenomena.

We consider 2-layer CNN changed into the state feedback in the output feedback from other layer. The circuit equations governing the changed 2-layer CNN are following.

$$\dot{x}_{1,k} = -x_{1,k} + a_1 y_{1,k} + c_1 x_{2,k} \quad (25)$$

$$\dot{x}_{2,k} = -x_{2,k} + a_2 y_{2,k} + c_2 x_{1,k} + d_2 y_{1,(k-1)} + d_2 y_{1,(k+1)} \quad (26)$$

Result of simulation in changed 2-layer CNN is shown Fig. 8. In Fig. 8, phase propagation phenomena similar to Fig.6 is observed.

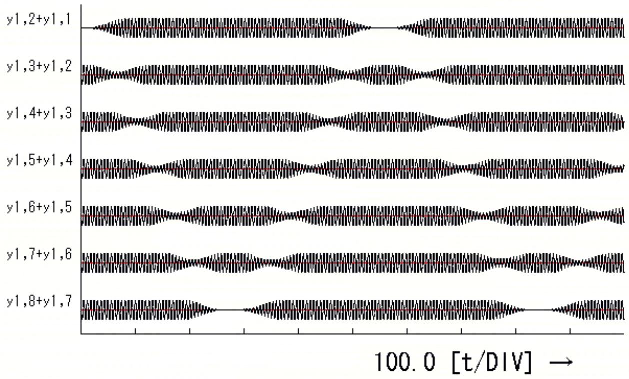

Figure 8: Result of simulation in changed 2-layer CNN

5. Conclusions

In this study, we investigated the relation between CNN constructed by two cells and van der Pol oscillator and found that an array of van der Pol oscillators coupled by inductors can be modeled by two-layer CNN. We expect that phase propagation phenomena will be generated in 2-layer CNN coupled the CNN Oscillators. Though, 2-layer CNN given same parameter of coupled van der Pol oscillators couldn't generate phase propagation phenomena. We observed that 2-layer CNN changed into the state feedback in the output feedback from other layer is generated phase propagation phenomena.

In future works, we will try to clarify phase propagation phenomena by comparing two CNNs, so that we can control the phase propagation for applications.

References

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[3] M. Yamauchi, M. Wada, Y. Nishio and A. Ushida, " Wave propagation phenomena of phase states in oscillators coupled by inductor as a ladder, " IEICE Trans. Fundamentals, vol.E82-A, no.11, pp.2592-2598, 1999.