Diffusion Analysis of Direction-Preserving Small-World CNN

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ABSTRACT: Recently, the authors have proposed Small-World Cellular Neural Network (SWCNN), which is constructed by introducing some random couplings between cells of the original Chua-Yang CNN. In this article, the Direction-Preserving SWCNN is proposed and the features are investigated.

1. Introduction

Studies of network map are very important, because they help us to understand the basic features and requirements of various systems. So far many connection topologies of network assumed to be either completely regular or completely random have been studied in the past. Cellular Neural Network (CNN) model invented by Chua and Yang in 1988 [1] is a typical of those completely local connectivities, which is presented as a preferred implementation of locally and regularly coupled neural networks. The CNN has been successfully used for various high-speed parallel signals processing applications such as image processing, pattern recognition as well as modeling of various phenomena in nonlinear systems [1]-[4]. However, in many cases in real life, many network topologies such as biological, technological and social networks are known to be not completely random nor completely local but somewhere in between. This was modeled in an interesting work by Watts and Strogatz in 1998 [5] as the small-world model. The model is a network consisting of many local links and fewer long range 'short cuts'. Therefore, it has a high clustering coefficient like regular lattices and a short characteristic path length of typical random networks. Interesting examples are shown by collaboration of movie stars, connectivity of internet web pages or neural nets, etc.

Recently, the authors have proposed Small-World Cellular Neural Network (SWCNN) [6], which is constructed by introducing some random couplings between cells of the original Chua-Yang CNN. In [6] we have reported some basic results using the concept of SWCNN. In this article, we propose a Direction-Preserving SWCNN and investigate the features.

2. Network Topologies of the Direction-Preserving SWCNN

In this section, we describe the connection topology of the Direction-Preserving SWCNN composed of a two-dimensional M by N array structure.

The Direction-Preserving SWCNN is obtained by rewiring some couplings with direction keeped between cells in the original Chua-Yang CNN. Namely, we choose a cell and a coupling that connects it to its nearest neighbour. We reconnect this coupling to the another same direction cell chosen at random over the network. We repeat this process for all cells with the probability p_c. As a result, we get a map which is more similar to the small-world network in the Ref. [5]. Figure 1 shows a sketch map of the Direction-Preserving SWCNN consisting of 4×4 cells. Obviously, when $p_c = 0$, the SWCNN is completely the same with the original CNNs, and the maps correponding to $0 < p_c < 1$ and $p_c = 1$ are respectively shown in the middle part and the right hand side of Fig. 1.

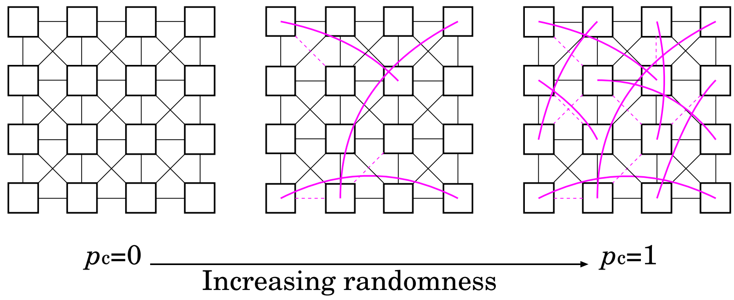

Figure 1: Direction Preserved SWCNN architecture.

The state equation of each cell $c(i, j)$ of the SWCNN is formulated by Eq. (1), and the output equation is defined by Eq. (2).

$$\dot{x}_{ij}(t) = -x_{ij}(t) + I + \sum_{c(k,l)\in N_r(i,j)} A(i,j;k,l)y_{kl}(t) + \sum_{c(k,l)\in N_r(i,j)} B(i,j;k,l)u_{kl}(t) \tag{1}$$

$$+w_c M(i,j;p,q)y_{pq}(t)$$

$$y_{ij}(t) = \frac{1}{2}(|x_{ij}(t)+1| - |x_{ij}(t)-1|) \tag{2}$$

$$i = 1, 2, ..., M, \ j = 1, 2, ..., N.$$

where $N_r(i, j)$ denotes the neighbor cells of radius r of a cell $c(i, j)$; A, B, and I are real constants called as feedback template, control template and bias current, respectively; x_{ij}, y_{ij}, u_{ij} denotes the state, input and output of the cell, respectively; $M(i, j; p, q)$ describes the small-world map that is randomly created by program with indicating the probability p_c in advance, if there is a coupling between one cell $c(i, j)$ and another cell $c(p, q)$, then the $M(i, j; p, q)$ is equal to 1, otherwise is zero; and w_c stands for the coupling weight between the randomly coupled cells.

In order to investigate the features of the network, we calculated the characteristic path length L(p_c) and the clustering coefficient C(p_c) as varying p_c. The characteristic path length L(p_c) is defined as the number of edges in the shortest path between two vertices, averaged over all pairs of vertices [5]. The clustering coefficient C(p_c) is defined as follows; Suppose that a vertex v has k_v neighbours; then at most $k_v(k_v - 1)/2$ edges can exist between them. Let C_v denote that fraction of these allowable edges that actually exist. Define C(p_c) as the average of C_v over all v [5].

The results are shown in Fig. 2. Because we make a restriction such that one cell has at most one random coupling, the clustering coefficient does not approach zero even if p_c becomes 1.0. However, the characteristic path length becomes shorter and the network possesses the feature of the small-world networks.

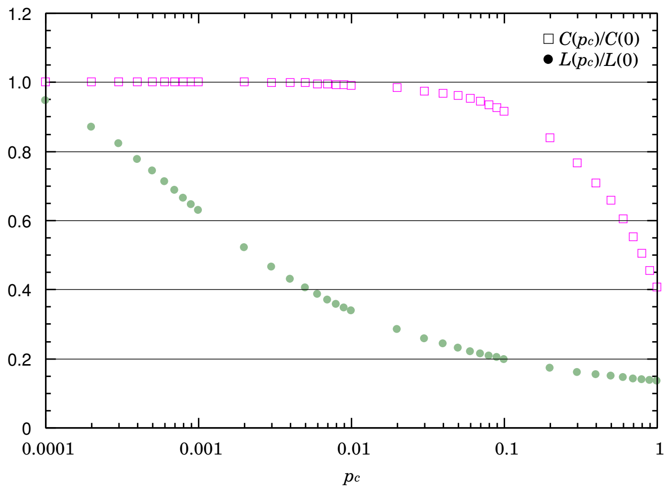

Figure 2: Characteristic path length and clustering coefficient of Direction-Preserving SWCNN.

3. Diffusion analysis

Image processing is one of important applications of the CNN. As well known, many templates and algorithms used for this purpose have been developed. In this section, we investigate the effect of applying the Direction-Preserving SWCNN to diffusion analysis.

The diffusion template is shown as follows.

$$\mathbf{A} = \begin{bmatrix} 0.1 & 0.15 & 0.1 \\ 0.15 & 0 & 0.15 \\ 0.1 & 0.15 & 0.1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad I = 0. \tag{3}$$

This is a propagating template, the computing time is proportional to the length of the image. In order to evaluate the ability of the SWCNN, we investigated the convergence speed of the SWCNN for this task with various p_c. The result is shown in following Figures.

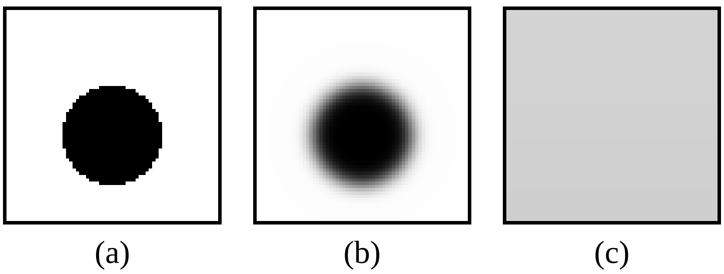

Figure 3: Diffusion simulation by the original CNN. (a) Initial state image. (b) Output image at 5 times. (c) Steady state output image.

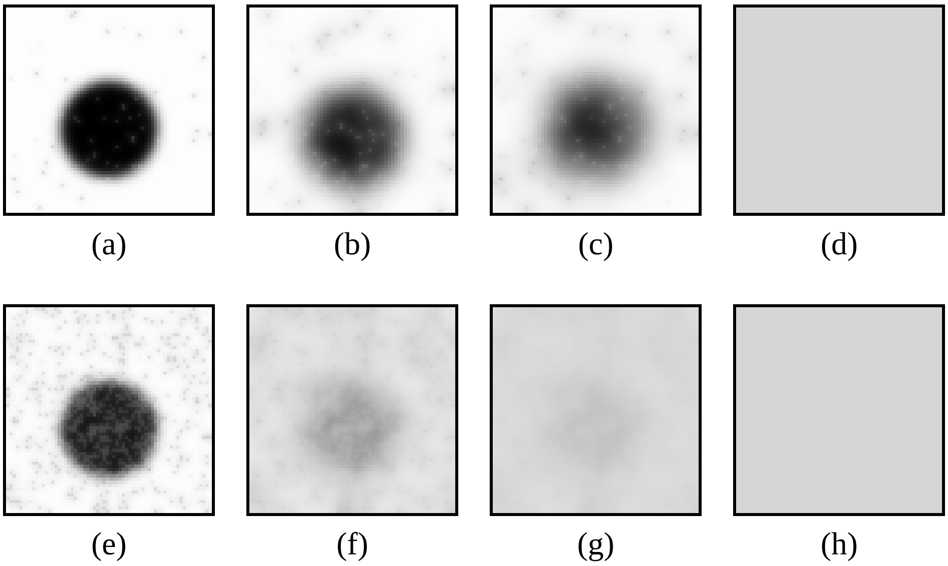

Figure 4: Diffusion simulation of Direction-Preserving SWCNN. (a),(b),(c) Output image at 5, 30 and 50 times with $p_c = 0.1$. (d) Steady state output image with $p_c = 0.1$. (e),(f),(g) Output image at 5, 30 and 50 times with $p_c = 1.0$. (h) Steady state output image with $p_c = 1.0$.

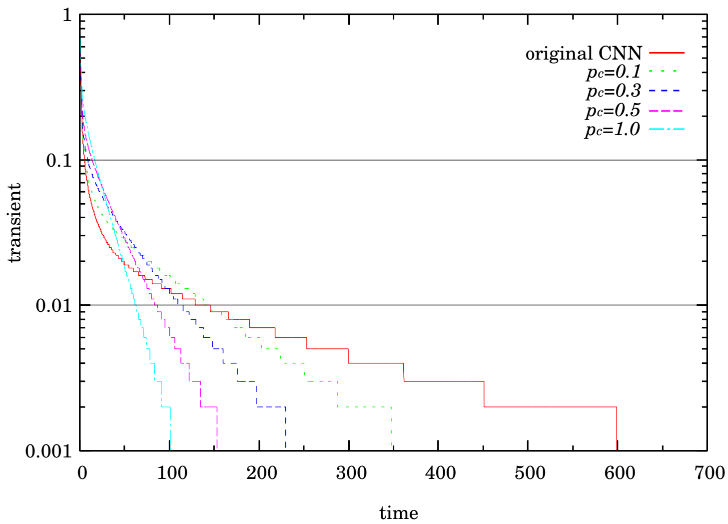

Figure 5: Transient with various p_c.

From the fig. 5, we can say that the Direction-Preserving SWCNN converges more quicker than the original CNN. Moreover, in lower p_c, we can confirm that the change of convergence speed is greater, in fig.6. However, in fig. 4, the convergence speed become higher with the p_c increases, but the noise has occurred, so it may not work well as diffusion task. So we adjust the random coupling weight w_c. Basically, the Direction-Preserved SWCNN is obtained by reconnect the couplings, here, we don't reconnect but add the couplings, and adjust the rate of the parameter between the parameter of A-*template* and w_c in a shortcut cell. The results are shown in fig. 7 and fig. 8. In ifg. 7, the parameter is set of 0.8(the parameter of A-$Template$) to 0.2(the random coupling parameter w_c).

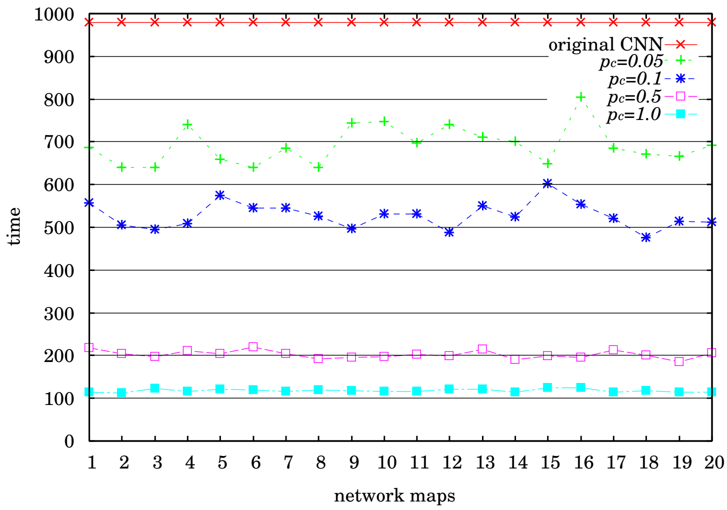

Figure 6: Convergence speed in various network maps.

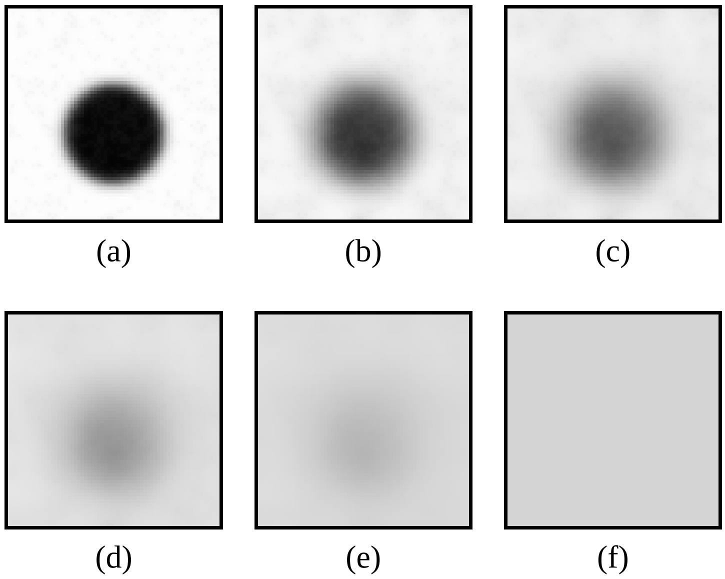

Figure 7: Diffusion simulation by improved method. (a)-(e) Output images at 5, 30, 50, 100 and 150 times with $p_c = 1.0$, respectively. (f) Steady state output image.

4. Conclusions

In this article, we have proposed Direction-Preserving network topologies of the SWCNN and investigate the features of the SWCNN. We have applied the Direction-Preserving SWCNN to diffusion. Because we used the templates designed for the original CNN, the obtained outputs were not sufficient for the tasks. However, we confirmed that the SWCNN could improve the convergence speed.

Our important future researches are design of the template for the SWCNN and analysis of the stability of the SWCNN.

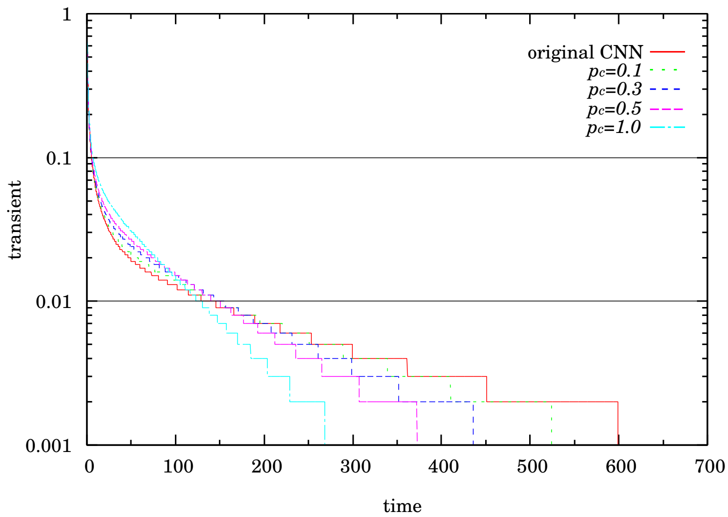

Figure 8: Improved results of transient with various p_c.

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