2005 RISP International Workshop on Nonlinear
Circuit and Signal Processing (NCSP'05)
Hawaii, USA, Mar. 4-6, 2005.

Asymmetric Alternating Propagation in Coupled Oscillators

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1. Introduction

Many kinds of digital circuits for various applications have been designed and developed. These circuits use signals generated by oscillators. Therefore, oscillators always keep an important position, and various oscillators are still continued to be designed and are put to practical use. Various researches about systems of coupled oscillators have been carried out up to now. And, a lot of interesting phenomena have been reported. However, discoveries of many new phenomena still continue. Therefore, coupled oscillators catch interests of researchers [1]-[4].

Recently, we discovered continuously existing wave of changing phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase in coupled van der Pol oscillators by inductors as a ladder. This phenomenon is observed in steady state. We call this phenomenon as "phase-inversion wave." And, the mechanisms of "propagation," "disapearance," "reflection in the middle of the array" and "reflection at an edge of the array," which are the basic characters of the phase-inversion waves, have been clarified [5]. Further, when even-number of oscillator ladders are coupled as a star and the phase-inversion waves are generated from the edges of the half numbers of ladders, the phase-inversion waves propagate to the center oscillator, do not reflect and penetrate to other ladders. We call this phenomenon as "alternating propagation"[6]-[7].

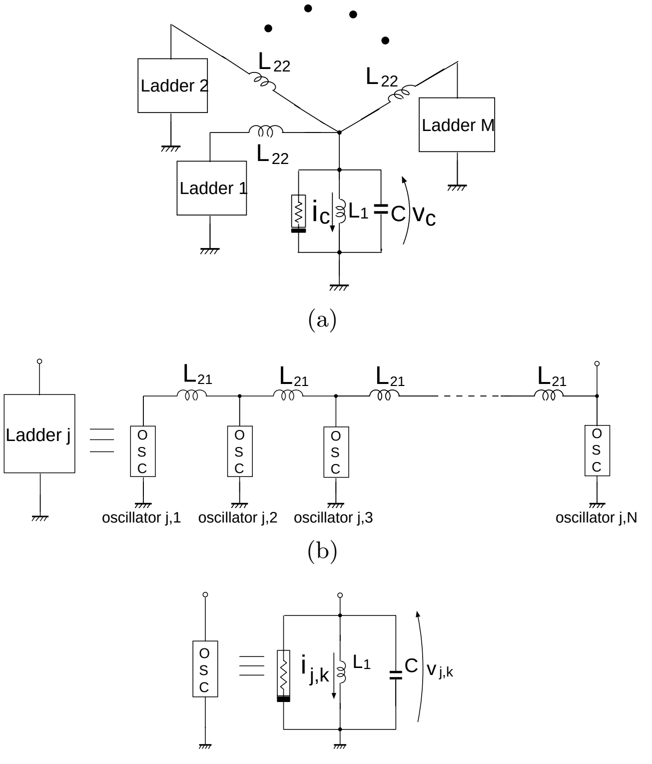
In this study, many ladders which are composed by van der Pol oscillators are coupled at the center oscillator. The van der Pol oscillators in each ladder are coupled by inductors. The phase-inversion waves are observed by computer calculations in this system. We investigate asymmetric alternating propagation of the phase-inversion waves by changing parameters, the number of generating phase-inversion waves and the number of the ladders. Further, we investigate this phenomenon by changing the value of some inductors used in the coupling between the center oscillator and the ladders.

2. Circuit Model

The circuit model used in this study is shown in Fig. 1. In each ladder, N van der Pol oscillators are coupled by coupling inductors L_{21} as Fig. 1(b). For computer calculations, we assume the $v-i$ characteristics of the nonlinear negative resistors in all the oscillators as the following function.

$$i_r(v) = -g_1 v + g_3 v^3 \quad (g_1, g_3 > 0) \tag{1}$$

The circuit equations governing the circuit in Fig. 1 are written as:



(a)

(b)

(c)

Figure 1: Circuit Model. (a) Oscillator ladders coupled at the center oscillator. (b) Coupled oscillators as a ladder. (c) van der Pol oscillator.

[Center Oscillator]

$$\dot{x}_c = y_c \tag{2}$$

$$\dot{y}_c = -x_c + \alpha_1 \left(\sum_{i=1}^{M} x_{i,N} - M x_c \right) + \varepsilon \left(y_c - \frac{1}{3} y_c^3 \right)$$

[Edge Oscillators] $(j = 1 \sim M)$

$$\dot{x}_{j,1} = y_{j,1} \tag{3}$$

$$\dot{y}_{j,1} = -x_{j,1} + \alpha_1 \left(x_{j,2} - x_{j,1} \right) + \varepsilon \left(y_{j,1} - \frac{1}{3} y_{j,1}^3 \right)$$

[Middle Oscillators] $(j = 1 \sim M, \ \ k = 2 \sim N-1)$

$$\dot{x}_{j,k} = y_{j,k} \tag{4}$$

$$\begin{aligned}\dot{y}_{j,k} = &-x_{j,k} + \alpha_1 \left(x_{j,k+1} - 2x_{j,k} + x_{j,k-1} \right) \\ &+ \varepsilon \left(y_{j,k} - \frac{1}{3} y_{j,k}^3 \right)\end{aligned}$$

[Adjacent Oscillators of Center Oscillator] $(j = 1 \sim M)$

$$\dot{x}_{j,N} = y_{j,N} \tag{5}$$

$$\begin{aligned}\dot{y}_{j,N} = &-x_{j,N} + \alpha_1 \left(x_{j,N-1} - x_{j,N} \right) + \alpha_2 (x_c - x_N) \\ &+ \varepsilon \left(y_{j,N} - \frac{1}{3} y_{j,N}^3 \right)\end{aligned}$$

where

$$t = \sqrt{L_1 C}\tau, \ i_{j,k} = \sqrt{\frac{C g_1}{3 L_1 g_3}} x_{j,k}, \ i_c = \sqrt{\frac{C g_1}{3 L_1 g_3}} x_c,$$

$$v_{j,k} = \sqrt{\frac{g_1}{3 g_3}} y_{j,k}, \ v_c = \sqrt{\frac{g_1}{3 g_3}} y_c, \ \alpha_1 = \frac{L_1}{L_{21}},$$

$$\alpha_2 = \frac{L_1}{L_{22}}, \ \varepsilon = g_1 \sqrt{\frac{L_1}{C}}, \ \frac{d}{d\tau} = \text{“}\cdot\text{”}.$$

It should be noted that α_1 and α_2 correspond to the coupling of the oscillators and ε corresponds to the nonlinearity of the oscillators. Throughout the paper, we fix $N = 8$, $\alpha_1 = 0.050$, $\varepsilon = 0.250$ and $\Delta\tau = 0.01$ and calculate (2)-(5) by using the fourth-order Runge-Kutta method.

3. Alternating Propagation of the Phase-Inversion Waves

Figure 2 shows an alternating propagation of two pairs phase-inversion waves on four ladders. Vertical axes are sum of two voltages of adjacent oscillators and horizontal axes are time. White regions in the diagram correspond to the states that sum of voltages of the two oscillators are close to zero, namely adjacent two oscillators synchronized at anti-phase. While, black regions correspond to the states that sum of voltages of the two oscillators with large amplitude, namely adjacent two oscillators synchronized at in-phase.

In this figure, we can see that two pairs of phase-inversion waves in the 1st and the 2nd ladders arrive at the center oscillator, and that the waves do not reflect and penetrate to the other two ladders; the 3rd and the 4th ladders. After reflecting at the edges of the 3rd and the 4th ladders, the waves return to the 1st and the 2nd ladders via the penetration at the center oscillator. When even-number of ladders are coupled and the phase-inversion waves are generated from the edges of the half number of ladders, this phenomenon can be observed. In other words, when Mg, the number of the ladders in which the waves are generated, is equal to $M/2$, the alternating propagation can be observed. The range of α_2 for which this phenomenon can be observed is 0.071∼0.216.

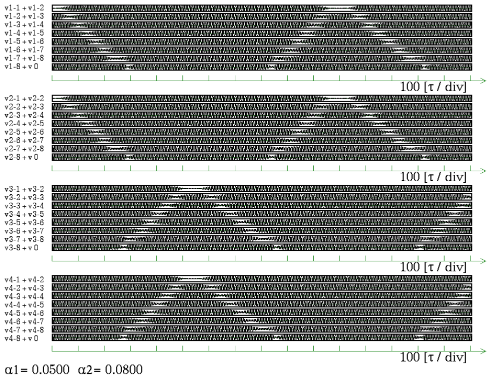


Figure 2: Alternating propagation for $M = 4$. Waves are generated from $\mathrm{OSC}_{1,1}$ and $\mathrm{OSC}_{2,1}$.

4. Asymmetric Alternating Propagation of the Phase-inversion Waves

In this study, we investigate the alternating propagation of the phase-inversion waves for the case of $Mg \neq M/2$. We call this type of phenomenon as asymmetric alternating propagation.

4.1. Asymmetric alternating propagation 1

In this section, we observe asymmetric alternating propagation phenomena, when many ladders are coupled. Figure 3 shows an asymmetric alternating propagation of the phase-inversion waves, which are generated from 5 ladders, on nine ladders, namely $M = 9$ and $Mg = 5$. When five pairs of phase-inversion waves arrive the center oscillator, the phase-inversion waves do not reflect and penetrate to the other four ladders (when around 250[τ]). The range of α_2 at which this phenomenon is observed are around 0.104∼0.186.

Table 1 shows the number of Mg for which symmetric and asymmetric alternating propagations are observed. As the number of the ladders increases, asymmetric alternating propagation becomes easier to be observed.

4.2. Asymmetric alternating propagation 2

In this section, the values of some L_{22} are replaced by L_{23}. Namely, the coupling structure becomes asymmetric. The

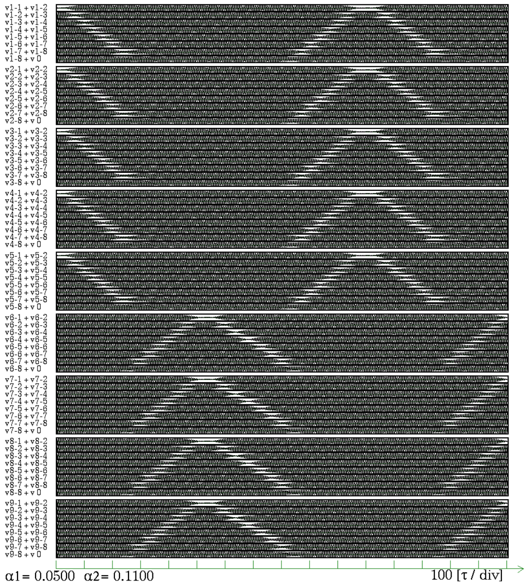


Figure 3: Asymmetric alternating propagation for $M = 9$ and $Mg = 5$.

parameter of α_3 is newly added for L_{23} as follows;

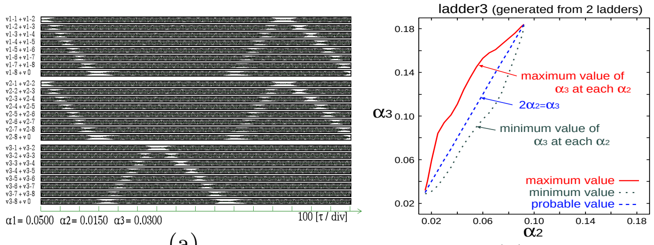
$$\alpha_3 = \frac{L_1}{L_{23}}. \tag{6}$$

4.2.1. M=3 and Mg=2

Figure 4 shows an asymmetric alternating propagation observed for $M = 3$. The coupling parameter between the third ladder and the center oscillator is changed to α_3. Phase-inversion waves are generated from 2 ladders.

(a) Two pairs of phase-inversion waves propagate to the center oscilator and penetrate to another ladder.

(b) This graph shows the ranges of α_2 and α_3 generating asymmetric alternating propagation. The upper line indicates the maximum value of α_3 for each α_2. The lower line indicates the minimum value of α_3 for each α_2. Namely, this phenomenon can be observed when α_3 is about $2\alpha_2$.



(a) (b)

Figure 4: Asymmetric alternating propagation. (a) $M = 3$ and $Mg = 2$. (b) Range where the phenomenon can be observed.

Table 1: Number of Mg for which alternating propagation is observed.

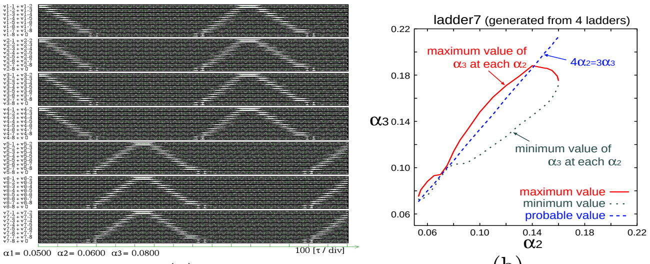
M	Symmetric	Asymmetric
3	×	×
4	2	×
5	×	×
6	3	×
7	×	×
8	4	×
9	×	4 , 5
10	5	×
11	×	5 , 6
12	6	×
13	×	6 , 7
14	7	6 , 8
15	×	7 , 8
16	8	7 , 9
17	×	8 , 9
18	9	8 , 10
19	×	9 , 10
20	10	9 , 11
21	×	9 , 10 , 11 , 12
22	11	10 , 12
23	×	10 , 11 , 12 , 13
24	12	11 , 13
25	×	11 , 12 , 13 , 14
26	13	12 , 14
27	×	12 , 13 , 14 , 15
28	14	13 , 15
29	×	13 , 14 , 15 , 16
30	15	13 , 14 , 16 , 17
31	×	14 , 15 , 16 , 17
32	16	14 , 15 , 17 , 18
33	×	15 , 16 , 17 , 18
34	17	15 , 16 , 18 , 19
35	×	16 , 17 , 18 , 19
36	18	16 , 17 , 19 , 20
37	×	16 , 17 , 18 , 19 , 20 , 21
38	19	17 , 18 , 20 , 21
39	×	17 , 18 , 19 , 20 , 21 , 22
40	20	18 , 19 , 21 , 22
···	···	···

4.2.2. M=7 and Mg=4

Figure 5 shows an asymmetric alternating propagation observed for $M = 7$. The coupling parameters between the ladder 5∼7 and the center oscillator are changed to α_3 respectively. Phase-inversion waves are generated from 4 ladders.

(a) Four pairs of phase-inversion waves propagate to the center oscillator and penetrate to the other three ladders.

(b) This graph shows the ranges of α_2 and α_3 generating asymmetric alternating propagation. The upper line indicates the maximum value of α_3 for each α_2. The lower line indicates the minimum value of α_3 for each α_2. Namely, this phenomenon can be observed when α_3 is about $4/3$ α_2,



(a) (b)

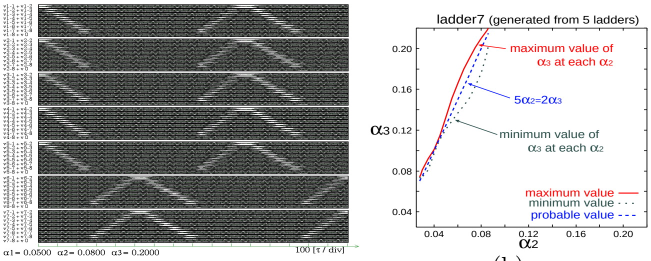
Figure 5: Asymmetric alternating propagation. (a) $M = 7$ and $Mg = 4$. (b) Range where the phenomenon can be observed.

4.2.3. M=7 and Mg=5

Figure 6 shows an asymmetric alternating propagation observed for $M = 7$. The coupling parameters between the ladder 6,7 and the center oscillator are changed to α_3 respectively. Phase-inversion waves are generated from 5 ladders.

(a) Five pairs of phase-inversion waves propagate to the center oscillator and penetrate to the other two ladders.

(b) This graph shows the ranges of α_2 and α_3 generating asymmetric alternating propagation. The upper line indicates the maximum value of α_3 for each α_2. The lower line indicates the minimum value of α_3 for each α_2. Namely, this phenomenon can be observed when α_3 is about $5/2$ α_2. It should be noted that the range where this phenomenon can be observed is narrower than the range in Fig. 5(b).



(a) (b)

Figure 6: Asymmetric alternating propagation. (a) $M = 7$ and $Mg = 5$. (b) Range where the phenomenon can be observed.

5. Conclusions

In this study, many ladders which were composed by van der Pol oscillators were coupled by an oscillator and many inductors. We investigated asymmetric alternating propagation of the phase-inversion waves by changing parameters, number of generating phase-inversion waves and the number of ladders.

As the number of the ladders increases, asymmetric alternating propagation becomes easier to be observed. Further, the asymmetric alternating propagation was able to be observed by changing the coupling parameters between the center oscillator and some ladders.

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