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Several Phase Synchronization Modes in Coupled Multi-State Chaotic Circuits

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Abstract—Nonlinear dynamics on coupled chaotic oscillatiors is considerable interesting for a wide variety of systems in several scientific fields and applications. This paper presents a novel type of several phase synchronization modes in coupled asynchronous multimode chaotic oscillators. Each chaotic circuit can individually behave both chaotic or periodic oscillations in the same parameters asynchronously. In this study, such chaotic circuits coupled by some inductors as a ring are proposed and classifications of phase synchronization modes are investigated. In numerical simulation, many types of phase synchronization modes are confirmed.

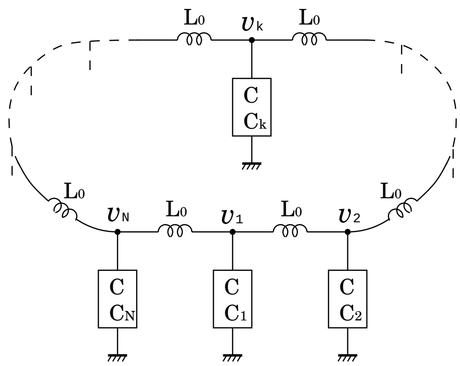

Figure 1: Coupled model of chaotic circuits by several inductors as a ring.

1. Introduction

Many types of coupled systems have been widely studied in order to clarify inherent features and many researchers have already proposed and investigated them. Coupled chaotic systems are as one of them which have several varieties of interesting behavior with emergent properties. The dynamics of chaotic multimode oscillations or chaotic itinerancy on several coupled systems is still considerable interest from the viewpoint of both natural scientific fields and several applications. They have been confirmed in several systems; e.g., coupled van der Pol oscillators[1], laser systems[2], and so on. As interesting phenomena, there are famous chaotic attractors such a double-scroll family[3], n-double scroll[4]–[6] and scroll grid attractors[7]. If the active elements including in the systems have complexity constructed by compound some nonlinear elements, it can be easily consider that they yield several interesting features. The circuit which can individually behave both chaotic or periodic oscillations in the same parameters had been shown[8]. This type of circuit was called a multi–state chaotic circuit(abbr. MSCC). Multimode oscillations in coupled two multi–state chaotic circuits had also been investigated[9]. Such complex and strange nonlinear structures yield a wide variety of chaotic phenomena. It is known that complex behavior can be confirmed such chaotic itinerancy and spatio-temporal chaos on the large scale coupled networks.

In this study, a novel type of phase pattern or multimode asynchronous oscillations on the coupled MSCCs as a ring is investigated. The schematic diagram of the coupled circuits is shown in Fig. 1. In the past our works, we had presented and confirmed that multi-state oscillations both chaotic and non–chaotic (limit cycle) can be generated asynchronously in the same parameters on the computer simulations [8] and also that realization on the real circuits [10]. In this paper, firstly the design scheme of a MSCC is shown briefly. Secondary both chaotic and periodic oscillations in the same parameters which can be confirmed in numerical simulations and circuit experiment are also shown. Finally phase synchronization and classification of several phase patterns in some MSCCs coupled by inductors are investigated. Several types of phase synchronization modes are confirmed asynchronously, but all circuit parameters are the same.

2. Model Description

The circuit shown in Fig. 2 is modified chaotic circuit from a change model in a well–known three dimensional chaotic circuit proposed by Inaba and Saito[11]. The original circuit consists of three memory elements, some diodes and designed negative resistors. It is well known that it can behave as Rössler type chaotic motions. We substitute a symmetrical continuous five segments piecewise lin-

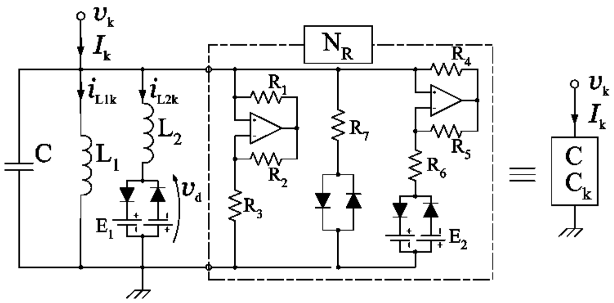

Figure 2: Proposed chaotic circuit with five–segment piecewise linear resistors.

ear resistor for the negative active resistor including in the original chaotic circuit. Further this circuit possesses another symmetrical piecewise nonlinear resistor with respect to the origin.

At first, we approximate the $i - v$ characteristics in the part of both diodes and E_1 by the following three-segment piecewise linear functions $v_d(i_{L2k})$.

$$v_d(i_{L2k}) = \frac{1}{2}\left(r_s i_{L2k} + V_d - |r_s i_{L2k} - V_d|\right) \qquad (1)$$

where threshold voltage V_d is realized by the total of threshold of the diodes and supply DC voltage E_1, and r_s is a resistance value at the parallel diode in while off state. The variable $v_d(i_{L2k})$ determines their chaotic dynamics.

We now consider the coupled model which combined N chaotic circuits are connected by inductors L_0 as a ring structure. The chaotic circuits are composed by all the same parameters. Therefore when we choose a threshold voltage value V_d as a criterion, the circuit equation of coupled MSCCs can be normalized by changing the following variables and parameters,

$$\begin{aligned} &i_{L1k} = \sqrt{\frac{C}{L_1}} V_d\, x_k\ ,\ \ i_{L2k} = \sqrt{\frac{C}{L_1}} V_d\, y_k\ , \\ &v_k = V_d\, z_k\ ,\ \ t = \sqrt{L_1 C}\, d\tau\ ,\ \ \text{“}\cdot\text{”} = \frac{d}{d\tau}\ , \\ &\alpha = \frac{L_1}{L_0}\ ,\ \ \beta = \frac{L_1}{L_2}\ ,\ \ \gamma = g\sqrt{\frac{L_1}{C}}\ ,\ \ \delta = r_s\sqrt{\frac{C}{L_1}} \end{aligned} \qquad (2)$$

where g is a temporary parameter as a linear negative conductance value of N_R if we consider the negative resistor as an ideal linear function. The differential equation concerned with L_0 can be eliminated by using KCL at the loop of the inductors L_0 and L_{1k}, then the circuit equations in each system are reduced to three–dimensional equations. Consider that the part of negative resistance N_R in Fig. 2 replaces to the function $h(z_k)$ of a voltage source z_k, then the circuit equations can be rewritten by

$$\begin{cases} \dot{x_k} &= z_k \\ \dot{y_k} &= \beta\big(z_k - f(y_k)\big) \\ \dot{z_k} &= \alpha(x_{k-1} - 2x_k + x_{k+1}) \\ &\qquad -(x_k + y_k) - h(z_k) \end{cases} \qquad (3)$$

$$f(y_k) = \frac{1}{2}\Big\{|\delta y_k + 1| - |\delta y_k - 1|\Big\}. \qquad (4)$$

The function $h(z)$ which can be designed by symmetrical five–segment piecewise linear with respect to the origin for the parameters four breakpoints at $\{\pm Bp_1, \pm Bp_2\}$ and five slopes by $\{m_0, m_1, m_2, m_1, m_0\}$ is described with a canonical form as follows.

$$\begin{aligned} h(z) \triangleq m_0\gamma^* z + \frac{\gamma^*}{2}\Big\{&(m_0 - m_1)\big(|z - Bp_2| - |z + Bp_2|\big) \\ &+ (m_1 - m_2)\big(|z - Bp_1| - |z + Bp_1|\big)\Big\} \end{aligned} \qquad (5)$$

where the parameter γ^* is used for a basic value, hence the values m_k(k = 0,1,2) mean the ratio to the value γ^*.

We can realize the MSCC on the real circuit. In order to realize the nonlinear characteristic of N_R, we designed a piecewise linear resistor constructed by using some operational amplifiers(op amps) and resistors. The details of a construction are described in [12].

Our proposed circuit can behave both chaotic and periodic oscillations in the same parameters when we supply with different initial conditions. Figure 3 shows some snapshots obtained from circuit experiment. The detailed schematic design had been explained in Ref. [8][9][10] and the circuit settings are put in the caption. As a result, both chaotic and periodic attractors can be observed in the same circuit parameters.

Figure 4 also shows a typical chaotic attractor obtained from computer calculation results in the case of N = 1 for the parameters $\beta = 10.0$, $\gamma^* = 0.78$, $\delta = 100$, with piecewise linear characteristics realized by breakpoints Bp_1 = 0.30, Bp_2 = 0.56, slopes $m_0 = -1.0$, $m_1 = 0.65$ and $m_2 = -0.2$. We can confirm that both chaotic and periodic attractors coexist in the same parameters. In order to know structure of these attractor, the 3-D shape attractor is also drawn. Further a bifurcation diagram by changing the parameter γ^* is shown in Fig. 5. As increasing γ^* periodic attractor bifurcates to chaos in the following routes while keeping the limit cycle at around the origin. Oscillation of symmetrical 1-period $\rightarrow$ asymmetrical 1-period $\rightarrow$ bifurcates to 2^n period $\rightarrow$ asymmetrical slight chaos $\rightarrow$ symmetrical fluttered chaos. We can observe that both two oscillation modes exist separately in the same parameters. However, chaotic attractor disappear and limit cycle is only observed when γ^* is larger than around 0.876($\equiv \gamma^\infty$). The reason is that circuit dynamics will stabilize and settle in a limit cycle if trajectory becomes larger and is soon drawn into inside area.

3. Numerical Simulations

In this section, the model of coupled MSCCs by inductors are investigated. We show some computer calculation results by using 4-th order Runge–Kutta method with time step size $\Delta t = 0.001$ for the circuit equation (3), (4) and (5) in some cases of $N = 2 \sim 7$ as follows.

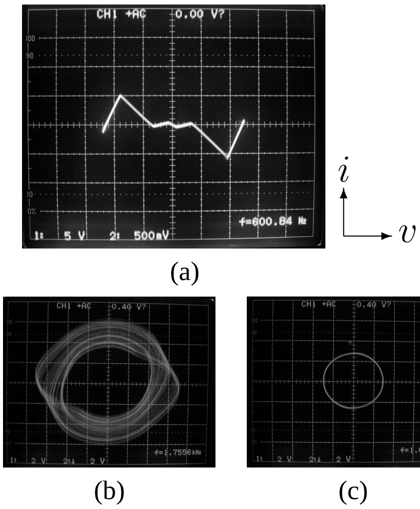

Figure 3: Some snapshots of the circuit experiment and numerical simulation. (a) v–i characteristics of the designed piecewise linear resistor, horizontal: 5V/div. (b) chaotic attractor, (c) limit cycle, horizontal: 2V/div. Circuit parameters settings: L_1 = 123.1mH, L_2 = 10.2mH, C = 68.7nF, R_1 = 33.2kΩ, R_2 = 21.7kΩ, R_3 = 1.22kΩ, R_4 = 196Ω, R_5 = 333Ω, R_6 = 1.47kΩ, R_7 = 10.3kΩ, E_1 = 4.80V, E_2 = 2.78V. Threshold voltage of one diode $v_{th} \simeq$ 0.78V.

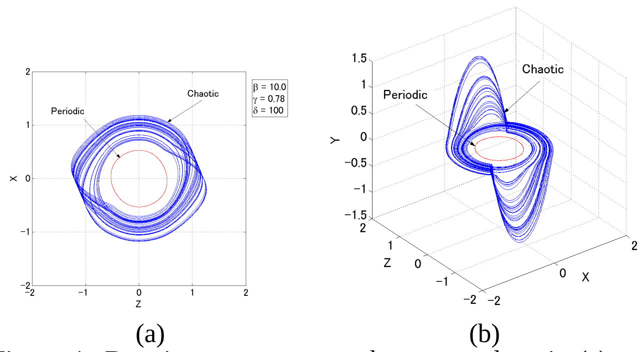

Figure 4: Drawing attractor onto the $z - x$ plane in (a), and 3-D trajectories in (b) for the parameters $\beta = 10.0$, $\gamma^* = 0.78$ and $\delta = 100$. $h(z)$: [Bp_1, Bp_2, m_0, m_1, m_2] = [0.35, 0.55, -1.0, 0.65, -0.20]

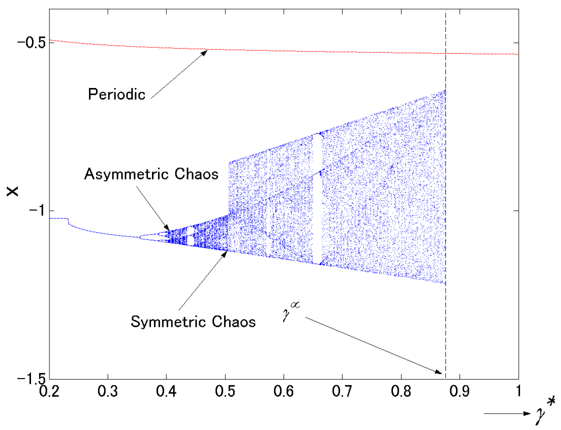

Figure 5: Bifurcation diagram by changing the parameter γ^* from 0.2 to 1.0 for β = 10.0 and δ = 100.

3.1. Two subcircuits case N = 2

Now we consider that the number of the coupled MSCCs is two. This case corresponds to the model in [9]. Though the detail results are omitted, we can confirm several types of phase synchronization modes in this model. In this case, four asynchronous oscillation modes could be confirmed consequently by numerical simulations when the initial conditions are varied. We could observe an in-phase synchronous limit cycle, an anti-phase synchronous limit cycle, an anti-phase chaotic synchronous state, and a double-mode oscillations in all the same parameters. In this coupled two MSCCs, double–mode chaotic oscillations were confirmed.

3.2. Three subcircuits case N = 3

In this section, we consider the case of $N = 3$. The circuit parameters in each MSCC are set as all the same parameters in the section 2 with additional parameter $\alpha = 0.50$. Compare with the case $N = 2$, several different synchronization phenomena can be found. Because all types of the results can not be represented, some simulation results are only shown here. Figure 6(a) shows a case of three phase synchronization of three limit cycles in while keeping $2\pi/3$ phase difference. From top of the figure, attractors drawing onto z–x plane, synchronization state of z_k–z_{k+1} plane, and waveform of difference between the two variables $z_k - z_{k+1}$. Figure 6(b) shows in-phase synchronization of them. They are corresponding normally to three phase synchronization in generic oscillators. The figure (c) shows a multimode oscillation of both chaotic and periodic attractors. Further (d) shows a new type of synchronization mode applicable to no other one. We could confirm to coexist with several types of synchronization modes.

3.3. Discussion of Coupled MSCCs $N \geqq 4$

In large coupled systems for $N \geqq 4$, it is easily expected to be confirmed more complex behavior. We now show only some results in Fig. 7 for the case of N = 4 and 7. In the case of $N = 4$, several types of synchronization modes are confirmed. On the other hand, in the case of $N = 7$, no much many synchronization modes have confirmed in such parameter settings. However when the number of N is large and the circuit parameters should be set appropriately, a certain kind of phase propagation phenomena may be confirmed. Furthermore several complex behavior could be also confirmed in a large number of N.

4. Conclusions

In this study, we have investigated several synchronization modes in coupled multi–state chaotic circuits. Coexistence of several types oscillation modes have been confirmed in coupled MSCCs by inductors as a ring for several cases. On large scale coupled chaotic circuits, we con-

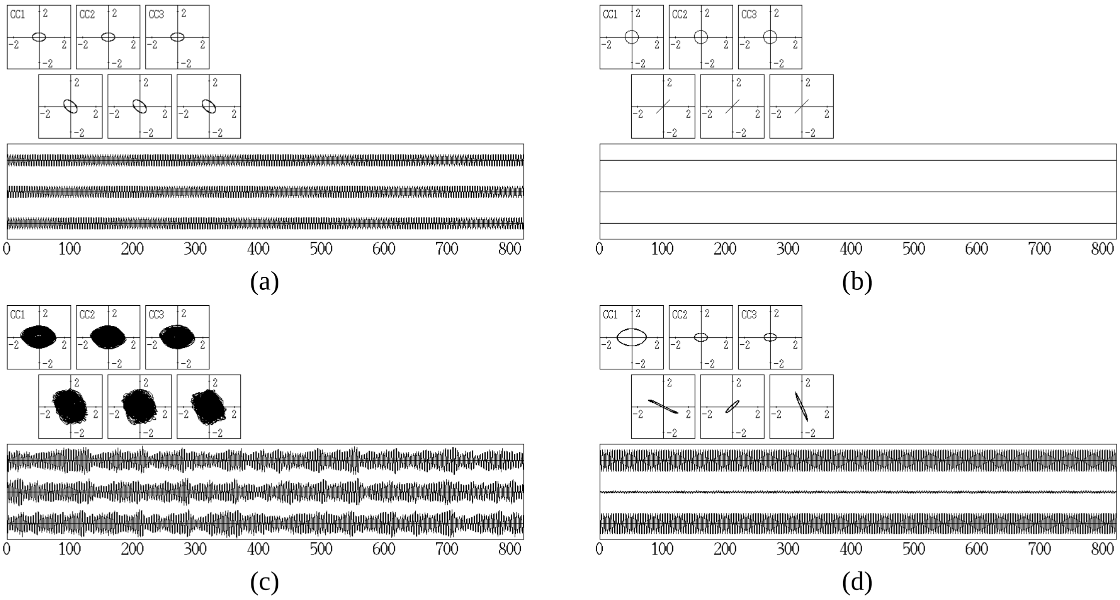

Figure 6: Some simulation results in the case of three MSCCs coupled by inductors. α = 0.50, β = 10.0, γ^* = 0.68, δ = 100. $h(z)$: [Bp_1, Bp_2, m_0, m_1, m_2] = [0.35, 0.55, -1.0, 0.65, -0.20]. (a) three phase synchronization, (b) in-phase synchronization, (c) multimode synchronization of both chaotic and periodic oscillations and (d) other type of synchronization modes.

sider that several types of complex behavior are expected to yield novel chaotic phenomena e.g., chaotic itinerancy, spatio–temporal chaos, multi–agent systems, soliton like wave propagation phenomena, and inherent emergent property, in which concerned with other current topics.

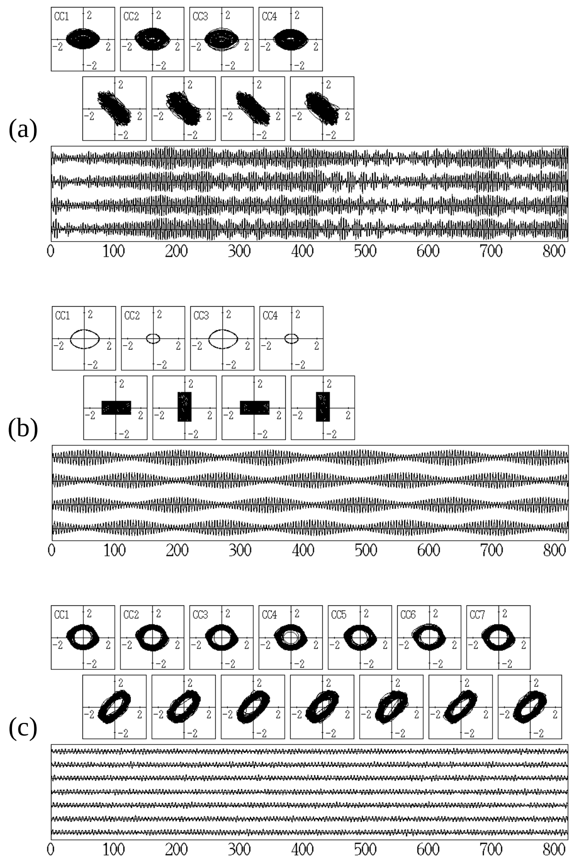

Figure 7: For some cases of three MSCCs coupled by inductors. The parameters are given by all the same settings as in Fig. 6 except for the number of circuits.

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