INVESTIGATION OF PHASE-WAVE PROPAGATION PHENOMENA IN SECOND ORDER CNN ARRAYS

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ABSTRACT

The work is concerned with the phase-wave propagation phenomena in second order Cellular Neural Network (CNN) arrays. We investigate the phase relationship between the adjacent cells, observe that how the phase difference is propagated in the second order arrays, and find the regular pattern of the propagation phenomena.

1. INTRODUCTION

Dynamic property of networks of oscillatory and chaotic elements is one of the very lively studied topics. Many papers of international conferences and journals are devoted uniquely to studies of spatially extended systems, active media, and coupled lattices showing important areas where studies of dynamic phenomena in coupled oscillators find potential applications. Since CNNs were invented in 1988, the CNN paradigm provides a flexible framework (or universal model) to describe spatio-temporal dynamics in discrete space and - perhaps more importantly from a practical point of view - allows for efficient VLSI implementation of analogue, array-computing structure. Many nonlinear phenomena such as pattern formation and autowaves have been reproduced in the CNNs, and the related literatures have been reported [1]–[5].

On the other hand, Investigation on phase relationship of mutually coupled oscillators or chaotic circuits is also very important work, which is well known by physicians but little explited in CNN. Endo and Mori [6]–[7] have studied several modes of synchronization in several van der Pol oscillators arrays. We have also discovered from pheas difference approach a very interesting phase-wave propagation phenomena in van der Pol oscillators coupled by inductors as a ladder [8]. Unfortunately, the basic circuits of the above models contain inductive and high-order nonlinearly resistive devices resulting in circuit sophistication, so that it is not suitable to develop the implementation of the large-scale arrays of these circuits.

In this paper, we investigate these dynamic behaviors encountered in a simpler circuit array consisted of second order CNN cells, in which there exists linear couplings between cells and each cell has a well-known piecewise-linear nonlinear output function at the output stage. Depending on the application of sevreal difference conditions, we investigate the phase relationship between the adjacent cells, and observe the regular pattern of the phase difference propagation phenomena, and discuss how to control the phase propagation phenomena.

2. TWO-LAYER CNN ARRAY

In this section, we describe the circuit array used for investigating propagation phenomena of phase difference between adjacent cells. This array is a one-dimensional two-layer CNN with a set of constant template. Its circuit topology simpler than those reported in literatures [6]–[8]. The state equation of each cell is simply described as follows:

$$\left.\begin{aligned} \dot{x}_{1,ij} &= -x_{1,ij} + I_1 \\ &\quad + \textstyle\sum_{C(k,l)\in N_r(i,j)} A_1(i,j;k,l) y_{1,kl} \\ &\quad + \textstyle\sum_{C(k,l)\in N_r(i,j)} B_1(ij;kl) u_{1,kl} \\ &\quad + \textstyle\sum_{C(k,l)\in N_r(i,j)} C_1(ij;kl) y_{2,kl} \\ \dot{x}_{2,ij} &= -x_{2,ij} + I_2 \\ &\quad + \textstyle\sum_{C(k,l)\in N_r(i,j)} A_2(ij;kl) y_{2,kl} \\ &\quad + \textstyle\sum_{C(k,l)\in N_r(i,j)} B_2(ij;kl) u_{2,kl} \\ &\quad + \textstyle\sum_{C(k,l)\in N_r(i,j)} C_2(ij;kl) y_{1,kl} \end{aligned}\right\} \quad (1)$$

with output equation

$$\left.\begin{aligned} y_{1;ij} &= 0.5(|x_{1;ij}+1| - |x_{1;ij}-1|) \\ y_{2;ij} &= 0.5(|x_{2;ij}+1| - |x_{2;ij}-1|) \end{aligned}\right\} \quad (2)$$

$$i = 1, 2, ..., N.$$

where A, B, C, and I are respectively the feedback template, control template, coupled template, and bias current, and the subscripts 1 and 2 stand for the first layer and the second layer of the two-layer CNN array. In this paper, we restrict our discussions to the following template.

$$A_1 = \begin{bmatrix} 0 & d_1 & 0 \\ d_1 & a_1 & d_1 \\ 0 & d_1 & 0 \end{bmatrix}, \; A_2 = \begin{bmatrix} 0 & d_2 & 0 \\ d_2 & a_2 & d_2 \\ 0 & d_2 & 0 \end{bmatrix},$$

$$C_1 = c_1, \; C_2 = c_2, \; B_1 = B_2 = 0, \; I_1 = I_2 = 0. \quad (3)$$

It has been proved in references [9] that for a particular choice of the parameters a_1, a_2, c_1, and c_2, this nonlinear second-order cell in absence of couple to its neighbor cells oscillates with a limit cycle centered to the origin and symmetric with respect to the phase plane axis.

3. DEFINITION OF PHASE DIFFERENCE

The phase-waves in an array of oscillators were firstly introduced in [8], which are observed from the changes of phase difference between adjacent oscillators in a ladder composed of van der Pol oscillators coupled by inductors. Assume that y_k and y_{k+1} in Fig. 1 are the outputs of the adjacent cells k and $k+1$, then the phase difference in our case is defined as follows:

$$\phi_{i,i+1}(n) = 2\pi \times \frac{t_i(n) - t_{i+1}(n)}{t_i(n) - t_i(n-1)} \quad (4)$$

where $t_i(n)$ is the time when the state $x_{1;i}$ crosses 0[V] at n-th period.

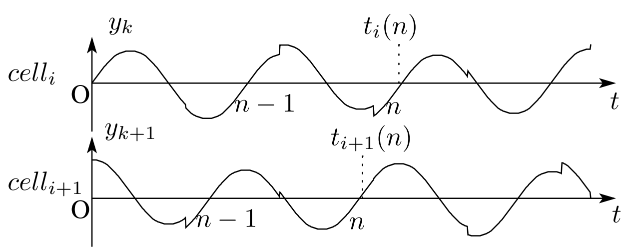


Fig. 1. Definition of phase difference.

4. AUTOWAVE AND PHASE-WAVE PROPAGATION PHENOMENA

Since the middle of the last decade, the investigations into the spatio-temporal dynamics in CNNs have been widely carried out. They have discussed various autowaves such as excitability waves, concentration and so on. We have also observed these propagation phenomena by using the simple second order array. Figure 2 shows an example of autowave with two concentric circular waves under the array parameters: $a_1 = 1.1$, $a_2 = 1.04$, $c_1 = -1$, $c_2 = 1$, $d_1 = 0.1$, and $d_2 = -0.01$. All of these propagation phenomena are observed from the output amplitude of cell varying with the

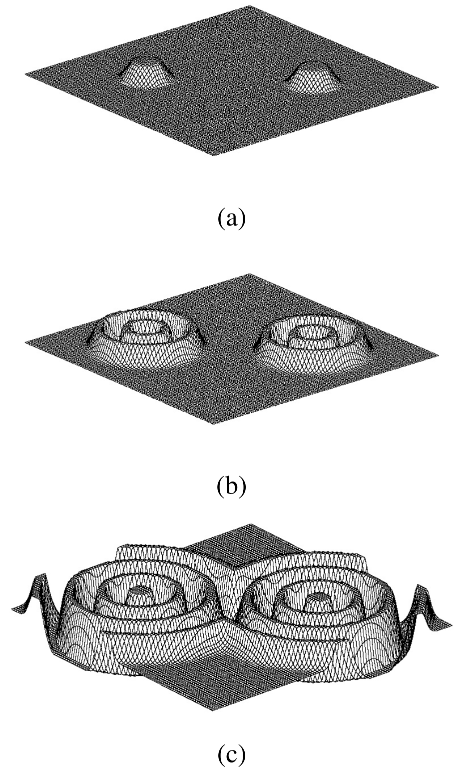


Fig. 2. A simulation for autowaves with two concentric circular waves. (a) is initial state; (b) and (c) show two snapshots observed in different iterates.

integration time. However, the investigation on phase relationship between adjcent cells is also an important work, which is well known by physicians but little explited in CNN so far. Here, we observe the propagation phenomena from the changes of phase difference between adjacent cells, we found a very interesting phase propagation phenomena. For simplicity, we consider one-dimensional second order array. Figure 4 show a simulation example for the one-dimensional array composed of 14 cells under the initial condition shown in Fig. 3, zero-flux boundary condition and the system parameter: $a_1 = 1.2$, $a_2 = 1.1$, $c_1 = -2$, $c_2 = 4$, $d_1 = 0.1$, and $d_2 = -0.1$. In fig. 4, the vertical axes are the sum of the first layer outputs of adjacent cells, and the horizontal axes are the time. Hence, the diagrams show qualitatively how phase differences between adjacent cells change as time goes, where white regions correspond to the state that two adjacent cells/oscillators are anti-phase synchronization, and black regions to the in-phase synchronization. Analysis of this simulation indicates that the anti-phase between adjacent cells is transferred gradually from one end to the other end of the one-dimensional CNNs array, and reflected at the end of array. This transfer process of phase continuously exists in the CNN array, so called "phase-inversion-wave".

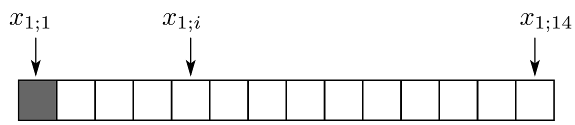


Fig. 3. A initial condition for one-dimensional array of 14 cells.

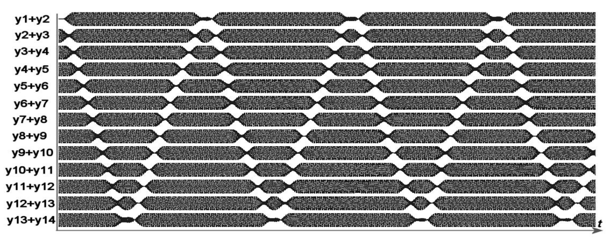
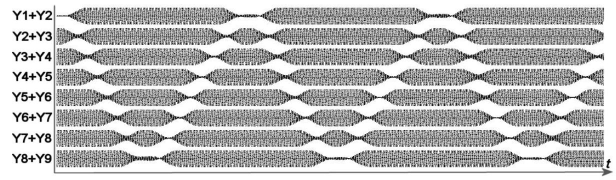
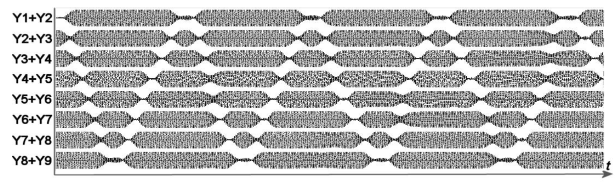


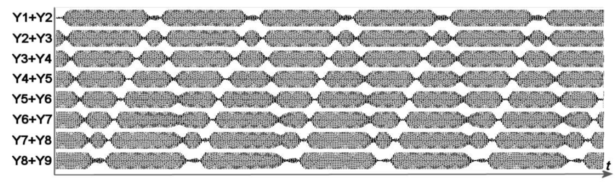
Fig. 4. Phase-inversion-wave in an one-dimensional array of 14 cells.



(a)



(b)



(c)

Fig. 5. Influence of couple parameters on phase-inversion-wave

5. INVESTIGATIONS OF PHASE-WAVES

In this section, we investigate the influence of various system parameters on the propagation of phase-waves; find their general regular pattern, so that we can control the phase wave propagation as well.

5.1. Phase Waves in In-Phase Synchronization Mode

In this subsection, the influence of coupling parameters between adjacent cells on phase inversion wave is investigated in a one-dimensional array composed nine two-layer CNN cells with zero-flux boundary condition. The parameter set is the same with the last simulation except parameters d_1 and d_2. We respectively carry out three simulations by adopting parameters $d_1 = 0.1$ and $d_2 = -0.1$, $d_1 = 0.15$ and $d_2 = -0.15$, and $d_1 = 0.2$ and $d_2 = -0.2$. The simulation results are respectively shown in Figs. 5(a), (b) and (c). As we have seen that, the propagation speed of phase-wave is directly proportional to the coupling parameters d_1 and d_2 between adjacent cells.

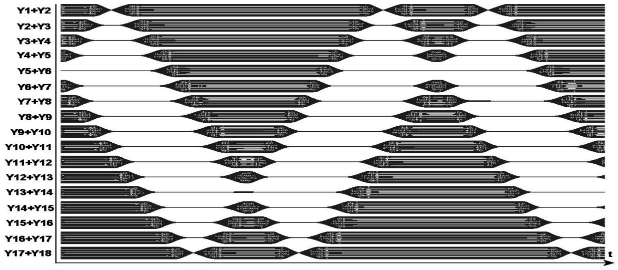
Figure 6 shows the influence of different initial states on the phase wave propagation in a one-dimensional array composed of 18 cells. The analysis of the simulation indicates that the anti-phase point is propagates toward both ends of the array, and is reflected at both ends or the position that two phase waves collide with each other.

5.2. Phase Waves in In-Anti Phase Synchronization Mode

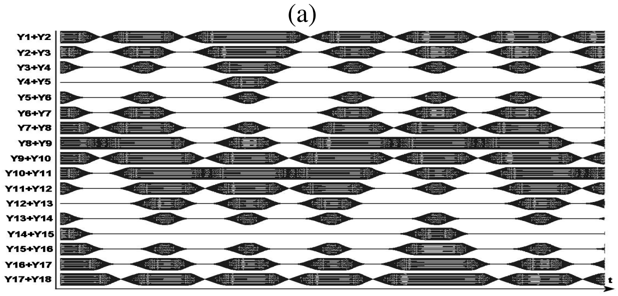
In this one-dimensional array, We also observe another type of phase wave propagation in in-anti phase synchronization mode [10]. In this mode, in-phase and anti phase between adjacent cells exist alternately, morwever the edge oscillators and their adjacent oscillators cannot be synchronized at in-phase. Hence, this Synchronization state can be observed only when the number of oscillators is even. We observe the phase inversion waves in this mode by using three different initial states. Figure 7 shows a simulation result of this mode, which is carried out in a one-dimensional array of 8 cells under the parameter: $a_1 = 1.2$, $a_2 = 1.1$, $c_1 = -1$, $c_2 = 2.5$, $d_1 = 0.1$, and $d_2 = -0.1$. Figure 7(a) shows the in-anti phase synchronization state, where no phase wave exist; but in Figs. 7(b) and (c), the phase waves are generated, and they are similarly reflected at both ends of the array.

6. CONCLUSIONS

In this study, we have proposed a simpler circuit structure – one-dimensional arrays composed of second order CNN cells for investigating the phase-wave propagation phenomena. We found that the phase wave propagation phenomena in two kinds of phase synchronization mode can be reproduced in the CNN array composed of almost numbers of



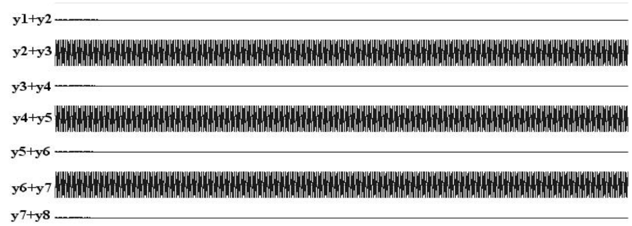
(a)



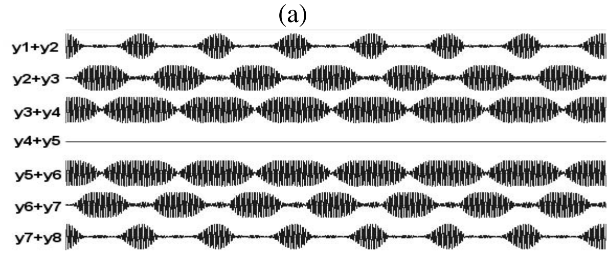
(b)

Fig. 6. Influence of initial condition on phase-inversion-wave

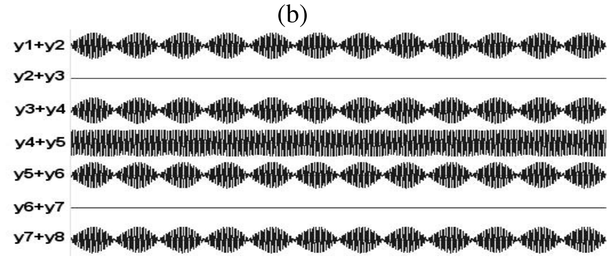
cells. Moreover, we have investigated qualitatively the influences of the coupling parameters and the initial states on these propagation phenomena. In future works, we will try to find out the patterns of these dynamics and clarify their mechanism, so that we can control the phase propagation for applications.



(a)



(b)



(c)

Fig. 7. Influence of three kinds of initial conditions on phase-inversion-wave

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