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Multimode Asynchronous Oscillations in Coupled Multi–State Chaotic Circuits

Masahiro WADA† *and* Yoshifumi NISHIO‡

† Dept. of Information Science and Systems Engineering, Konan University
8–9–1 Okamoto, Kobe 658–8501, Japan.
Phone/FAX: +81–78–435–2531, E-mail: wada–m@konan–u.ac.jp
‡ Dept. of Electrical and Electronic Engineering, Tokushima University

Abstract— In this paper, novel types of multimode asynchronous oscillations in the coupled multi–state chaotic circuits are investigated. Each chaotic circuit can behave asynchronously both chaotic or periodic oscillation at the same parameters. Because it has a hard nonlinearity of the resistor with five segments piecewise linear regions, the both different oscillation modes are separated. A design scheme of the piecewise linear elements constructed by using electrical parts is proposed. Further results of some types of multimode oscillations in numerical simulation and circuit experiment are also shown.

1. Introduction

The dynamics of chaotic multimode oscillations is still considerable interest from the viewpoint of both natural scientific fields and several applications. They have been confirmed in several systems, e.g., coupled van der Pol oscillators[1], laser systems[2], and so on. On the other hand, many types of chaotic systems and circuits have already been proposed and investigated in detail. If the active elements including in the systems have complexity constructed by compound nonlinear elements, it can be easily consider that they yield several interesting features. There are famous chaotic attractors such a double-scroll family[3], *n*-double scroll[4]–[6] and scroll grid attractors[7]. The purpose of our study is to clarify coexistence of both chaotic and non–chaotic behavior in the chaotic systems. Further complex behavior in the large scale network of the coupled chaotic circuits are also investigated.

In this study, we investigate a novel type of multimode asynchronous oscillations on the coupled multi–state chaotic circuits. There is a typical three dimensional autonomous chaotic system proposed by Inaba[8], which consists of three memory elements, some diodes and designed negative resistors. It is well known that it can behave as Rössler type chaotic motions. We substitute a symmetrical continuous five segments piecewise linear resistor for the negative active resistor including in the chaotic circuit. This proposed circuit can behave both chaotic and periodic motions at the same parameters when we supply with different initial conditions. Although it may be considered that this is similar to the chaotic systems[9][10] at a glance, those systems are different from stability at the origin. Hence it is excited at the origin because of negative resistance. Generally it can be constructed a concept for complicated structure of chaotic attractors to make many equilibrium points. In the past our works, we had presented and confirmed that multi-state oscillations both chaotic and non–chaotic (limit cycle) can be generated asynchronously at the same parameters on the computer simulations [11] and that realization on the real circuits [12]. In this paper, firstly the design scheme of piecewise nonlinear segments constructed by electrical elements is explained. Secondary both chaotic and periodic oscillations at the same parameters which can be confirmed in numerical simulations and circuit experiment are shown. Further, the results for several types of multimode oscillations on the coupled chaotic circuits are also shown.

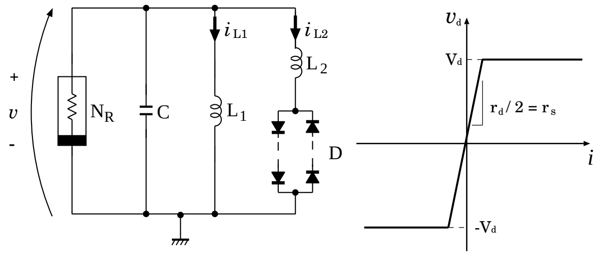

Figure 1: Schematic figure for the chaotic circuit, and a typical characteristic of the diode model with polarities.

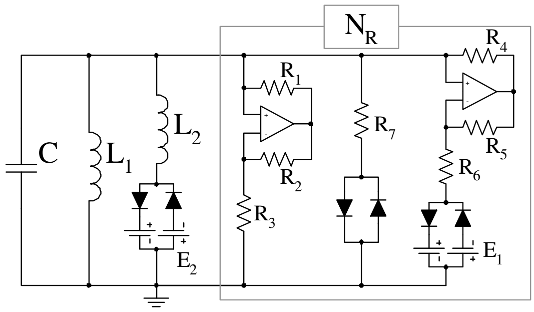

Figure 2: Proposed chaotic circuit with piecewise linear resistors.

2. Model Description

A well–known chaotic circuit proposed by Inaba and Saito[8] is shown in Fig. 1 and i–v characteristic of a typical diode model with polarities also is also shown. The variable $v_d(i_{L2})$ is a function depending on the current through their diodes D in Fig. 1, which determines their chaotic dynamics.

In this study, we substitute a symmetrical continuous five segments piecewise linear resistor for the negative active re-

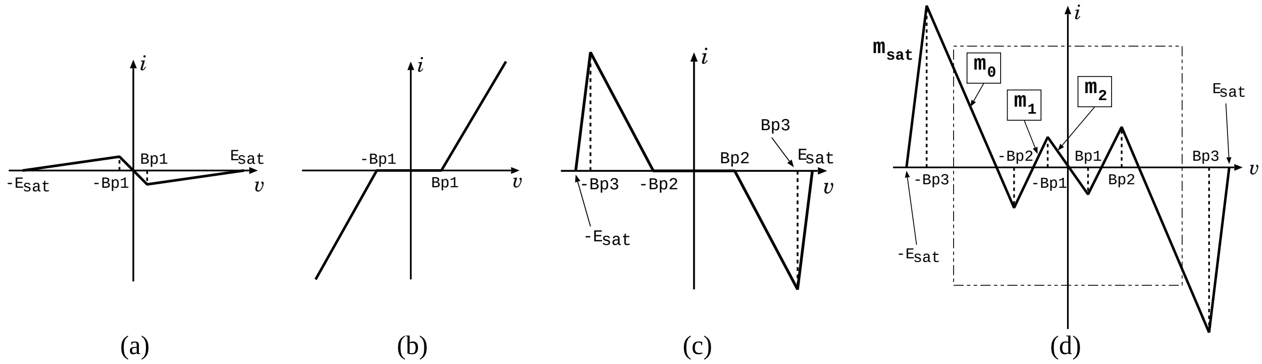

Figure 3: Design method for v–i characteristic of a piecewise linear resistor in the chaotic circuit. Characteristics of (a) N_{r1}, (b) diodes and one resistor, (c) N_{r2} and ideal diodes with polarities, (d) compound characteristics of N_R connected by (a) to (c) in parallel.

sistor including in the chaotic circuit. The designed chaotic circuit is shown in Fig. 2. The piecewise linear resistor can be easily constructed by a connection of some components in parallel as shown in Fig. 3(a)–(c), and then compound characteristics of N_R can be illustrated as shown in (d).

The circuit equation can be normalized when we chose V_d for a threshold voltage value of the diodes by changing the following variables and parameters as follows.

$$\begin{aligned} & i_{L1} = \sqrt{\frac{C}{L_1}} V_d\, x \ , \ \ i_{L2} = \sqrt{\frac{C}{L_1}} V_d\, y \ , \\ & v = V_d\, z \ , \ \ t = \sqrt{L_1 C}\, d\tau \ , \ \ \text{“}\cdot\text{”} = \frac{d}{d\tau} \ , \\ & \beta = \frac{L_1}{L_2} \ , \ \ \gamma = g\sqrt{\frac{L_1}{C}} \ , \ \ \delta = r_s\sqrt{\frac{C}{L_1}} \end{aligned} \tag{1}$$

where g is a linear negative conductance value of N_R if we consider the negative resistor as an ideal. Further let us consider that the part of negative resistance in Fig. 1 replaces to $h(z)$ as a function of voltage source z, then the circuit equations can be rewritten by

$$\left\{ \begin{array}{lcl} \dot{x} & = & z \\ \dot{y} & = & \beta\big(z - f(y)\big) \\ \dot{z} & = & -(x+y) - h(z) \end{array} \right. \tag{2}$$

$$f(y) = \frac{1}{2}\Big\{|\delta y + 1| - |\delta y - 1|\Big\}. \tag{3}$$

where $f(y)$ is a function of the current y and $h(z)$ is a function of the voltage z, respectively. The function $h(z)$ which can be designed by symmetrical five segments piecewise linear with respect to the origin for the parameters four breakpoints at $\{\pm Bp_1,\ \pm Bp_2\}$ and five slopes by $\{m_0, m_1, m_2, m_1, m_0\}$ is described with a canonical form as follows.

$$\begin{aligned} h(z) \triangleq m_0\gamma^* z + \frac{\gamma^*}{2}\Big\{ & (m_0 - m_1)\big(|z - Bp_2| - |z + Bp_2|\big) \\ & + (m_1 - m_2)\big(|z - Bp_1| - |z + Bp_1|\big)\Big\} \end{aligned} \tag{4}$$

where γ^* is a basic variable parameter, hence values $m_k(k = 0,1,2)$ mean the ratio to the value γ^*.

3. Circuit Experiment and Simulation Results

The circuit experiment results by using real implemented circuit are shown. In order to realize the nonlinear characteristic of N_R, we designed a piecewise linear resistor constructed by using some OP amps(TL082CP) and resistors. The details technique is explained in Ref. [13]. The nonlinear resistor is realized by two OP amps, some diodes and DC voltage which is used for setting the threshold voltage strictly, and resistors. The circuit parameters chosen in this simulation are as follows.

L_1 = 123.1[mH], L_2 = 10.2[mH], C = 68.7[nF],
R_1 = 33.2 [kΩ], R_2 = 21.7 [kΩ], R_3 = 1.22 [kΩ],
R_4 = 196 [Ω], R_5 = 333 [Ω], R_6 = 1.47 [kΩ],
R_7 = 10.3 [kΩ], E_1 = 2.78 [V], E_2 = 4.80 [V].

Normally, the voltage v of the circuit oscillates in the area of the threshold voltage between around $\pm\ V_d$ ($V_d \simeq \pm 5.6$[V]) because of a breakpoint at the part of L_2 and diodes. Chaotic and periodic attractors are shown in Fig. 4. Both chaotic and periodic attractors can be observed in the same circuit parameters.

Some numerical simulation results are also shown. The other parameters are β = 10.0, $\gamma^* = 0.76$, δ = 100, and construction of the piecewise linear characteristics form are realized by Bp_1 = 0.30, Bp_2 = 0.56, $m_0 = -1.0$, $m_1 = 1.0$ and $m_2 = -0.5$. It is noticed that values of these breakpoints are normalized by threshold voltage V_d of the diodes with polarities connected to L_2. If we chose V_d between breakpoints Bp_2 and Bp_3 and it is assumed that the value of breakpoint Bp_3 is enough larger than V_d, it can be ignored the area over the voltage $\pm Bp_3$ in the computer calculation. Therefore five segments regions drawn in Fig. 3(d) are used dynamically and this canonical form of five segments piecewise linear function was also only described in Eq. (4). In order to be accuracy numerical calculation, the boundary of switching of the diodes is calculated by using bisection method. Figure 4(d) and (e) show attractors for γ^* = 0.76 when the initial conditions are changed differently. Limit cycle (e) and chaotic attractor (d) can be confirmed coexistence at the same parameters. When γ^* increases, this oscillation mode bifurcates to chaos from one periodic state via period doubling bifurcation in the outside region the following route while keeping the limit cycle in the inside region. Oscillation of symmetrical 1-period → asymmetrical 1-period → bifurcates to 2^n period → asym-

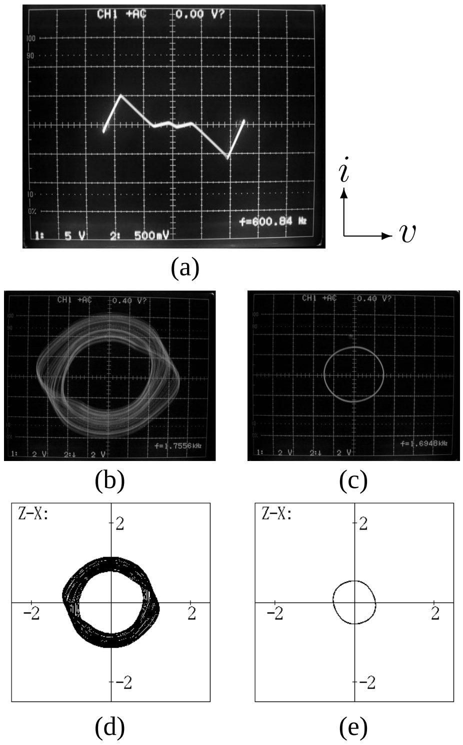

Figure 4: Some snapshots of the circuit experiment and numerical simulation. (a) v–i characteristics of the designed piecewise linear resistor, horizontal: 5[V/div]. (b) chaotic attractor, (c) limit cycle, horizontal: 2[V/div]. Threshold voltage of one diode $v_{th} \simeq 0.78$ [V]. Corresponding results (d) and (e) obtained by numerical calculation onto the z–x plane for the parameters $\beta = 10.0$, $\gamma^* = 0.76$, $\delta = 100$, Bp_1 = 0.30, Bp_2 = 0.56, $m_0 = -1.0$, $m_1 = 1.0$ and $m_2 = -0.5$.

metrical slight chaos → symmetrical fluttered chaos. We can observe that both two oscillation modes exist separately at the same parameters. Hereby we call this this circuit a multi–state chaotic circuit(abbr. MSCC).

4. Multimode Oscillations in Coupled MSCCs

Here we consider the case of the coupled two MSCCs by an inductor as shown in Fig. 5. By using KCL at the loop of the inductors L_0, L_{11} and L_{21}, the circuit equations can be reduced to the six–dimensional differential equations. Further we use a new parameter as $\alpha = L_1/L_0$, by changing parameters and variables similar to Eq. (2), the circuit equation can be described as follows.

$$\left\{ \begin{array}{lcl} \dot{x}_k & = & z_k \\ \dot{y}_k & = & \beta\big(z_k - f(y_k)\big) \\ \dot{z}_k & = & (-1)^k \alpha(x_1 - x_2) - (x_k + y_k) - h(z_k). \end{array} \right. \quad (5)$$

where k is a number of the circuit(k = 1,2.), and α corresponds to the strength of coupling.

Some numerical and circuit experiment results are shown. We can change values γ^* and m_k as control parameters. In this case, four asynchronous oscillation modes could be confirmed consequently by numerical simulations when the initial conditions are varied. These simulation results are shown in Fig. 6 for the case of the parameter α = 0.80, γ^* = 0.480, and other parameters are the same used in the section 3, from the left attractor of z_1–x_1, attractor of z_2–x_2 and synchronization state of z_1–z_2 projection respectively. Figure 6(a) shows an in-phase synchronous state of small size periodic attractors. Figure 6(b) shows an anti-phase synchronous state of small size periodic attractors. Figure 6(c) shows an anti-phase synchronous state of large size chaotic attractors. Figure 6(d) shows a double-mode asynchronous state of large size chaotic and small size periodic motions. These all simulation results are confirmed at the same parameters. Multimode oscillation means that it consists several types of oscillation simultaneously. The multimode oscillations were confirmed in the proposed coupled MSCCs.

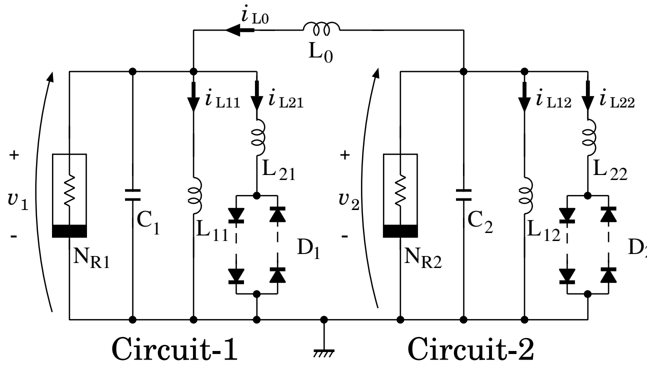

Figure 5: Circuit model of two MSCCs coupled by an inductor.

Some circuit experimental results are shown in Fig. 7. Attractors between v_1 and i_{L11}, synchronization state between v_1 and v_2 are shown. Several types of oscillation modes can be observed. Figure 8 shows a time waveform of v_1 and its FFT spectrum in the case of double-mode oscillations. This one MSCC oscillates around 1.73[kHz] as a fundamental frequency. It can be easily calculated from the circuit parameters. We can confirm that two peaks and wide band frequencies are observed from the FFT result. The highest peak is certainly located at around this point. Therefore chaotic and non-chaotic oscillations in the two circuits are generated alternatively, we can observe the double-mode oscillations in the coupled circuits. Besides we can also confirm several types of multimode oscillations, i.e., in–phase and anti–phase synchronization of periodic oscillations, double-mode chaotic oscillations, unfortunately except for an anti–phase chaotic synchronization mode from the circuit experiment.

5. Conclusions

In this study, we have investigated multimode oscillations in the coupled multi–state chaotic circuits by numerical simulation and circuit experiment. Further coexistence of four types oscillation modes, i.e., in-phase limit cycle, anti-phase limit cycle, anti-phase chaotic synchronization, and double-mode asynchronous of both chaotic and periodic oscillations have been observed in coupled MSCCs by an inductor. These chaotic behavior observed in this study are expected to yield new chaotic phenomena in several types of coupled chaotic systems, e.g., chaotic itinerancy, spatio–temporal chaos, multi–agent systems, and inherent emergent

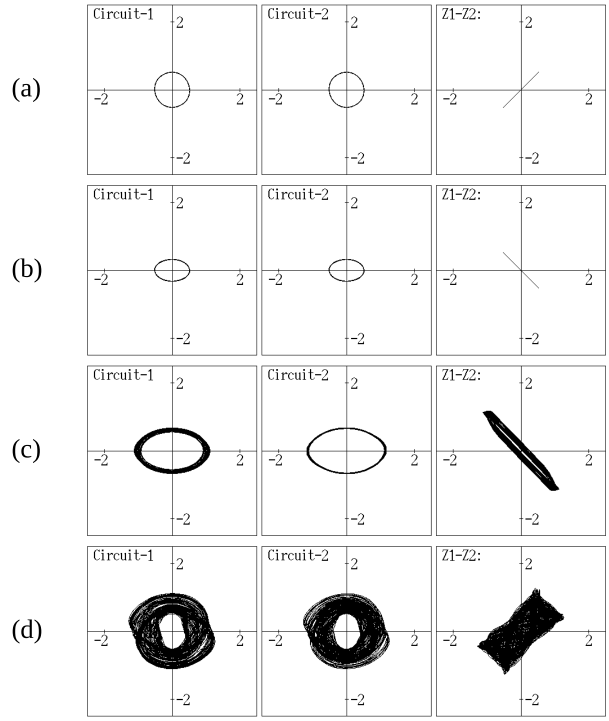

Figure 6: Four oscillation modes exist at the same parameters: α = 0.80, β = 10.0 and γ^* = 0.480. (a) in-phase synchronous limit cycle, (b) anti-phase synchronous limit cycle, (c) anti-phase chaotic synchronous attractor, (d) double-mode oscillations both of two different attractors chaotic and small size limit cycle.

property, in which concerned with current topics.

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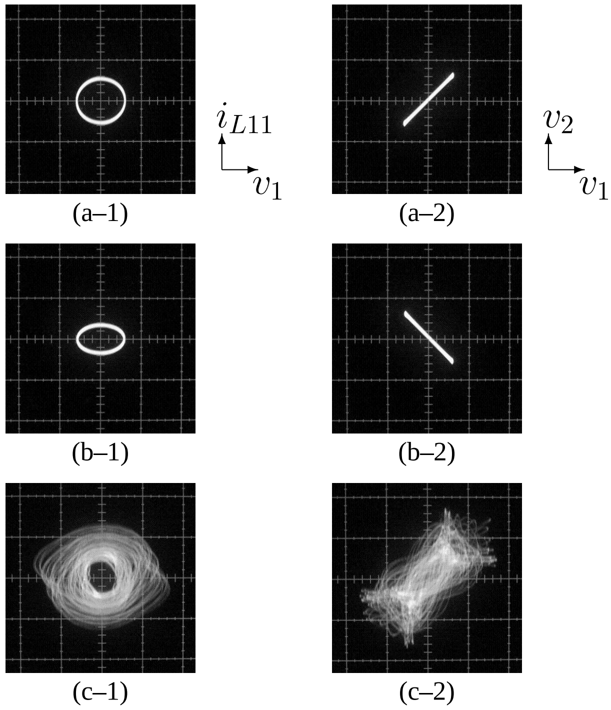

Figure 7: Some snapshots from circuit experiment results for the following parameters; L_0 = 183.3[mH], L_1 = 123.1[mH], L_2 = 10.2[mH], C = 68.7[nF], R_1 = 33.2 [kΩ], R_2 = 21.7 [kΩ], R_3 = 1.22 [kΩ], R_4 = 196 [Ω], R_5 = 333 [Ω], R_6 = 1.47 [kΩ], R_7 = 10.3 [kΩ], E_1 = 2.78 [V], E_2 = 4.80 [V], horizontal: 5[V/div]. (a) in-phase synchronous limit cycle, (b) anti-phase synchronous limit cycle, (c) double-mode oscillations.

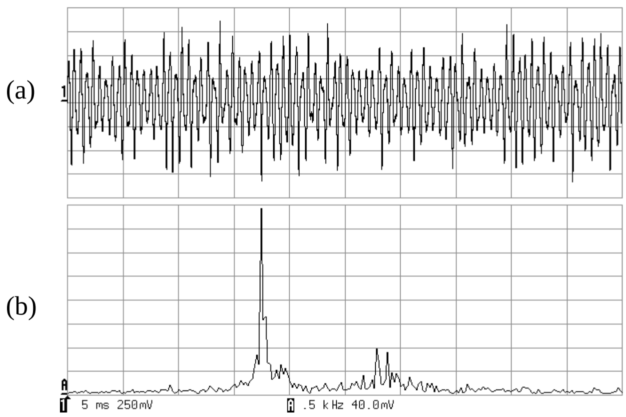

Figure 8: Snapshots of a time waveform and a FFT spectrum at the double-mode chaotic oscillations. (a) time waveform, horizontal: 5[msec/div], vertical: 2.5[V/div], (b) FFT, horizontal: 500[Hz/div], vertical: 400[mV/div].