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Complex Behavior in Coupled Chaotic Circuits Related with Intermittency

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Abstract—In this study, a complex behavior in two coupled chaotic circuits related with intermittency is investigated. When each chaotic circuit generates intermittency chaos near the three-periodic window, three different synchronization states appear and disappear in a chaotic manner. This complex behavior is modeled by a first-order Markov chain with four states. Further, the phenomenon is confirmed by circuit experiments.

1. Introduction

Synchronization and the related bifurcation of chaotic systems are good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. In particular, the breakdown of chaos synchronization has attracted many researchers' attentions and their mechanisms have been gradually made clear [1]-[5]. However, a lot of phenomena around chaos synchronization are still veiled as well as other nonlinear problems. Hence, in order to understand and exploit such phenomena, it is important to discover them, to model them, and to investigate them.

On the other hand, intermittency chaos [6] is deeply related to *the edge of chaos* [7] and many people suggest that such a behavior between order and chaos gains better performance for various kinds of information processing than fully developed chaos. Therefore, we consider that unveiling various roles of the intermittency chaos is important to exploit it for future engineering applications.

In this study, a complex behavior in two coupled chaotic circuits related with intermittency is investigated. At first, we observe three different types of synchronization states when each chaotic circuit generates three-periodic solution. Next, we vary a control parameter of each chaotic circuit to generate intermittency chaos near the three-periodic window. In that case, we can observe a complex behavior of the three synchronization states. Namely, intermittency bursts interrupt the synchronizations and different synchronizations reappear after the bursts settle down. We model this interesting complex behavior by a first-order Markov chain with four states. Transition probabilities between the states are obtained by counting all of the transitions in plenty of computer simulation. The stationary probability distribution and the expected sojourn time in each state are calculated from the transition probabilities. These statistical quantities are compared with those obtained from computer simulations. We emphasize that the synchronizations of the three-periodic solutions and those complex behavior caused by the intermittency are observed from both computer calculations and circuit experiments.

2. Circuit Model

Figure 1 shows the circuit model, which is the asymmetric version of the circuit investigated in [8]. In the circuit, two identical chaotic circuits are coupled by a resistor R.

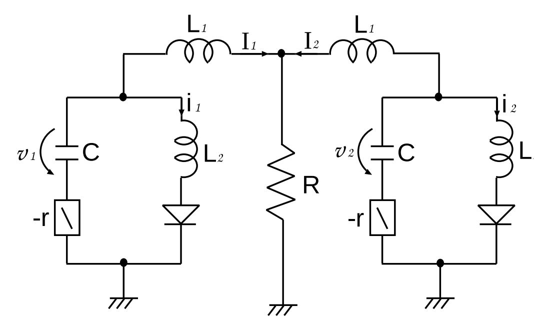

Figure 1: Circuit model.

At first, the $i-v$ characteristics of the diodes are approximated by two-segment piecewise-linear functions as

$$v_d(i_k) = \frac{1}{2}(r_d i_k + E - |r_d i_k - E|). \tag{1}$$

By changing the variables and parameters,

$$I_k = \sqrt{\frac{C}{L_1}}Ex_k, \quad i_k = \sqrt{\frac{C}{L_1}}Ey_k, \quad v_k = Ez_k,$$

$$t = \sqrt{L_1 C}\tau, \quad \alpha = \frac{L_1}{L_2}, \quad \beta = r\sqrt{\frac{C}{L_1}}, \tag{2}$$

$$\gamma = R\sqrt{\frac{C}{L_1}}, \quad \delta = r_d\sqrt{\frac{C}{L_1}},$$

the normalized circuit equations are given as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \beta(x_k + y_k) - z_k - \gamma \displaystyle\sum_{j=1}^{2} x_j \\ \dfrac{dy_k}{d\tau} = \alpha\beta(x_k + y_k) - z_k - f(y_k) \qquad (k = 1, 2) \\ \dfrac{dz_k}{d\tau} = x_k + y_k \end{cases} \tag{3}$$

where

$$f(y_k) = 0.5\ (\delta y_k + 1 - |\delta y_k - 1|). \tag{4}$$

3. Synchronization Phenomena

Figure 2 shows three-periodic attractor observed from the isolated subcircuit. For the computer calculations, the fourth-order Runge-Kutta method is used with step size $h = 0.001$.

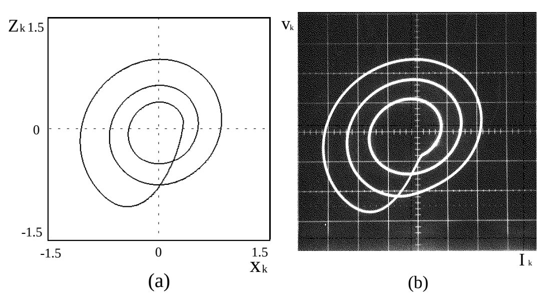

Figure 2: Three-periodic attractor observed form each subcircuit. (a) Computer calculated result. x_k vs. z_k. $\alpha = 7.0$, $\beta = 0.152$, $\gamma = 0.0$ and $\delta = 100.0$. (b) Circuit experimental result. I_k vs. v_k. $L_1 = 300$mH, $L_2 = 10$mH, $C = 33$nF, $r = 740\Omega$ and $R = 0.0\Omega$.

Figure 3 shows that three different types of synchronization states, when the two circuits generating the three-periodic attractors are coupled. These three synchronization states can be obtained by giving different initial conditions. As we can see from the figures, the two circuits tend to be synchronized in anti-phase. This is because the states minimizing the energy consumed by the coupling resistor R correspond to stable synchronization states. For three-periodic solutions there exist three different peaks in the waveform. Hence, three different synchronization states could coexist as shown in Fig. 3. We name the three synchronization states as the states T_1, T_2, and T_3.

We also confirm the generation of the three different synchronization states in circuit experiments as shown in Fig. 4.

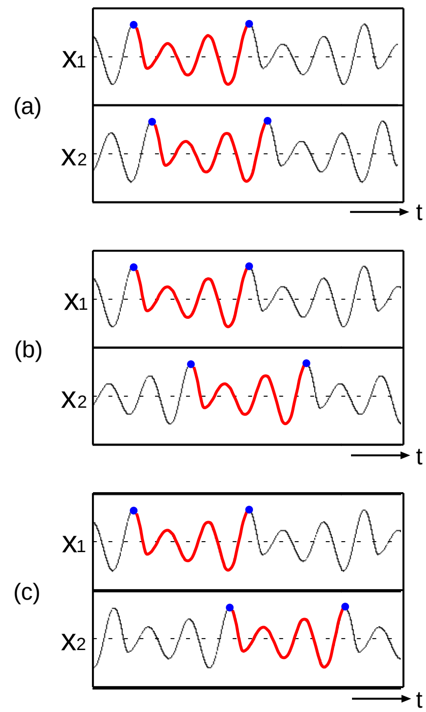

Figure 3: Time waveforms of three synchronization states (computer calculated results). $\alpha = 7.0$, $\beta = 0.152$, $\gamma = 0.005$ and $\delta = 100.0$. (a) State T_1, (b) State T_2, and (c) State T_3.

Next, we vary a control parameter of each chaotic circuit to generate intermittency chaos near the three-periodic window as shown in Fig. 5.

If we couple the two chaotic circuits when the intermittency chaos appear, we can observe a complex behavior of the three synchronization states. Namely, intermittency bursts disturb the synchronizations and different synchronizations appear and disappear in a chaotic way.

In order to investigate the complex phenomenon, we define the Poincaré section as $z_1 = 0$ and $x_1 < 0$. Further we plot the discrete data of x_2 on the Poincaré map when x_1 is smaller than -1.2. This threshold is introduced to extract only the data when x_1 takes the largest peak (dots in the waveform in Fig. 3). Figure 6 shows the discrete data of x_2 obtained by the above-mentioned method. We can see that the synchronization states are interrupted by the intermittent bursts and different synchronization states reappear after the bursts settle down. Although the results can not be shown in the same manner, we also confirmed the same phenomenon in the circuit experiments. The changing of the synchronization states can be shown in a picture as Fig. 7.

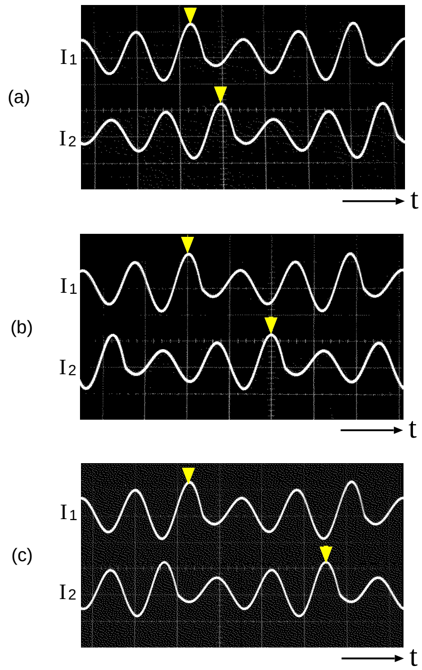

Figure 4: Time waveforms of three synchronization states (circuit experimental results). $L_1 = 300\text{mH}$, $L_2 = 10\text{mH}$, $C = 33\text{nF}$, $r = 740\Omega$ and $R = 40.0\Omega$. (a) State T_1, (b) State T_2, and (c) State T_3.

4. Markov Chain Modeling

In this section we model the interesting complex behavior in the last section by a first-order Markov chain with four states. The proposed Markov chain is shown in Fig. 8 where T_1. T_2, and T_3 are the three synchronization states introduced in the last section, B is the burst state appearing in the transition between the three. Further,

$$P(B|B) = 1 - \sum_{i=1}^{3} P(B|T_i). \tag{5}$$

must be satisfied.

From the state-transition diagram in Fig. 8, we can obtain the transition probability matrix $\mathbf{P}$ as

$$\begin{bmatrix} P(T_1|T_1) & 0 & 0 & P(B|T_1) \\ 0 & P(T_2|T_2) & 0 & P(B|T_2) \\ 0 & 0 & P(T_3|T_3) & P(B|T_3) \\ 1-P(T_1|T_1) & 1-P(T_2|T_2) & 1-P(T_3|T_3) & P(B|B) \end{bmatrix}. \tag{6}$$

The stationary probability distribution describing probability of the solution being in each phase state;

$$\mathbf{Q} = [Q(T_1), Q(T_2), Q(T_3), Q(B)]^T \tag{7}$$

can be calculated from the following equations

$$\mathbf{Q} = \mathbf{PQ} \quad \text{and} \quad \sum_{i=1}^{3} Q(T_i) + Q(B) = 1. \tag{8}$$

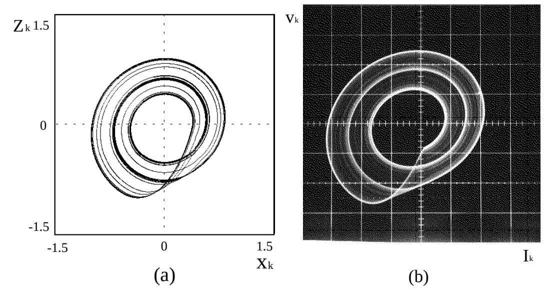

Figure 5: Intermittency chaos near the three-periodic window. (a) Computer calculated result. x_k vs. z_k. $\alpha = 7.0$, $\beta = 0.133682$, $\gamma = 0.0$ and $\delta = 100.0$. (b) Circuit experimental result. I_k vs. v_k. $L_1 = 300\text{mH}$, $L_2 = 10\text{mH}$, $C = 33\text{nF}$, $r = 735\Omega$ and $R = 0.0\Omega$.

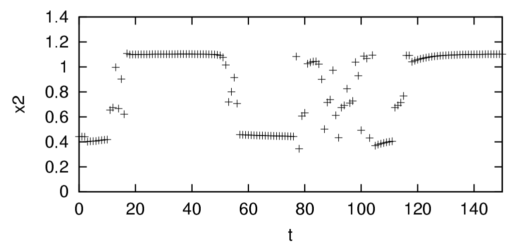

Figure 6: Time series of synchronization states disturbed by intermittency chaos (computer calculated re sults). $\alpha = 7.0$, $\beta = 0.133682$, $\gamma = 0.005$ and $\delta = 100.0$.

It is also possible to estimate expected sojourn time in each phase state by using the transition probabilities. For example, probability density function of the sojourn time in $Q(T_1)$ is given by

$$P_{ST}(Q(T_1)), n) = P(T_1|T_1)^{n-1}(1 - P(T_1|T_1)). \tag{9}$$

From (9) the expected sojourn time in $Q(T_1)$ is calculated as

$$\begin{aligned} E_{ST}(Q(T_1)) &= \sum_{n=1}^{\infty} \{n \times P_{ST}(Q(T_1), n)\} \\ &= (1 - P(T_1|T_1)) \sum_{n=1}^{\infty} nP(T_1|T_1)^{n-1} \\ &= \frac{1}{1 - P(T_1|T_1)}. \end{aligned} \tag{10}$$

5. Results and Discussions

Because we cannot get the transition probabilities theoretically, we obtain the data by counting all of the transitions in plenty of computer simulations. For the parameter values in Fig. 6, we obtained those as $P(T_1|T_1) = 0.91641$, $P(T_2|T_2) = 0.33261$, $P(T_3|T_3) = 0.79080$, $P(T_B|T_B) = 0.84255$, $P(T_B|T_1) = 0.03922$, $P(T_B|T_2) = 0.05680$, and $P(T_B|T_3) = 0.06142$. By using these probabilities, we calculated the stationary

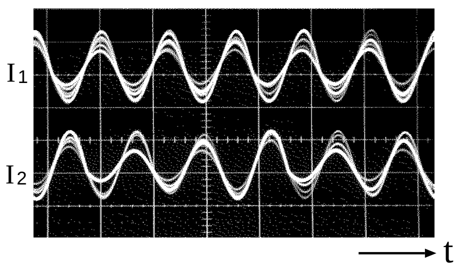

Figure 7: Time series of synchronization states disturbed by intermittency chaos (circuit experimental r esults). $\alpha = 7.0$, $\beta = 0.133682$, $\gamma = 0.0$ and $\delta = 100.0$.

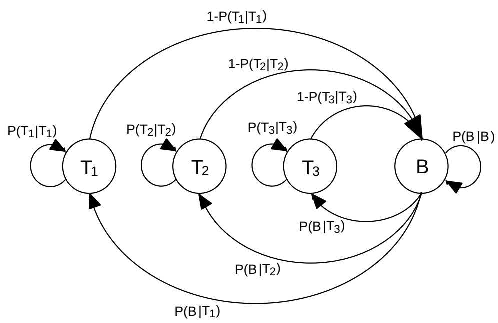

Figure 8: Markov chain model.

probability distribution and the expected sojourn time in each state.

The results are summarized in Tables 1 and 2. In the tables, "Simulation" is the data obtained by counting the states in computer simulations and "Markov" is the results calculated by using Eqs. (6)-(10).

We can see the results obtained from Markov chain agree very well with computer simulated results.

6. Conclusions

In this study, we have investigated a complex behavior in two coupled chaotic circuits related with intermittency. When each chaotic circuit generated three-periodic solution, three different types of synchronization states were observed. However, if a control parameter of each chaotic circuit was varied to generate intermittency chaos near the three-periodic window, intermittency bursts interrupted the synchronizations and different synchronizations reappeared after the bursts settle down. This complex behavior was modeled by a first-order Markov chain with four states. Further, the phenomenon was confirmed by circuit experiments.

Table 1: Stationary Probability $\mathbf{Q}$.

State	Simulation	Markov
$Q(T_1)$	0.2547	0.25397
$Q(T_2)$	0.0460	0.04606
$Q(T_3)$	0.1587	0.15888
$Q(B)$	0.5406	0.54116

Table 2: Expected sojourn times E_{ST}.

State	Simulation	Markov
$E_{ST}(Q(T_1))$	11.9632	11.9198
$E_{ST}(Q(T_2))$	1.4984	1.4984
$E_{ST}(Q(T_3))$	4.7801	4.7500
$E_{ST}(Q(B))$	6.3391	6.3490

Acknowledgments

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