Complex Behavior in a Ring of Chaotic Circuits Related with Intermittency

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ABSTRACT: In this study, a complex behavior in a ring of chaotic circuits related with intermittency is investigated. When each chaotic circuit generates three-periodic solution, various different types of synchronization states are observed. However, if a control parameter of each chaotic circuit is varied to generate intermittency chaos near the three-periodic window, intermittency bursts interrupt the synchronizations and different synchronizations reappear after the bursts settle down.

1. Introduction

Synchronization and the related bifurcation of chaotic systems are good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. In particular, the breakdown of chaos synchronization has attracted many researchers' attentions and their mechanisms have been gradually made clear [1]-[5]. However, a lot of phenomena around chaos synchronization are still veiled as well as other nonlinear problems. Hence, in order to understand and exploit such phenomena, it is important to discover them, to model them, and to investigate them.

On the other hand, intermittency chaos [6] is deeply related to *the edge of chaos* [7] and many people suggest that such a behavior between order and chaos gains better performance for various kinds of information processing than fully developed chaos. Therefore, we consider that unveiling various roles of the intermittency chaos is important to exploit it for future engineering applications.

In this study, a complex behavior in a ring of chaotic circuits related with intermittency is investigated. At first, we analyze behavior in a basic system of two coupled chaotic circuits. Next, we observe more complex behavior when the two coupled chaotic circuits are expanded to a ring of chaotic circuits. In that case, we observe various different types of synchronization states when each chaotic circuit generates three-periodic solution. And, we vary a control parameter of each chaotic circuit to generate intermittency chaos near the three-periodic window. So, we can observe a complex behavior of the various synchronization states. Namely, intermittency bursts interrupt the synchronizations and different synchronizations reappear after the bursts settle down.

2. Basic Coupled Circuit

Figure 1 shows the basic coupled circuit. Each subcircuit is three-dimensional autonous one and consists of three memory elements, one linear negative resistor and one diode. We can regard the diodes as pure resistive elements, because operation frequency is not too high. Figure 2 shows three-periodic attractor observed from each subcurcuit.

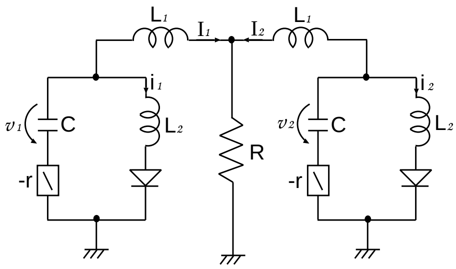

Figure 1: Basic coupled circuit.

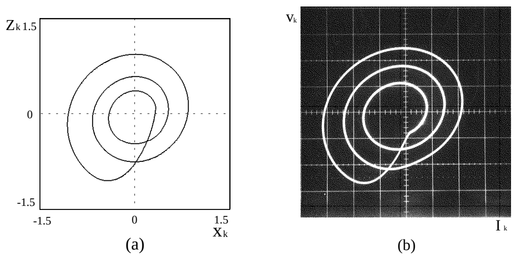

Figure 2: Three-periodic attractor observed form each subcircuit. (a) Computer calculated result. x_k *vs.* z_k*.* $\alpha = 7.0$*,* $\beta = 0.152$*,* $\gamma = 0.0$ *and* $\delta = 100.0$*. (b) Circuit experimental result.* I_k *vs.* v_k*.* $L_1 = 300mH$*,* $L_2 = 10mH$*,* $C = 33nF$*,* $r = 740\Omega$ *and* $R = 0.0\Omega$*.*

Figure 3(1) shows that three different types of synchronization states, when the two circuits generating the three-periodic attractors are coupled. These three synchronization states can be obtained by giving different initial conditions. As we can see from the figures, the two circuits tend to be synchronized in anti-phase. This is because the states minimizing the energy consumed by the coupling resistor R correspond to stable synchronization states. For three-periodic solutions there exist three different peaks in the waveform. Hence, three different synchronization states could coexist as shown in Fig. 3(1).

We also confirm the generation of the three different synchronization states in circuit experiments as shown in Fig. 3(2).

Next, we vary a control parameter of each subcurcuits to generate intermittency chaos near the three-periodic window as shown in Fig. 4.

If we couple the two chaotic circuits when the intermittency chaos appear, we can observe a complex behavior of the three synchronization states. Namely, intermittency bursts disturb the synchronizations and different synchronizations appear and disappear in a chaotic way.

In order to investigate the complex phenomenon, we define the Poincaré section as $z_1 = 0$ and $x_1 < 0$. The data of x_1 on the Poincaré map is defined as $\hat{x}_1$. Further we plot the discrete data of $\hat{x}_2$ on the Poincaré map when $\hat{x}_1$ is smaller than -1.2. This threshold is introduced to extract only the data when $\hat{x}_1$ takes the largest peak (bullets in the waveform in Fig. 3). Figure 5 shows the discrete data of $\hat{x}_2$ obtained by the above-mentioned method. We can see that the synchronization states are interrupted by the intermittent bursts and different synchronization states reappear after the bursts settle down. Although the results can not be shown in the same

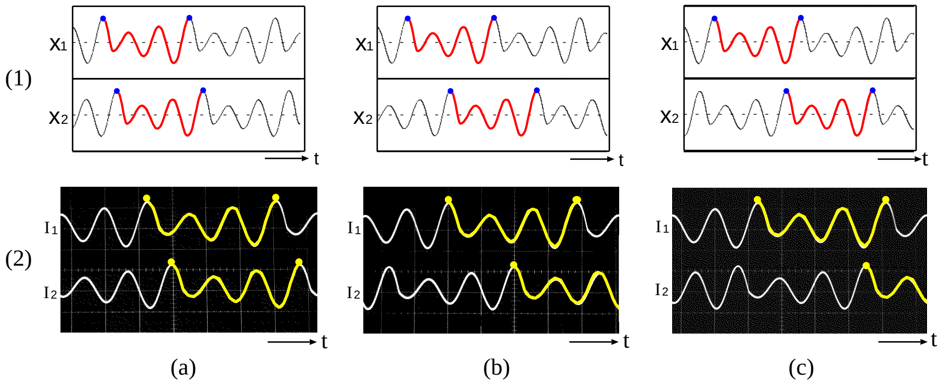

Figure 3: Time waveforms of three synchronization states. (1) shows Computer calculated results. $\alpha = 7.0$, $\beta = 0.152$, $\gamma = 0.005$ *and* $\delta = 100.0$. *(2) shows Circuit experimental results.* $L_1 = 300mH$, $L_2 = 10mH$, $C = 33nF$, $r = 740\Omega$ *and* $R = 40.0\Omega$.

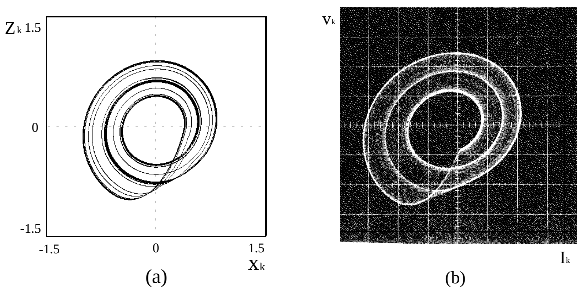

Figure 4: Intermittency chaos near the three-periodic window. (a) Computer calculated result. x_k *vs.* z_k. $\alpha = 7.0$, $\beta = 0.133682$, $\gamma = 0.0$ *and* $\delta = 100.0$. *(b) Circuit experimental result.* I_k *vs.* v_k. $L_1 = 300mH$, $L_2 = 10mH$, $C = 33nF$, $r = 735\Omega$ *and* $R = 0.0\Omega$.

manner, we also confirmed the same phenomenon in the circuit experiments. The changing of the synchronization states can be shown in a picture as Fig. 6.

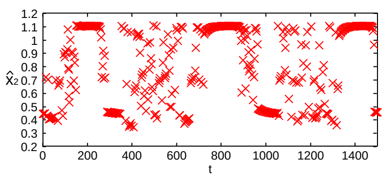

Figure 5: Time series of synchronization states disturbed by intermittency chaos (computer calculated results). $\alpha = 7.0$, $\beta = 0.133682$, $\gamma = 0.005$ *and* $\delta = 100.0$.

3. Ring of Chaotic Circuits

In this study, we consider a ring of the circuits as shown in Fig. 7. In the circuit adjacent two subcircuits are coupled by one resistor R. Because such coupling systems tend to minimize the energy consumed by the coupling resistors, every two adjacent subcircuits tend to synchronize with anti-phase.

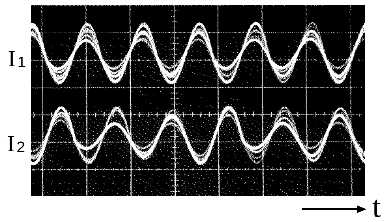

Figure 6: Time series of synchronization states disturbed by intermittency chaos (circuit experimental results). $\alpha = 7.0$, $\beta = 0.133682$, $\gamma = 0.0$ *and* $\delta = 100.0$.

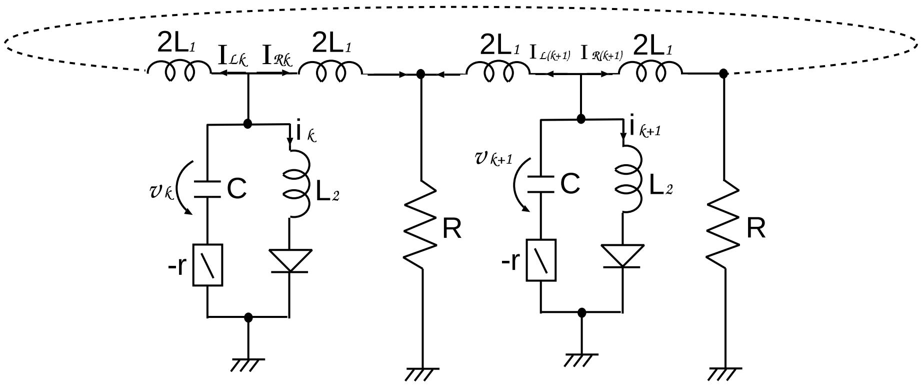

Figure 7: Ring of Chaotic Circuits.

At first, the $i - v$ characteristics of the diodes are approximated by two-segment piecewise-linear functions as

$$v_d(i_k) = \frac{1}{2}\,(r_d i_k + E - |r_d i_d - E|). \tag{1}$$

By changing the variables and parameters,

$$\begin{aligned} &I_{Rk} = \sqrt{\frac{C}{L_1}} E x_{Rk},\ I_{Lk} = \sqrt{\frac{C}{L_1}} E x_{Lk},\ i_k = \sqrt{\frac{C}{L_1}} E y_k,\ v_k = E z_k, \\ &t = \sqrt{L_1 C}\tau,\ \alpha = \frac{L_1}{L_2},\ \beta = r\sqrt{\frac{C}{L_1}},\ \gamma = R\sqrt{\frac{C}{L_1}},\ \delta = r_d\sqrt{\frac{C}{L_1}}, \end{aligned} \tag{2}$$

the normalized circuit equations are given as

$$\begin{cases} \dfrac{dx_{Rk}}{d\tau} = \dfrac{1}{2}\,\{\beta(x_{Rk} + x_{Lk} + y_k) - z_k - \gamma(x_{Rk} + x_{L(k+1)})\} \\ \dfrac{dx_{Lk}}{d\tau} = \dfrac{1}{2}\,\{\beta(x_{Rk} + x_{Lk} + y_k) - z_k - \gamma(x_{Lk} + x_{R(k+1)})\} \\ \dfrac{dy_k}{d\tau} = \alpha\{\beta(x_{Rk} + x_{Lk} + y_k) - z_k - f(y_k)\} \\ \dfrac{dz_k}{d\tau} = x_{Rk} + x_{Lk} + y_k \end{cases} \quad (k = 1, 2, 3, ..., N)) \tag{3}$$

where

$$f(y_k) = \frac{1}{2}\left(\delta y_k + 1 - |\delta y_k - 1|\right). \tag{4}$$

and

$$x_{LN} = x_{L1},\ x_{R0} = x_{RN}. \tag{5}$$

Note that when the coupling parameter γ, which is in proportion to R, is equal to zero, the coupling term in (3) vanishes. For all of computer calculations, the fourth-order Runge-kutta method is used with step size $h = 0.005$.

We carried out computer simulations for the case of $N = 4$ in a ring of chaotic circuits. At first, we observed various different types of synchronization state by changing initial value when each chaotic circuit generates three-periodic solution. Figure 8 shows data of $\hat{x}_{R2} + \hat{x}_{L2}$, $\hat{x}_{R3} + \hat{x}_{L3}$ and $\hat{x}_{R4} + \hat{x}_{L4}$ on the Poincaré map when $\hat{x}_{R1} + \hat{x}_{L1}$ is smaller than -1.2.

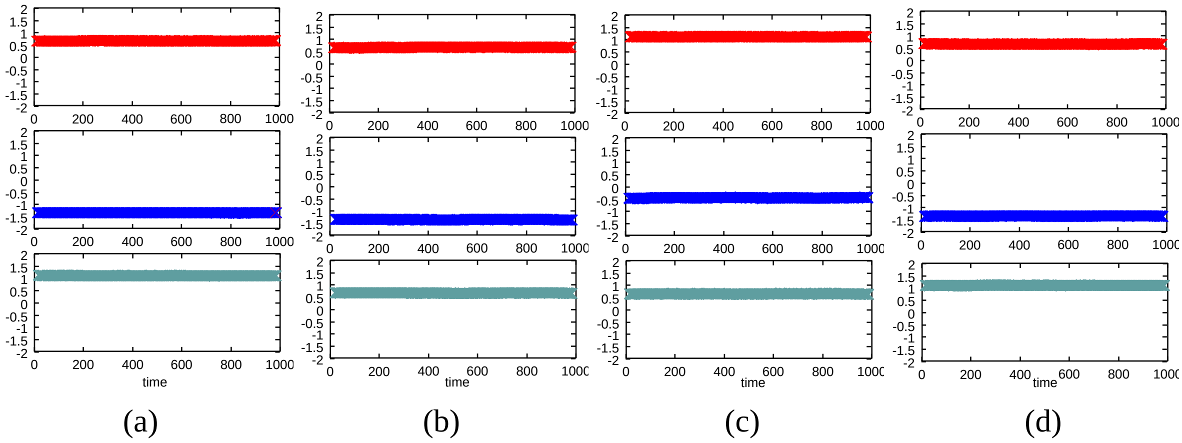

Figure 8: Various different types of synchronization state disturbed by changing initial value (computer calculated results). $\alpha = 7.0$, $\beta = 0.16$, $\gamma = 0.005$ *and* $\delta = 50.0$. *Upper figures:* $\hat{x}_{R2} + \hat{x}_{L2}$. *Middle figures:* $\hat{x}_{R3} + \hat{x}_{L3}$. *Lower figures:* $\hat{x}_{R4} + \hat{x}_{L4}$.

Next, we vary a control parameter of each chaotic circuit to generate intermittency chaos near the three-periodic window. We can observe a complex behavior of the various synchronization state in a ring of chaotic circuits when the intemittency chaos appear. Figure 9 shows data of $\hat{x}_{R2} + \hat{x}_{L2}$, $\hat{x}_{R3} + \hat{x}_{L3}$ and $\hat{x}_{R4} + \hat{x}_{L4}$ on the Poincaré map when $\hat{x}_{R1} + \hat{x}_{L1}$ is smaller than -1.2. We can see that the synchronization states are interrupted by the intermittent bursts and different synchronization sates reappear after the bursts settle down.

4. Conclusions

In this study, we investigated a complex behavior in two coupled chaotic circuits related with intermittency chaos near the three periodic window. At first, we observed three different types of synchronization states when each chaotic circuit generates three-periodic solution. Next, we varied a control parameter of each chaotic circuit to generate intermittency chaos near the three-periodic window. In that case, we observed a complex behavior of the three synchronization states. Namely, intermittency bursts interrupt the synchronizations and different synchronizations reappear after the bursts settle down. Furthermore, we observed these complex behavior when two coupled chaotic circuits is expand to a ring of chaotic circuits.

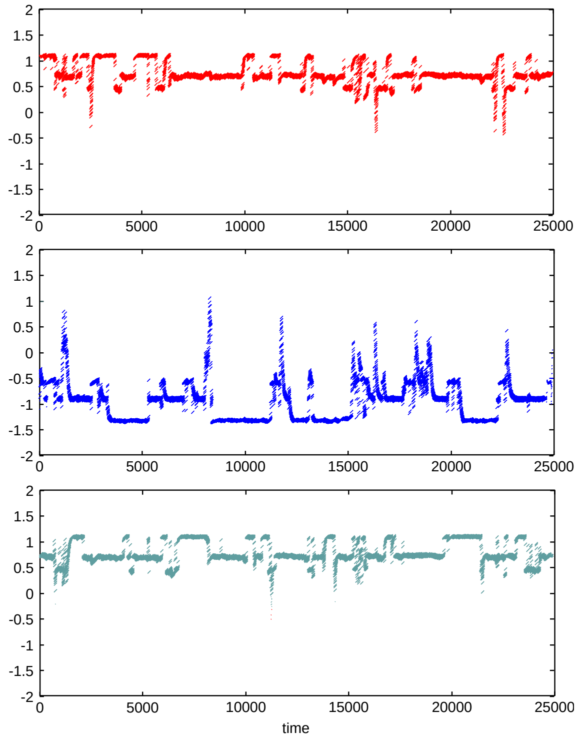

Figure 9: Time series of synchronization states disturbed by intermittency chaos. $\alpha = 7.0$,$\beta = 0.152$,$\gamma = 0.005$ *and* $\delta = 50.0$. *Upper figures:* $\hat{x}_{R2} + \hat{x}_{L2}$. *Middle figures:* $\hat{x}_{R3} + \hat{x}_{L3}$. *Lower figures:* $\hat{x}_{R4} + \hat{x}_{L4}$.

Acknowledgement

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References

[1] N. Platt, E.A. Spiegel and C. Tresser, "On-Off Intermittency: A Mechanism for Bursting," *Phys. Rev. Lett.*, vol. 70, no. 3, pp. 279-282, 1993

[2] E. Ott and J.C. Sommerer, "Blowout Bifurcations: the Occurrence of Riddled Basins and On-Off Intermittency," *Phys. Lett.* vol. A, 188, pp. 39-47, 1994.

[3] P. Ashwin, J. Buescu and I. Stewart, "Bubbling of Attractors and Synchronisation of Chaotic Oscillators," *Phys. Lett.* vol. A, 193, pp. 126-139, 1994.

[4] T. Kapitaniak and L.O. Chua, "Locally-Intermingled Basins of Attraction in Coupled Chua's Circuits," *Int. J. Bifurcation and Chaos*, vol. 6, no. 2, pp. 357-366, 1996.

[5] M. Wada, Y. Nishio and A. Ushida, "Analysis of Bifurcation Phenomena on Two Chaotic Circuits Coupled by an Inductor," *IEICE Trans. Fundamentals*, vol. E80-A, no. 5, pp. 869-875, May 1997.

[6] Y. Pomeau and P. Manneville, "Intermittent Transition to Turbulence in Dissipative Dynamical Systems," *Comm. Math. Phys.*, vol. 74, pp. 189-197, 1980.

[7] C.G. Langton, "Computation at the Edge of Chaos: Phase Transitions and Emergent Computation," *Physica D*, vol. 42, pp. 12-37, 1990.