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Analysis of Chaotic Circuits using Wien Bridge Oscillator and a Resonator

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Abstract

In this study, we analyze chaotic circuits using a Wien bridge oscillator and a resonator. A Wien bridge oscillator and a resonator are coupled by diodes. These circuits are designed by our designing method of a chaotic circuit. Until now, we proposed and analyzed some chaotic circuits using this method. We consider that this study fortifies our designing method.

1. Introduction

Recently, designing principles of chaotic circuits are studied in actively. These studies aim to understand chaos and to apply chaos. Because electric circuit is superior in its simplicity, repeatability and response in the experiment, a electric circuit is a very useful tool for understanding chaos. On the other hand, many researchers developing chaotic circuit applications use a few popular chaotic circuits. In order to develop the excellent application, a chaotic circuit which has suitable characteristics should be used for the application. In our previous studies, a designing method of chaotic circuits was proposed. This method is to couple an oscillator and a resonator with diodes. We investigated this method applied to some oscillators and LC resonators. However, we have confirmed only one node per one oscillator.

In this study, we investigated combinations of a Wien bridge oscillator and two type LC resonators. In particular, we paid attention to a node that is coupled with diodes.

2. Designing method

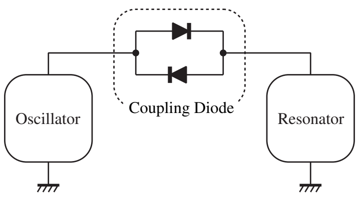


Figure 1: Designing method.

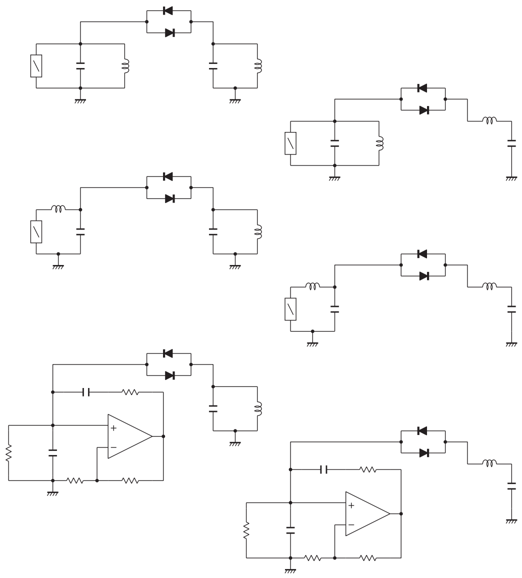


Figure 2: Chaotic circuits developed by our designing method.

Figure 1 shows a designing method proposed by us. One oscillator and one resonator are coupled with a coupling diode which is coupled two diodes each other. We consider that these diodes play a role of nonlinearity which is essential for the generation of chaos. Thus, circuits models of oscillators and resonators are modeled as linear model in our studies. Figures 2 show chaotic circuit developed by our designing method. van del Pol oscillators (parallel and series type), and Wien bridge oscillator are used as a oscillator. LC resonators (parallel and series type) are used as a resonator.

3. Circuit Model

3.1. Wien bridge oscillator

Figure 3 shows the circuit schematic of a Wien bridge oscillator. The oscillating frequency is decided by C_1, R_1, C_2

and R_2. The amplitude is decided by R_3 and R_4. We investigate combinations of coupling nodes shown as nodes 1, 2, 3 and 4 in Fig. 3.

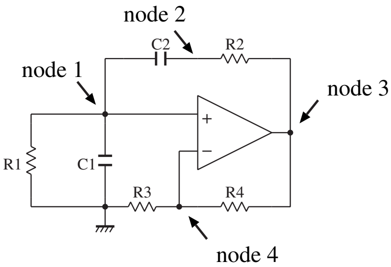
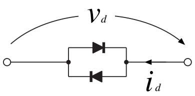
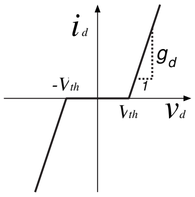


Figure 3: Wien bridge oscillator.

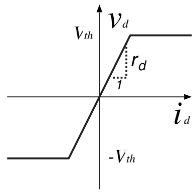
3.2. LC resonator and diodes.



(a) Coupling diode model.



(b) Voltage controlled type.



(c) Current controlled type.

Figure 4: Coupling diode model and its characteristics

Series type LC resonator and parallel type LC resonator are used as the resonator. We assume that an inductor and a capacitor are ideal elements.

Figure 4 shows coupling diode model. In the case of coupled with parallel type LC resonator, voltage controlled type model shown in Fig. 4 (b) is used. In the case of coupled with series type LC resonator, current controlled type model shown in Fig. 4 (c) is used.

3.3. Circuit models

We investigate the circuits combining a node 1, 2, 3 or 4 with a series/parallel type LC resonator via a coupling diode. At first, circuit experiments were carried out. In the case of using node 3 or 4, an oscillation on the saturation region of an operational amplifier and the convergence to an equilibrium point are observed. Chaotic phenomena could not be observed. Therefore, the case of nodes 1 and 2 are investigated in this study. In the case of using node 2, exchanging C_2 and R_2 means constructing another circuit. Therefore, six combinations as shown in Table 1 are investigated.

Table 1: Combinations of a node and a resonator

no.	node	resonator
1	node 1	parallel
2	node 1	series
3	node 2	parallel
4	node 2	series
5	node 2 (exchanged C_2 and R_2)	parallel
6	node 2 (exchanged C_2 and R_2)	series

4. Circuit experiments and computer calculations

4.1. Circuit equations

In order to carry out computer calculations, circuit equations of the circuits as shown in Table 1 are derived using models of section 3. For example, the case of circuit no. 1 is shown as follows.

$$\begin{cases} \dfrac{\mathrm{d}v_1}{\mathrm{d}t} = \dfrac{1}{C_1}\left(\dfrac{R_4}{R_2R_3} - \dfrac{1}{R_1}\right)v_1 - \dfrac{1}{R_2C_1}v_2 - \dfrac{1}{C_1}i_d, \\ \dfrac{\mathrm{d}v_2}{\mathrm{d}t} = \dfrac{R_4}{C_2R_2R_3}v_1 - \dfrac{1}{R_2C_2}v_2, \\ \dfrac{\mathrm{d}v_3}{\mathrm{d}t} = \dfrac{1}{C_3}(-i_1 + i_d), \\ \dfrac{\mathrm{d}i_1}{\mathrm{d}t} = \dfrac{1}{L_1}v_3, \end{cases} \tag{1}$$

where,

$$i_d = g_d \left\{ v_1 - v_3 + (|v_1 - v_3 - V_{th}| - |v_1 - v_3 + V_{th}|)/2 \right\} \tag{2}$$

Changing variable and parameter as follows.

$$\begin{array}{l} v_1 = V_{th}x_1,\ \ v_2 = V_{th}x_2,\ \ v_3 = V_{th}x_3, \\ i_1 = x_4V_{th}/\sqrt{R_1R_2}\ \ i_d = y_dV_{th}/\sqrt{R_1R_2} \\ \alpha = \sqrt{R_1/R_2},\ \ \beta = R_4/R_3,\ \ \gamma = C_1/C_3, \\ \delta = C_1R_1R_2/L_1,\ \ \varepsilon = \sqrt{R_1R_2}g_d, \\ C_1 = C_2,\ \ \frac{d}{d\tau} = \text{“ } \cdot \text{ ”},\ \ t = C_1\sqrt{R_1R_2}\tau, \end{array} \tag{3}$$

Normalized circuit equation is shown as follows.

$$\begin{cases} \dot{x_1} = (\alpha\beta - \dfrac{1}{\alpha})x_1 - \alpha x_2 - \varepsilon y_d, \\ \dot{x_2} = \alpha\beta x_1 - \alpha x_2, \\ \dot{x_3} = -\gamma x_4 + \gamma\varepsilon y_d, \\ \dot{x_4} = \delta x_3, \end{cases} \tag{4}$$

where,

$$y_d = x_1 - x_3 + (|x_1 - x_3 - 1| - |x_1 - x_3 + 1|)/2. \quad (5)$$

4.2. Circuit experiments and Computer calculations

Using normalized circuit equations, computer calculations of each circuits are carried out. Figures 5 show the circuit experimental results and computer calculated results of circuit no. 1. Fig. 5 (a) show the attractors of circuit experimental results, Fig. 5 (b) show the attractors of computer calculated results, and Fig. 5 (c) show the Poincaré maps obtained by computer calculation. We choose β as control parameter. A periodic orbit (1), quasi-periodic orbit (2), chaotic attractors (3) (4) (6) and a large periodic window (5) are observed. Figures 6, 7, 8, 9 and 10 show the circuit experimental results and computer calculated results of circuit no. 2, 3, 4, 5 and 6, respectively. The Figs. (a) show the attractors of circuit experimental results, and the Figs. (b) show the attractors of computer calculated results.

We can observe almost same attractors and the same bifurcation scenario in circuit experimental results and computer calculated results. These result mean that our linearized model of an oscillator does not lose features needed for chaos generation. Namely, this mean that Wien bridge oscillator behave as simple divergently oscillating part and that only the nonlinearity of the coupling diode controls the amplitude. In other words, the oscillator plays a role of expanding, while the coupling diode plays folding. These are known as the essence of generating chaos.

5. Conclusions

In this study, we investigated circuits combining a oscillator with a resonator about chaotic circuits proposed by us. The circuit consist of one Wien bridge oscillator, one resonator and one coupling diode. In particular, we paid attention to a node that is coupled with a coupling diode. As a result, we confirmed six chaotic oscillators.

6. Acknowledgment

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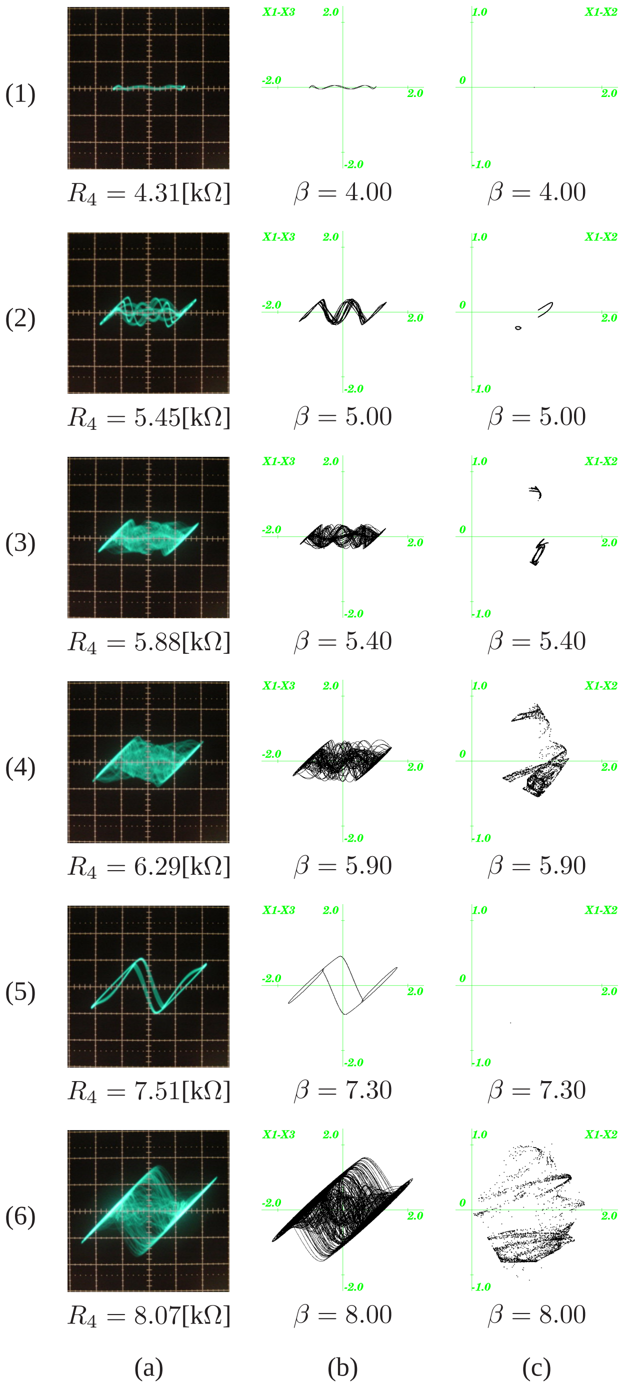
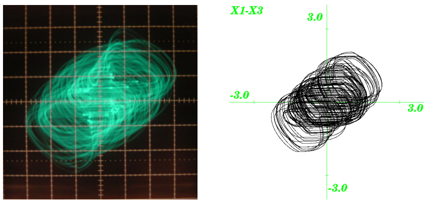


Figure 5: (a)circuit experimental results, (b)computer calculated results, (c) Poincar/'e maps. (a)horizontal axis: v_1[0.5v/div], vertical axis:v_3[0.5v/div], $R_1 = 2.0[\mathrm{k}\Omega]$, $R_2 = 5.6[\mathrm{k}\Omega]$, $R_3 = 1.0[\mathrm{k}\Omega]$, $C_1 = C_2 = C_3 = 33$[nF], $L_1 = 10$[mH] R_4. (b)horizontal axis: x_1, vertical axis: x_3, $\alpha = 0.6$, $\gamma = 1.1$, $\delta = 43$, $\varepsilon = 30$.

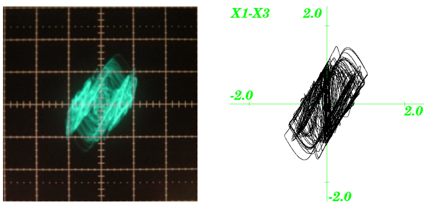
References

[1] Y. Hosokawa, Y. Nishio and A. Ushida, "A Design Method of Chaotic Circuits Using an Oscillator and a Resonator," *Proc. of ISCAS2001*, Vol. 3, pp. 373–376, 2001.



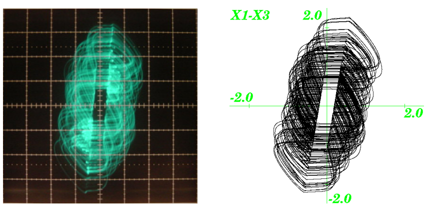
(a) (b)

Figure 6: circuit experimental result (a) and computer calculated result (b) of circuit no. 2. (a)horizontal axis: v_1[0.5v/div], vertical axis:v_3[0.5v/div], $R_1 = 3.3$[kΩ], $R_2 = 2.4$[kΩ], $R_3 = 1.0$[kΩ], $R_4 = 2.7$[kΩ], $C_1 = C_2 = C_3 = 33$[nF], $L_1 = 50$[mH]. (b)horizontal axis: x_1, vertical axis: x_3, $\alpha = 1.2$, $\beta = 2.7$, $\gamma = 1.0$, $\delta = 7$, $\varepsilon = 20$.



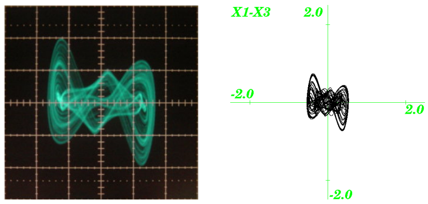
(a) (b)

Figure 7: circuit experimental result (a) and computer calculated result (b) of circuit no. 3. (a)horizontal axis: v_1[0.5v/div], vertical axis:v_3[0.5v/div], $R_1 = 5.6$[kΩ], $R_2 = 5.6$[kΩ], $R_3 = 1.0$[kΩ], $R_4 = 5.0$[kΩ], $C_1 = C_2 = C_3 = 33$[nF], $L_1 = 50$[mH]. (b)horizontal axis: x_1, vertical axis: x_3, $\alpha = 1.0$, $\beta = 4.5$, $\gamma = 1.0$, $\delta = 20$, $\varepsilon = 40$.



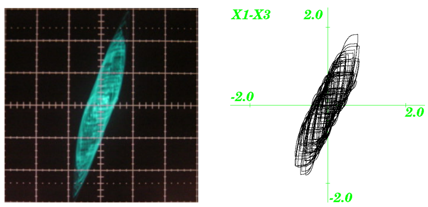
(a) (b)

Figure 8: circuit experimental result (a) and computer calculated result (b) of circuit no. 4. (a)horizontal axis: v_1[0.5v/div], vertical axis:v_3[0.5v/div], $R_1 = 3.3$[kΩ], $R_2 = 2.7$[kΩ], $R_3 = 1.0$[kΩ], $R_4 = 3.1$[kΩ], $C_1 = C_2 = C_3 = 33$[nF], $L_1 = 50$[mH]. (b)horizontal axis: x_1, vertical axis: x_3, $\alpha = 1.6$, $\beta = 2.1$, $\gamma = 1.0$, $\delta = 6$, $\varepsilon = 20$.



(a) (b)

Figure 9: circuit experimental result (a) and computer calculated result (b) of circuit no. 5. (a)horizontal axis: v_1[0.2v/div], vertical axis:v_3[0.2v/div], $R_1 = 5.6$[kΩ], $R_2 = 5.6$[kΩ], $R_3 = 1.0$[kΩ], $R_4 = 3.1$[kΩ], $C_1 = C_2 = C_3 = 33$[nF], $L_1 = 50$[mH]. (b)horizontal axis: x_1, vertical axis: x_3, $\alpha = 1.0$, $\beta = 3.0$, $\gamma = 1.0$, $\delta = 20$, $\varepsilon = 40$.



(a) (b)

Figure 10: circuit experimental result (a) and computer calculated result (b) of circuit no. 6. (a)horizontal axis: v_1[0.5v/div], vertical axis:v_3[0.5v/div], $R_1 = 3.3$[kΩ], $R_2 = 3.3$[kΩ], $R_3 = 1.0$[kΩ], $R_4 = 4.5$[kΩ], $C_1 = C_2 = C_3 = 33$[nF], $L_1 = 50$[mH]. (b)horizontal axis: x_1, vertical axis: x_3, $\alpha = 1.1$, $\beta = 3.4$, $\gamma = 1.0$, $\delta = 15$, $\varepsilon = 20$.