

Interconnect Modeling Using Numerical Inversion of Laplace Transforms

Junji Kawata¹, Yuichi Tanji², Yoshifumi Nishio³ and Akio Ushida¹

1	Tokushima Bunri University Phone: +81-87-894-5111	1314-1 Shido, Sanuki, Japan Email: kawata@fe.bunri-u.ac.jp
2	Kagawa University	2217-20 Hayashi, Takamatsu, Japan
3	Tokushima University	2-1 Mimami-josanjima, Tokushima, Japan

Abstract

In this paper, the admittance matrix of RLCG interconnects is modeled by partial fractions composed of the exact poles of the admittance matrix and the corresponding residues. The exact poles can be obtained by applying the Leverrier-Faddeeva algorithm. In this paper, a new method for calculating the residues is proposed. The residues can be calculated by using numerical inversion of Laplace transforms and the least squares method so that the impulse or pulse response of the partial fraction is matched with the one of each element of the exact admittance matrix in the time-domain. From the partial fraction representation, asymptotic equivalent circuit models suitable for SPICE-like simulators can be synthesized.

1. Introduction

The analysis and design of high speed LSI chips are becoming more and more important, because the sub-circuits coupled with interconnects embedded in the substrate sometimes cause the fault switching operations due to the signal delays, crosstalks, reflections and so on [1]-[5]. One of the requirements for the detailed analysis of these effects (i.e. signal integrity analysis) is to provide high-speed interconnect models suitable for general-purpose circuit simulators. The purpose of this study is to provide a passive macromodel for RLCG interconnects.

From the telegrapher's equation of the interconnect, we first derive the admittance matrix at the near and far ends [1]. Next, each of the elements of the admittance matrix is approximated by partial fraction using some dominant poles around the origin and corresponding residues. The poles can be calculated by a computational method proposed in our previous study [18]. The residues can be subsequently evaluated by using the numerical inversion of Laplace transforms and the least squares method so that the time-domain response of the partial fraction matches with the one of each element of the exact admittance matrix. We show the numerical methods for calculating the exact poles and the corresponding residues in Sect. 2 and Sect. 3, respectively. Section 4 gives illustrative examples.

2. Calculation of Exact Poles

Now, consider a uniform N coupled RLCG interconnect described by the following telegrapher's equations:

$$\left. \begin{aligned} \frac{d\mathbf{V}(x, s)}{dx} &= -(\mathbf{R} + s\mathbf{L})\mathbf{I}(x, s) \\ \frac{d\mathbf{I}(x, s)}{dx} &= -(\mathbf{G} + s\mathbf{C})\mathbf{V}(x, s) \end{aligned} \right\} \quad (1)$$

The input and output relations at the near and far ends are described by the admittance matrix as follows [1]:

$$\begin{bmatrix} \mathbf{I}(0, s) \\ \mathbf{I}(d, s) \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{11}(s) & \mathbf{Y}_{12}(s) \\ \mathbf{Y}_{21}(s) & \mathbf{Y}_{22}(s) \end{bmatrix} \begin{bmatrix} \mathbf{V}(0, s) \\ \mathbf{V}(d, s) \end{bmatrix} \quad (2)$$

where, using the eigenvalues $\mathbf{\Gamma}(s) = \text{diag}[\gamma_i(s)]$, we have the following relations:

$$\left. \begin{aligned} \mathbf{Y}_{11}(s) &= \mathbf{Y}_{22}(s) \\ &= \mathbf{P}_c(s)\text{diag}[\coth \gamma_i(s)d]\mathbf{P}_v(s)^{-1} \\ \mathbf{Y}_{12}(s) &= \mathbf{Y}_{21}(s) \\ &= -\mathbf{P}_c(s)\text{diag}\left[\frac{1}{\sinh \gamma_i(s)d}\right]\mathbf{P}_v(s)^{-1} \end{aligned} \right\} \quad (3)$$

where

$$\left. \begin{aligned} \text{diag}[\gamma_i(s)^2] &= \mathbf{P}_v(s)^{-1}(\mathbf{R} + s\mathbf{L})(\mathbf{G} + s\mathbf{C})\mathbf{P}_v(s) \\ \text{diag}[\gamma_i(s)^2] &= \mathbf{P}_c(s)^{-1}(\mathbf{G} + s\mathbf{C})(\mathbf{R} + s\mathbf{L})\mathbf{P}_c(s) \end{aligned} \right\} \quad (4)$$

Thus, we have

$$\mathbf{P}_c(s) = (\mathbf{R} + s\mathbf{L})^{-1}\mathbf{P}_v(s)\mathbf{\Gamma}(s) \quad (5)$$

$$\mathbf{P}_c^T(s) = \mathbf{P}_v(s)^{-1} \quad (6)$$

Observe that all the poles of admittance matrix can be found at the locations satisfying $\sinh \gamma_i(s)d = 0$, $i = 1, 2, \dots, N$. Thus, we have the following theorem for calculation of the exact poles.

Theorem 1: *The locations of poles satisfying the relations (3) are found by solving the following equation:*

$$\left| (\mathbf{R} + s\mathbf{L})(\mathbf{G} + s\mathbf{C}) + \left(\frac{n\pi}{d}\right)^2 \mathbf{I} \right| = 0, \quad n = 1, 2, \dots \quad (7)$$

where d is the length of the interconnect.

Proof: In the case of $n \neq 0$, we have from (3) that the poles satisfy the following relations:

$$|\mathbf{P}_v(s)\text{diag}[\sinh \gamma_i(s)d]\mathbf{P}_c(s)^{-1}| = 0 \quad (8)$$

$$|\mathbf{P}_v(s)\text{diag}[\tanh \gamma_i(s)d]\mathbf{P}_c(s)^{-1}| = 0 \quad (9)$$

Since the transfer matrices $\mathbf{P}_v(s)$ and $\mathbf{P}_c(s)$ are nonsingular for the nonzero eigenvalues, the poles satisfying the above two relations are given by

$$\gamma_i(s)d = jn\pi, \quad i = 1, 2, \dots, N, \quad n = 1, 2, \dots$$

Therefore, the characteristic equation from the telegrapher's equation (1) needs to satisfy the relation (7).

Q.E.D.

Corollary 1.1: *The poles at $n = 0$ satisfy the following relation:*

$$|\mathbf{R} + s\mathbf{L}| = 0 \quad (10)$$

Proof: We have $\gamma_i = 0$ at $n = 0$. Thus, using the relation of (5), we have the following relation;

$$\begin{aligned} & \lim_{\gamma_i \rightarrow 0} \mathbf{P}_c(s)\text{diag}\left[\frac{1}{\sinh \gamma_i(s)d}\right] \mathbf{P}_v(s)^{-1} \\ &= (\mathbf{R} + s\mathbf{L})^{-1} \mathbf{P}_v(s)\text{diag}[\gamma_i(s)] \frac{1}{\text{diag}[\gamma_i(s)d]} \mathbf{P}_v(s)^{-1} \\ &= \frac{1}{d}(\mathbf{R} + s\mathbf{L})^{-1} \end{aligned} \quad (11)$$

Q.E.D.

The exact poles for (7) can be calculated by using the Leverrier-Faddeeva algorithm [17]. The detail has been shown in [18].

We choose the dominant poles located around the origin of the complex plane, because the poles have large impact on the transient responses. Thus, an arbitrary (i, j) -element of the admittance matrix given by (3) is approximated by partial fraction composed of the dominant poles and the corresponding residues, in the following form:

$$\hat{Y}(s) = \sum_{i=1}^N \left\{ \frac{k_{0,i}}{s - p_{0,i}} + \sum_{n=1}^M \frac{b_{1,i,n}s + b_{0,i,n}}{s^2 + a_{1,i,n}s + a_{0,i,n}} \right\} \quad (12)$$

where N is the number of conductors and M shows the number of complex conjugate pairs for each $\gamma_i(s)$. We can derive asymptotic equivalent circuit models that satisfy the relation (12), but omit in this paper due to space limitation (refer to [18] for detail).

3. Calculation of Residues with Numerical Inversion of Laplace Transforms

The residues have to be evaluated in order to approximate each element of the exact admittance matrix (3) with partial fraction (12). In our previous study [18], the residues are calculated in the frequency-domain so that the frequency response curve of the partial fraction matches to the one of each

element of the exact admittance matrix. In this paper, we propose an another method to compute the residues by using the numerical inversion of Laplace transforms and the least squares method.

Given a Laplace transform $V(s)$, the time domain response $v(t)$ can be calculated by the following formula:

$$v(t_n) = \frac{e^{at}}{T} \times \left[\text{Re} \left\{ \sum_{k=0}^{K-1} V \left(a + \frac{jk\pi}{T} \right) e^{j2k(\pi/K)n} \right\} - \frac{1}{2}V(a) \right] \quad (13)$$

In this section, a novel algorithm for the residues evaluation using the time-domain responses of the admittance matrix is presented. With simplified notation for the partial fraction (12), the frequency-domain response of the partial fraction for an external input $E(s)$ is given by

$$\begin{aligned} \hat{I}(s) &= \hat{Y}(s)E(s) \\ &= \left\{ \sum_{m=1}^N \frac{k_{0,m}}{s - p_{0,m}} + \sum_{m=1}^{NM} \frac{b_{1,m}s + b_{0,m}}{s^2 + a_{1,m}s + a_{0,m}} \right\} E(s) \\ &= \sum_{m=1}^N \hat{I}_{0,m}(s) + \sum_{m=1}^{NM} \hat{I}_{1,m}(s) \end{aligned} \quad (14)$$

Then the time-domain response for (14) can be obtained by

$$\hat{i}(t) = \sum_{m=1}^N \hat{i}_{0,m}(t) + \sum_{m=1}^{NM} \hat{i}_{1,m}(t) \quad (15)$$

where

$$\begin{aligned} \hat{i}_{0,m}(t) &= \mathcal{L}^{-1} \left\{ \hat{I}_{0,m}(s) \right\}, \\ \hat{i}_{1,m}(t) &= \mathcal{L}^{-1} \left\{ \hat{I}_{1,m}(s) \right\} \end{aligned}$$

and these responses in (15) are calculated by the numerical inversion of Laplace transforms (13). If a suitable set of residues is found, the time-domain response $\hat{i}(t)$ coincides with one of the exact admittance matrix $i(t) = \mathcal{L}^{-1}\{Y(s)E(s)\}$. Thus, assigning $\hat{I}_{1,m}(s)$ and $\hat{I}_{0,m}(s)$ into (13), rearranging with respect to the residues $\{k_{0,m}, b_{1,m}, b_{0,m}\}$, and considering the condition $i(t_n) = \hat{i}(t_n)$ to be satisfied for any sampled time t_n ($n = 1, \dots, K-1$), we have

$$\mathbf{AX} = \mathbf{B} \quad (16)$$

where \mathbf{X} is unknown vector containing the residues and \mathbf{B} is column vector consists of the time-domain responses for each element of the exact admittance matrix, which have to be calculated by the numerical inversion of Laplace transforms (13) in advance.

$$\mathbf{X} = [k_{0,1} \dots k_{0,N} \ b_{1,1} \ b_{0,1} \dots b_{1,NM} \ b_{0,NM}]^T$$

$$\mathbf{Y} = [i(t_0) \ i(t_1) \ \dots \ i(t_{K-1})]^T$$

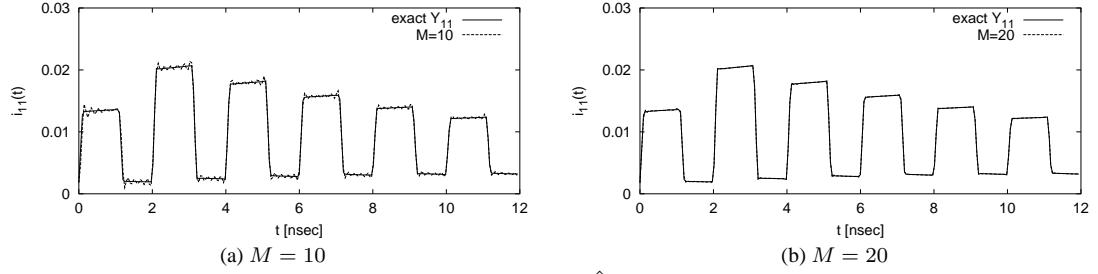


Figure 1: Pulse response waveforms $i_{11}(t)$ and $\hat{i}_{11}(t)$ for single conductor interconnect.

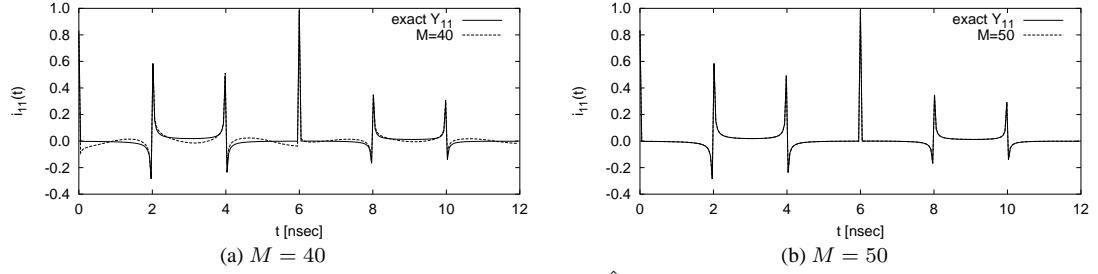


Figure 2: Impulse response waveforms $i_{11}(t)$ and $\hat{i}_{11}(t)$ for single conductor interconnect.

Each element of \mathbf{A} is given as:

$$A_{n,m} = \frac{e^{at_n}}{T} \times \left[\text{Re} \left\{ \sum_{k=0}^{K-1} \frac{E(s_k)e^{j2k(\pi/K)n}}{s_k - p_{0,m}} \right\} - \frac{1}{2} \cdot \frac{E(a)}{a - p_{0,m}} \right], \quad m = 1, \dots, N$$

$$\left. \begin{aligned} A_{n,(N+2m-1)} &= \frac{e^{at_n}}{T} \times \left[\text{Re} \left\{ \sum_{k=0}^{K-1} \frac{E(s_k)e^{j2k(\pi/K)n}s_k}{s_k^2 + a_{1,m}s_k + a_{0,m}} \right\} - \frac{1}{2} \cdot \frac{E(a)a}{a^2 + a_{1,m}a + a_{0,m}} \right] \\ A_{n,(N+2m)} &= \frac{e^{at_n}}{T} \times \left[\text{Re} \left\{ \sum_{k=0}^{K-1} \frac{E(s_k)e^{j2k(\pi/K)n}}{s_k^2 + a_{1,m}s_k + a_{0,m}} \right\} - \frac{1}{2} \cdot \frac{E(a)}{a^2 + a_{1,m}a + a_{0,m}} \right] \end{aligned} \right\}, \quad m = 1, \dots, NM$$

where $s_k = a + j\pi k/T$. The least squares solution \mathbf{X} can be found by using QR factorization so that $\|\mathbf{B} - \mathbf{AX}\|_2$ is minimized.

4. Illustrative Examples

First, we compute the exact poles by means of the method described in Sect. 2. The parameters of interconnects are set as follows:

$$R = 0.5[\Omega/\text{mm}], \quad L = 10[\text{nH/mm}], \quad C = 4[\text{pF/mm}], \\ G = 0.5[\text{mS/mm}], \quad d = 5.0[\text{m}]$$

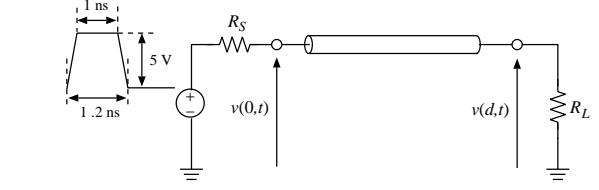


Figure 3: Simple linear circuit with single conductor interconnect. ($R_S = 10[\Omega]$, $R_L = 10[\Omega]$)

and the parameters of the numerical inversion of Laplace transforms are fixed as $a = 0.2$, $K = 500$ throughout this paper. Using the exact poles, we calculated the residues with our proposed method. As the external input $E(s)$, both cases of pulse and impulse are considered. It is assumed that the input pulse and the period for calculating the time-domain responses are the same as those used in the transient simulation. The pulse response waveforms for the partial fraction and the exact admittance matrix with varying the value of M are shown in Fig. 1. Where, the time-domain responses of the partial fractions are computed by (12) after evaluating the residues. The pulse response waveforms of the partial fractions approach those of exact admittance matrix as M increases, and the former almost matches to the latter for the case of $M = 20$. While on the other hand, the impulse responses of the partial fractions do not match to the one of exact admittance matrix until $M = 50$, as shown in Fig. 2. It can be expected that this is due to the bandwidth of input signal. Thus it is reasonable to employ the pulse input from the viewpoint of computational costs and hereinafter we focus on only the pulse responses.

Next the transient simulations are carried out. As an ex-

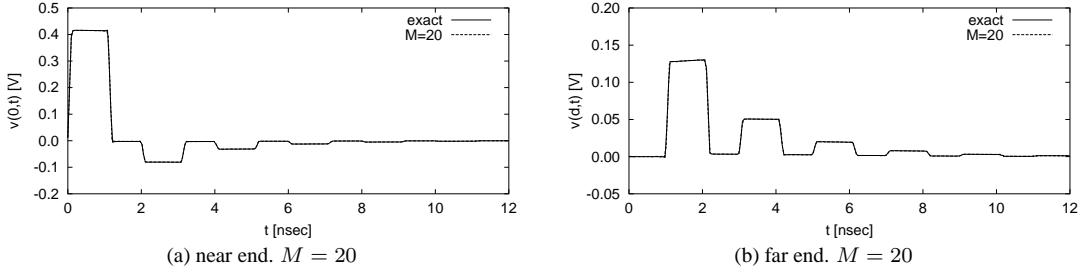


Figure 4: Transient responses for example circuit shown in Fig. 5.

ample, a simple linear circuit with a single conductor interconnect shown in Fig. 5 is simulated by using the numerical inversion of Laplace transforms for the exact admittance matrix (3) and the approximated partial fractions (12). Note that the numerical inversion of Laplace transforms can be applied to *only* linear circuits, but we use this method in order to investigate the effects of the reduction approximating the admittance matrix with the partial fractions. Fig. 6 shows the transient response waveforms, where the pulse input shown in Fig. 5 is applied for the residues calculations. We found that the transient responses for the partial fractions with the residues obtained by our proposed method approximately coincide with those of the exact admittance matrix.

5. Conclusions

In this paper, we have proposed an algorithm for calculating the residues which are required for partial fraction approximation of the admittance matrix of interconnects. The residues can be calculated by means of the numerical inversion of Laplace transforms and the least squares method so that the time-domain responses of the partial fraction match to those of the exact admittance matrix. In our simulations, it has been shown that lower order of approximation is possible fitting by pulse response waveforms.

In future, we will carry out the transient simulations by making use of the asymptotic equivalent circuits and compare with the results obtained using the SPICE simulator.

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