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Two Chaotic Self-Oscillatory Circuits Coupled by Diodes

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1. Introduction

Many researchers have been investigating chaotic circuits. For instance, proposing new chaotic circuit, analyzing generated phenomena by the chaotic circuit, development of its applications and so on. In particular, many three-dimensional chaotic self-oscillatory circuits are proposed[1][2]. These circuits are investigated in detail and there are many its applications. Most of these circuits include a diode or piecewise linear function circuit as a nonlinear element. A three-dimensional chaotic self-oscillatory circuit proposed by Shinriki et al[3] includes two diodes as nonlinear elements. The chaotic circuit can generate chaos in the case of disconnecting one of diodes. In this study, we propose a system coupled by disconnected diodes. This system consists of two three-dimensional chaotic self-oscillatory circuit. By circuit experiments and computer calculations, chaotic wandering and its control possibility are confirmed. Chaotic wandering control can be expected by its application field.

2. System Model

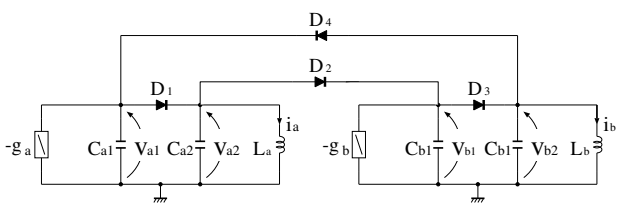

Figure 1: System Model.

Figure 1 shows a proposed system. The system consists of two chaotic self-oscillatory circuits as shown in Fig. 2. In this study, the diode D_b of the circuit is utilized as a coupling element. The diode is disconnected as shown in Fig. 2 and it is connected to the node which disconnected from the other diodes. Therefore, the proposed system consists of two circuit only. Each of negative resistors is approximated as a linear function. Each of diodes is approximated piecewise linear function as shown in Fig. 3.

We define the parameters and variables:

$$v_{a1} = E_{th}x_a,\ \ v_{a2} = E_{th}y_a,\ \ i_a = E_{th}\sqrt{C_{a1}/L_a}z_a,$$
$$v_{b1} = E_{th}x_b,\ \ v_{b2} = E_{th}y_b,\ \ i_b = E_{th}\sqrt{C_{a1}/L_a}Z_b,$$
$$t = \sqrt{L_aC_{a1}}\tau,\ \ \alpha = g_a\sqrt{L_a/C_{a1}},\ \ \beta = C_{a1}/C_{a2},$$
$$\gamma = g_b\sqrt{L_a/C_{b1}},\ \ \delta = C_{a1}/C_{b2},\ \ \varepsilon = L_a/L_b,$$
$$\zeta = g_d\sqrt{L_a/C_{a1}}$$

Normalized equations are described as follows:

$$\begin{cases} \dot{x}_a = & \alpha x_a - \zeta f'(x_a - y_a) + \zeta f'(y_b - x_a), \\ \dot{y}_a = & \beta\{-z_a + \zeta f'(x_a - y_a) - \zeta f'(y_a - x_b)\}, \\ \dot{z}_a = & y_a, \\ \dot{x}_b = & \gamma x_b - \zeta f'(x_b - y_b) + \zeta f'(y_a - x_b), \\ \dot{y}_b = & \delta\{-z_b + \zeta f'(x_b - y_b) - \zeta f'(y_b - x_a)\}, \\ \dot{z}_b = & \varepsilon y_b, \end{cases} \tag{1}$$

where $f'(X) = (X - 1 + |X - 1|)/2$.

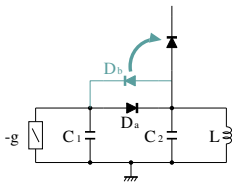

Figure 2: chaotic self-oscillatory circuits proposed by Shinriki et al.

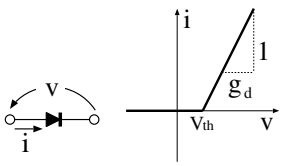

Figure 3: diode model.

3. Chaotic Wandering and its Control

3.1. Experimental Results

Some kind of chaotic attractors are observed in the circuit experiment. Figures 4, 5 and 6 show the circuit exper-

iment results. In Figs. 4,four attractors are observed. In this case, these results are observed on same parameters. The observed attractor depend on an initial state only. In Figs. 5, two attractors are observed. These results are also observed in same parameters. In another parameters, point symmetric attractors of Figs. 5 are observed. Figures 6 are observed attractors in another parameters. These results are observed in different parameters. In Figs. 6 (a), the orbit wanders chaotically among areas which are separated vertical and horizontal axis. Additionally, we found the phenomena that changing values of two negative resistors make it possible to control the chaotic wandering. We investigate this phenomena in the computer calculations.

3.2. Computer Calculated Results

Equations (1) are used for the computer calculations. First, we assume that parameters of two circuits is the same. In this case, attractors similar into Fig. 4, 5 and 6 are observed. However, controlled phenomena of chaotic wandering are not observed. Next, we assume that parameters of two circuits is not the same. Namely, β and δ are changed. β and δ are corresponding to values of capacitors. In this case, same phenomena as experimental results are observed. This result shows that the asymmetry of two circuit parameters is important. Figures 7, 8 and 9 show the computer calculated results. These results are corresponding to Fig. 4, 5 and 6, respectively. These results show that the system model is valid. In order to investigate the phenomena that changing values of two negative resistors make it possible to control the chaotic wandering, this system model and its parameters are used.

Figures 10, 11 show the relationship among α, γ and the orbit existence rate per each quadrant. Parameters α and γ are corresponding to values of two negative resistors in the system. Initial values are set first quadrant and third quadrant as follows:

In Fig. 10 : $x_{a1} = x_{a2} = x_{b1} = x_{b2} = 0.10$.

In Fig. 11 : $x_{a1} = x_{a2} = x_{b1} = x_{b2} = -0.10$.

We also investigated cases that initial values are set second and fourth quadrants. These results are similar to the case of first and third quadrant, respectively. Therefor, we compare Fig. 10 and Fig. 11. For $\alpha < 0.40$ and $\gamma < 0.40$, the orbit existence rate is 0. This means that the orbit staying in one quadrant. Namely, the orbit existence rate is 0 or 100[%], accurately.

Now, we pay attention to each area in figures.

- Area $\alpha = 0.20$-0.40 and $\gamma = 0.40$.
  The rate of first quadrant is 40[%] or more, the rate of third quadrant is 40[%] or less. This result shows that the orbit move between first quadrant and third quadrant and the rate of first quadrant is more than third quadrant. On the other hands,
- Area $\alpha = 0.40$ and $\gamma = 0.20$-0.40.
  The rate of first quadrant is less than third quadrant. Additionally, around these parameter area, the orbit can be also moved to second and fourth quadrant.
- Area $\alpha = 0.10$-0.20 and $\gamma = 0.40$-0.55.
  in the case of Fig. 10, the orbit movement is biased to first and second quadrants. in the case of Fig. 11, the orbit movement is biased to third and fourth quadrants. Additionally, both cases are biased to one of two quadrants by parameters.
- Area $\alpha = 0.40$-0.55 and $\gamma = 0.40$-0.55.
  The rates of second and fourth quadrants are 30[%] or more, the rates of first and third quadrants are 20[%] or less.
- The others.
  Some projecting points in first and third quadrants are observed. As a result of investigating these points, these points shows windows.

As a result, we can say that changing values of two negative resistors make it possible to control the chaotic wandering in some measure.

4. Conclusions

In this study, we have proposed a system which is consist of two chaotic self-oscillatory circuits coupled by diodes. As a result of circuit experiments and computer calculations, we could confirmed that changing values of two negative resistors make it possible to control the chaotic wandering in some measure. In order to observe these phenomena, the asymmetry of two circuit parameters is important.

References

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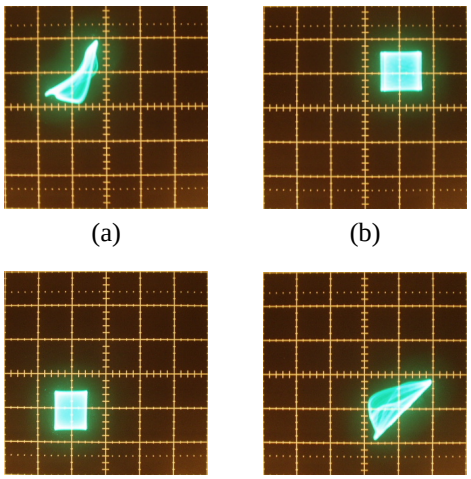
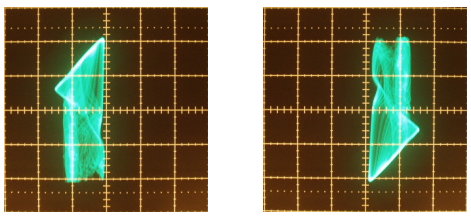


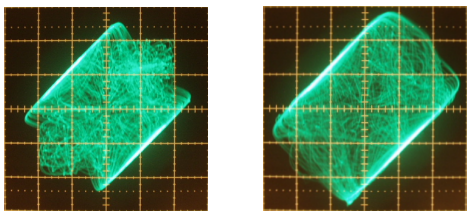
(a) (b) (c) (d)

Figure 4: Circuit experimental results 1. Horizontal axis: v_{a1}[0.5V/div]. Vertical axis: v_{b1}[0.5V/div]. $C_{a1} = 98.90$[nF], $C_{b1} = 103.02$[nF], $C_{a2} = 69.77$[nF], $C_{b2} = 69.68$[nF], $L_a = 51.32$[mH], $L_b = 52.27$[mH], $g_a = 0.5489$[mS] and $g_b = 0.6048$[mS].



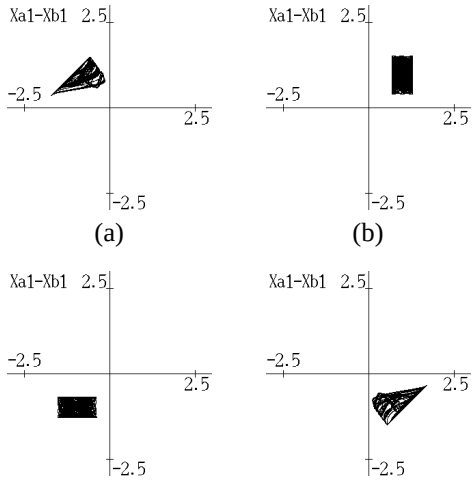
(a) (b)

Figure 5: Circuit experimental results 2. Horizontal axis: v_{a1}[0.5V/div]. Vertical axis: v_{b1}[0.5V/div]. $C_{a1} = 98.90$[nF], $C_{b1} = 103.02$[nF], $C_{a2} = 69.77$[nF], $C_{b2} = 69.68$[nF], $L_a = 51.32$[mH], $L_b = 52.27$[mH], $g_a = 0.3486$[mS] and $g_b = 0.8070$[mS].



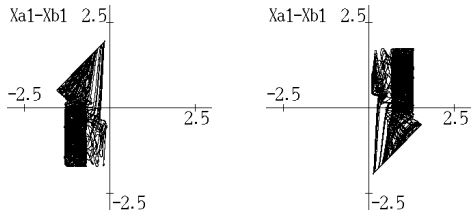
(a) (b)

Figure 6: Circuit experimental results 3. Horizontal axis: v_{a1}[0.5V/div]. Vertical axis: v_{b1}[0.5V/div]. $C_{a1} = 98.90$[nF], $C_{b1} = 103.02$[nF], $C_{a2} = 69.77$[nF], $C_{b2} = 69.68$[nF], $L_a = 51.32$[mH] and $L_b = 52.27$[mH]. (a): $g_a = 0.7407$[mS] and $g_b = 0.7407$[mS]. (b): $g_a = 0.8953$[mS] and $g_b = 0.8540$[mS].



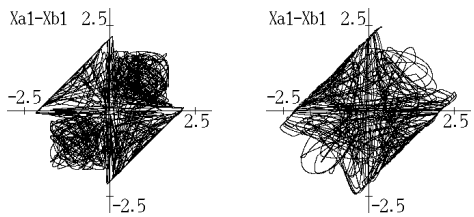
(a) (b) (c) (d)

Figure 7: Computer calculated results 1. Horizontal axis: x_a, Vertical axis: x_b. $\alpha = 0.30$, $\beta = 1.47$, $\gamma = 0.20$, $\delta = 1.30$, $\varepsilon = 1.00$ and $\zeta = 10.0$.



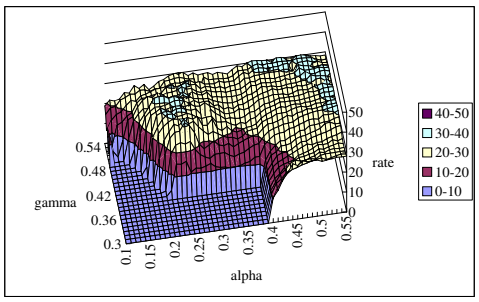
(a) (b)

Figure 8: Computer calculated results 2. Horizontal axis: x_a, Vertical axis: x_b. $\alpha = 0.20$, $\beta = 1.47$,$\gamma = 0.40$, $\delta = 1.30$, $\varepsilon = 1.00$ and $\zeta = 10.0$.

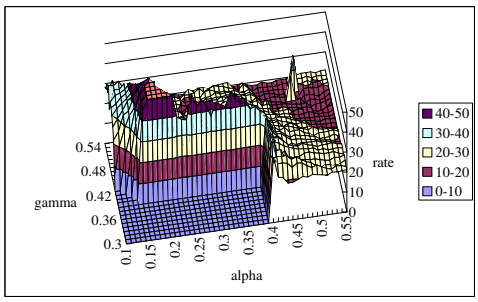


(a) $\alpha = 0.40$, $\gamma = 0.40$ (b) $\alpha = 0.50$, $\gamma = 0.45$

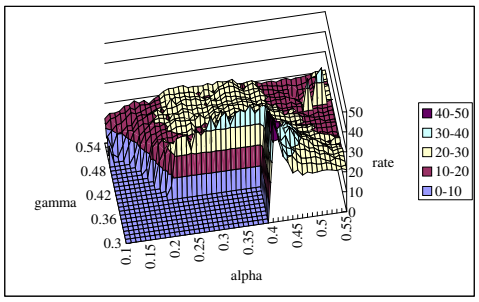
Figure 9: Computer calculated results 3. Horizontal axis: x_a, Vertical axis: x_b. $\beta = 1.47$, $\delta = 1.30$, $\varepsilon = 1.00$ and $\zeta = 10.0$.



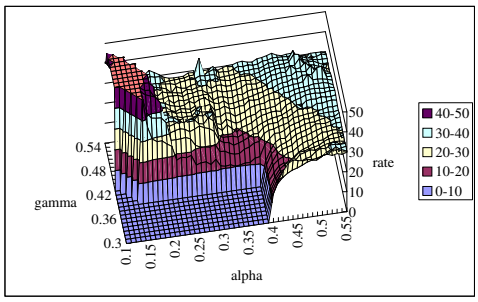
(a) fourth quadrant



(b) first quadrant

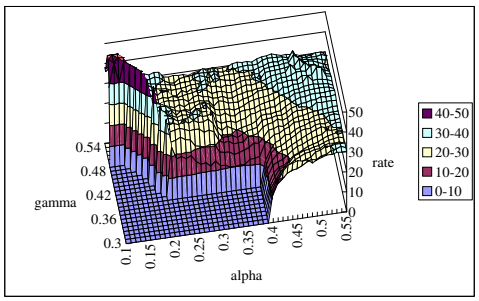


(c) third quadrant

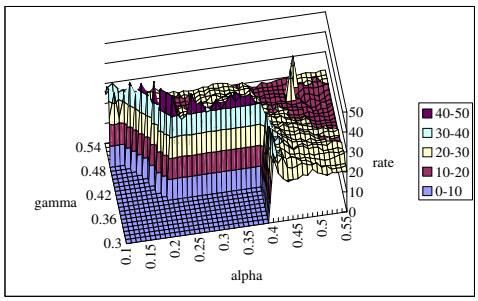


(d) second quadrant

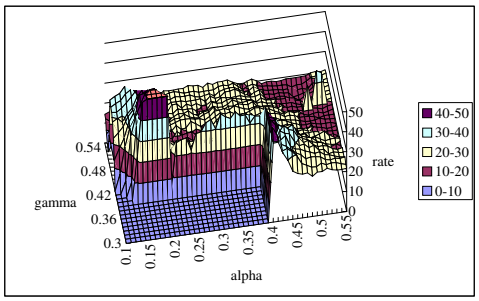
Figure 10: Relationship among two negative resistance values and the orbit existence rate per each quadrant. (initial values: first quadrant). horizontal axis: x_a. Vertical axis:x_b. $\beta = 1.47$, $\delta = 1.30$, $\varepsilon = 1.00$ and $\zeta = 10.0$.



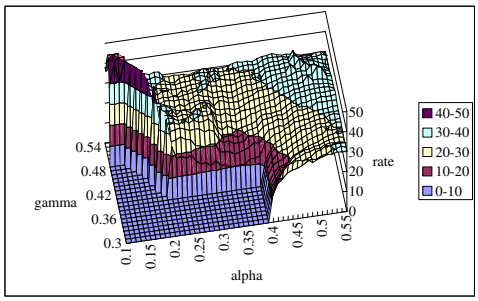
(a) fourth quadrant



(b) first quadrant



(c) third quadrant



(d) second quadrant

Figure 11: Relationship among two negative resistance values and the orbit existence rate per each quadrant. (initial values: third quadrant). Horizontal axis: x_a. Vertical axis: x_b. $\beta = 1.47$, $\delta = 1.30$, $\varepsilon = 1.00$ and $\zeta = 10.0$.