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Analysis of an Asymmetrical Coupled System Using Chaotic Circuits and van der Pol Oscillators

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Abstract

In our previous study, we observed the interesting phenomenon. It is that the synchronous rate of the whole system increases in spite of increasing parameter mismatches in the system.

In this study, in order to verifier the this phenomenon, an asymmetrical coupled system is proposed and investigated. The system is realized by connecting chaotic subcircuits and van der Pol oscillators. In the case of five subcircuits, we confirmed synchronous phenomena in computer calculations. Additionally, It was confirmed that synchronous rates of chaotic subcircuits are increased by increasing a parameter mismatch rate of van der Pol oscillators. We consider that this result is corresponding to results of previous study.

1. Introduction

Coupled systems of chaotic subsystems generate various kinds of complex higher-dimensional phenomena such as spatio-temporal chaotic phenomena, clustering phenomena, and so on. One of the most studied systems may be the coupled map lattice proposed by Kaneko[1]. The advantage of the coupled map lattice is its simplicity. However, many of nonlinear phenomena generated in nature would be not so simple. Therefore, it is important to investigate the complex phenomena observed in natural physical systems such as electric circuits systems[2]-[6].

In our previous study[**?**], synchronization phenomena in an asymmetrical coupled system of chaotic circuits were investigated. The system is coupled globally and the coupling elements are resistors. Each subcircuit has two coupling nodes and the asymmetrical coupled system is realized by selecting one of two coupling nodes. This system was investigated as non-ideal system. The small parameter mismatches were given to the subcircuits as an mismatch of the oscillation frequency. As a result of investigating this system, interesting phenomenon was observed. The phenomenon is that the synchronous rate of the whole system increases in spite of increasing parameter mismatches in the system. We suppose that the phenomenon can be explained as follows:

(1) The synchronizations of the A-node circuits group and B-node circuits group are constricted each other.

(2) Decreasing the synchronization of one group decrease an influence to the other group.

(3) Therefore, the synchronization of the other group increases.

In this study, the system which replaced the one side of chaotic circuits with van der Pol oscillators is investigated.

2. Asymmetrical Coupled Systems

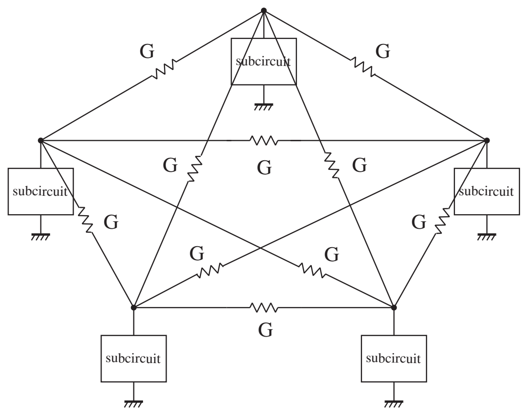


Figure 1: Asymmetrical coupled chaotic system.

The system is coupled globally and the coupling elements are resistors as shown in Fig. 1. Coupled subcircuits are chaotic circuits[8] as shown in Fig. 2 and van der Pol circuits. In our previous study, only chaotic circuits are used and asymmetry is realized as selecting one of two nodes in Fig. 2. In this study, circuits coupled with B-node are replaced van der Pol oscillators. Now, in order to carry out computer calculation, circuit equations are derived.

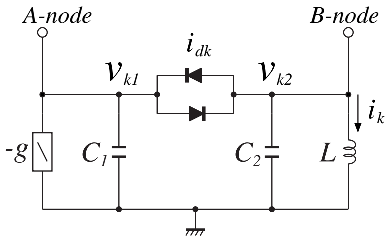
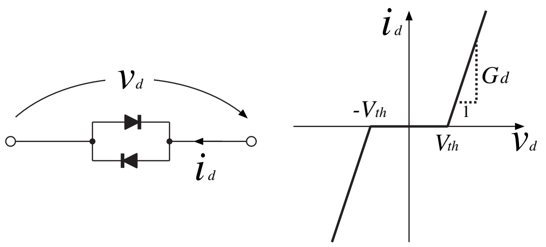


Figure 2: Chaotic subcircuit.



(a)circuit model (b)characteristics

Figure 3: Nonlinear resistor which consist of two diodes.

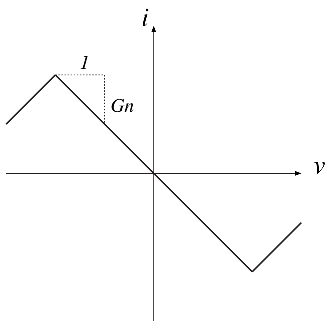


Figure 4: Characteristics of the nonlinear negative resistor on van der Pol oscillator.

chaotic circuit ($1 \leq k \leq m$):

$$\begin{cases} C_1 \dfrac{dv_{k1}}{dt} &= g v_{k1} - G_d f_1(v_{k1} - v_{k2}) \\ & \quad + G \left\{ \displaystyle\sum_{k=1}^{n} v_{i1} - (m+n) v_{k1} \right\}, \\ C_2 \dfrac{dv_{k2}}{dt} &= -i_{k3} + G_d f_1(v_{k1} - v_{k2}), \\ L_1 \dfrac{di_{k3}}{dt} &= (1+p_k) v_{k2}, \end{cases} \tag{1}$$

where,

$$f_1(v) = v + \frac{|v - V_{th}|}{2} - \frac{|v + V_{th}|}{2}.$$

Function $f_1(v)$ is characteristics of the nonlinear resistor shown in Fig. 3.

van der Pol oscillator ($m+1 \leq k \leq m+n$):

$$\begin{cases} C_3 \dfrac{dv_{k1}}{dt} &= -i_{k3} - G_n f_2(v_{k1}) \\ & \quad + G \left\{ \displaystyle\sum_{k=1}^{n} v_{i1} - (m+n) v_{k1} \right\}, \\ L_2 \dfrac{di_{k3}}{dt} &= (1+q_k) v_{k1}, \end{cases} \tag{2}$$

where,

$$f_2(v) = v - |v + V_{th}| + |v - V_{th}|.$$

Function $f_2(v)$ is characteristics of the nonlinear negative resistor shown in Fig. 4.

Using the following parameters and variables,

$$\begin{aligned} & x_{k1} = \frac{v_{k1}}{V_{th}}, \quad x_{k2} = \frac{v_{k2}}{V_{th}}, \quad x_{k3} = \frac{1}{V_{th}}\sqrt{\frac{L_1}{C_2}}, \\ & t = \sqrt{L_1 C_2}\tau, \quad \text{“}\cdot\text{”} = \frac{d}{d\tau}, \quad \alpha = \frac{C_2}{C_1}, \\ & \beta = g\sqrt{\frac{L_1}{C_2}}, \quad \gamma = G_d\sqrt{\frac{L_1}{C_2}}, \quad \delta = G\sqrt{\frac{L}{C_2}}, \\ & \varepsilon = \frac{C_2}{C_3}, \quad \zeta = G_n\sqrt{\frac{L_1}{C_2}}, \quad \eta = \frac{L_1}{L_2}, \end{aligned} \tag{3}$$

normalized equations are described as follows.

chaotic circuit($1 \leq k \leq m$):

$$\begin{cases} \dot{x}_k = & \alpha\beta x_k - \alpha\gamma f_1^{'}(x_k - y_k) \\ & + \alpha\delta \left\{ \displaystyle\sum_{i=1}^{m+n} x_i - (m+n) x_k \right\}, \\ \dot{y}_k = & -z_k + \gamma f_1^{'}(x_k - y_k), \\ \dot{z}_k = & (1+p_k) y_k, \end{cases} \tag{4}$$

where,

$$f_1^{'}(x) = x + \frac{(|x-1| - |x+1|)}{2}.$$

van der Pol oscillator($m+1 \leq k \leq m+n$):

$$\begin{cases} \dot{x}_k &= \varepsilon\zeta f_2^{'}(x_k) - \varepsilon z_k \\ & \quad + \delta\varepsilon \left\{ \displaystyle\sum_{i=1}^{m+n} x_i - (m+n) x_k \right\}, \\ \dot{z}_k &= (1+q_k)\eta x_k, \end{cases} \tag{5}$$

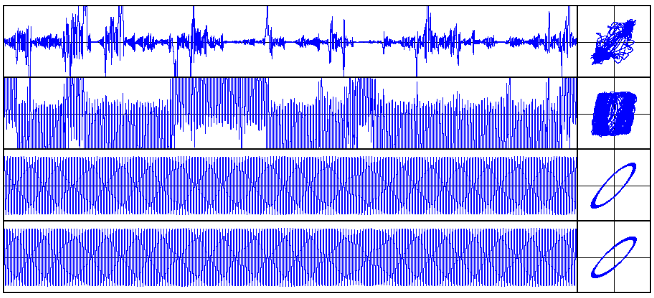
where,

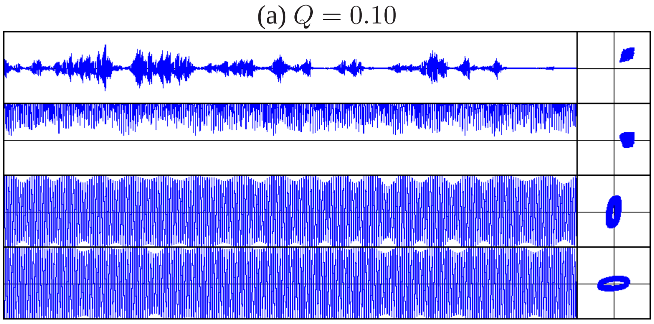
$$f_2'(x) = x - |x+1| + |x-1|.$$

We carry out computer calculations in the case of five subcircuits. The system consists of two chaotic circuits and three van der Pol oscillators. Figure 5 shows computer calculated results using following initial values and parameters.

$$\begin{cases} x_k(0) = y_k(0) = z_k(0) = 0.100, \ (k = 1, 2, 3, 4, 5), \\ \alpha = 0.400, \ \beta = 0.620, \ \gamma = 20.0, \ \delta = 0.080, \\ \varepsilon = 0.1, \ \zeta = 0.3, \ \eta = 0.2 \\ p_k = 0.001(k-1), \ q_k = Q(k-1). \end{cases} \tag{6}$$

The synchronization of van der Pol oscillators is very strong


(a) $Q = 0.10$


(b) $Q = 0.30$

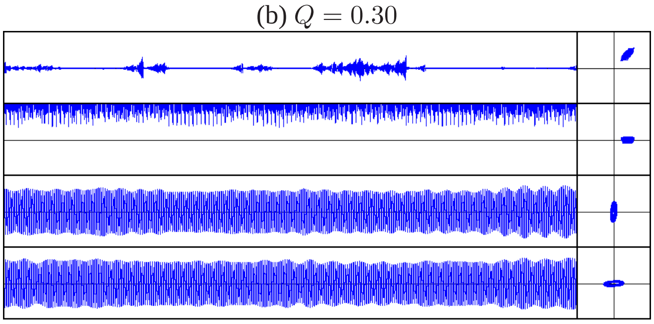

(e) $Q = 0.50$

Figure 5: Computer calculated results of proposed system.

against noise than synchronization of chaotic circuits. Accordingly, by using larger difference of q_k values than previous study, an asynchronous state is realized. Figs. 5 show computer calculated results. Left side figures show voltage differences between each subcircuits. Vertical axes show voltage differences and horizontal axes show time. Namely, in the case of synchronizing two subcircuits, the amplitude becomes zero. Right side figures show attractors of each subcircuits. The first graph shows the voltage difference between the two chaotic circuits. We can see that synchronization and un-synchronized burst appear alternately in a random way. The second graph shows the voltage difference between an chaotic circuit and a van der Pol oscillator. This graph is drawn on upper side and these two are not synchronized at all. The third and fourth graphs show the voltage differences between the van der Pol oscillators. Periodic frequency differences are observed.

In the case of previous study, corresponding results are shown in Fig. 6. From comparing two results, we can see as follows: Fig. 5(a) is similar to the case of higher negative resistor value than Fig. 6. Figs. 5(b) are similar to Fig. 6. In Figs. 5(c), synchronous rates are higher than Fig. 6. Additionally, voltage differences between van der Pol oscillators are lower than Fig. 5(b). Now, in order to investigate relation-

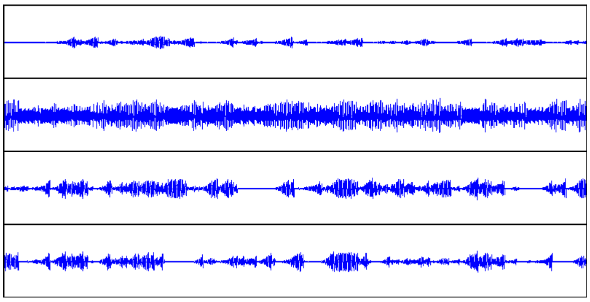


Figure 6: One of the computer calculated result of previous study.

ship between synchronous rates and parameter mismatches, we define the synchronous as shown in Fig. 7 and follows:

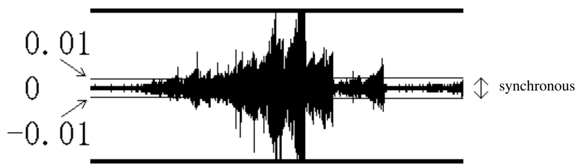


Figure 7: Definition of the synchronous.

$$|y_k - y_{k+1}| < 0.01 \tag{7}$$

where k is number of subcircuits.

Fig. 8 shows synchronous rate of chaotic circuits to Q which is corresponding to parameter mismatch rate. Fig. 9 shows a result of previous study corresponding to Fig. 8. A-node of Fig. 8 is corresponding to chaotic circuits and B-node is corresponding to van der Pol oscillators. In each of them, synchronous rate is increasing by increasing Q. We can consider to obtain the same phenomena. However, we can also consider that this result shows influence decreasing small amplitudes of van der Pol oscillators.

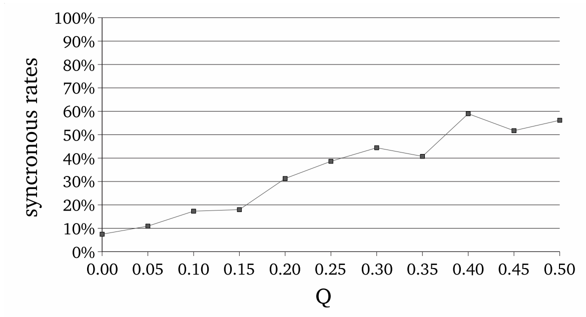


Figure 8: Synchronous rate of chaotic circuits to the parameter mismatch of van der Pol oscillators.

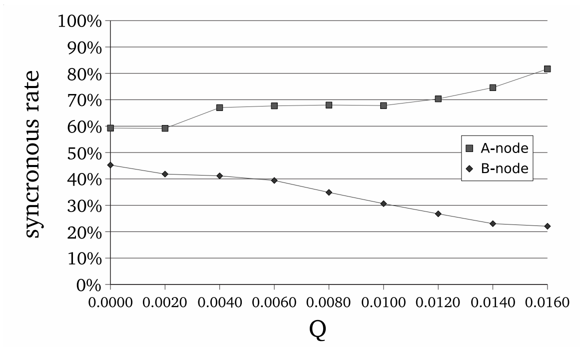


Figure 9: Synchronous rate of chaotic circuits to the parameter mismatch in previous study.

3. Conclusions

In this study, in order to verifier the phenomenon of previous study, an asymmetrical coupled system is proposed and investigated. The system is realized by connecting chaotic subcircuits and van der Pol oscillators. In the case of five subcircuits, we confirmed similar phenomena in computer calculations. Additionally, It was confirmed that synchronous rates of chaotic subcircuits are increased by increasing a parameter mismatch rate of van der Pol oscillators. We consider that this result is corresponding to results of previous study. However, we can also consider that this result shows influence of small amplitudes of van der Pol oscillators. In order to clarify this phenomena, we will investigate on the system which increased the number of subcircuits. In the system, it is possible to investigate the relationship between synchronous rate and the rate between numbers of chaotic circuits and van der Pol oscillators.

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