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Penetration of Phase-Inversion Waves on Coupled Oscillators by Inductors as a Cross

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1. Introduction

Large number of coupled limit-cycle oscillators are useful as models for a wide variety of systems in natural fields, for example, diverse physiological organs including gastrointestinal tracts and axial fiber of nervous systems, convecting fluids, arrays of Josephson junctions and so on. Hence, it is very important to analyze synchronization and the related phenomena observed in coupled oscillators in order to clarify mechanisms of generations or in order to control the generating-conditions of various phenomena in such natural systems. In the field of the electrical engineering, a lot of studies on synchronization phenomena of coupled van der Pol oscillators have been carried out up to now.

Recently, we discovered continuously existing wave of changing phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase in coupled van der Pol oscillators by inductors as a ladder [3]-[5]. This phenomenon is observed in steady state. We call this phenomenon as "phase-inversion wave". And, the mechanisms of "propagation," "disappearance," "reflection in the middle of the array" and "reflection at an edge of the array," which were the basic characters of the phase-inversion waves were clarified [3].

In this study, four ladders of van der Pol oscillators are coupled as a cross. Each ladder is composed by van der Pol oscillators which are coupled by inductors. The phase-inversion waves are observed in this system. We investigate various phenomena of the phase-inversion waves by changing initial values and parameters. Especially, we pay our attention to penetration of the phase-inversion waves at the crosspoint. Firstly, propagation mechanism of a pair of phase-inversion waves on a ladder is made clear by using the relationship of phase difference between adjacent oscillators and instantaneous frequency of each oscillator. And, penetration mechanism of two pairs of phase-inversion waves are made clear.

2. Circuit Model

The circuit model used in this study is shown in Fig. 1. N van der Pol oscillators are coupled by coupling inductors L_2. We carried out computer calculations for the cases of $N = 8$. In the computer calculations, we assume the $v - i$ characteristics of the nonlinear negative resistors in each circuit as the following function.

$$i_r(v_{j,k}) = -g_1 v_{j,k} + g_3 v_{j,k}^3 \quad (g_1, g_3 > 0) \tag{1}$$

The circuit equations governing the circuit in Fig.1 are written as:

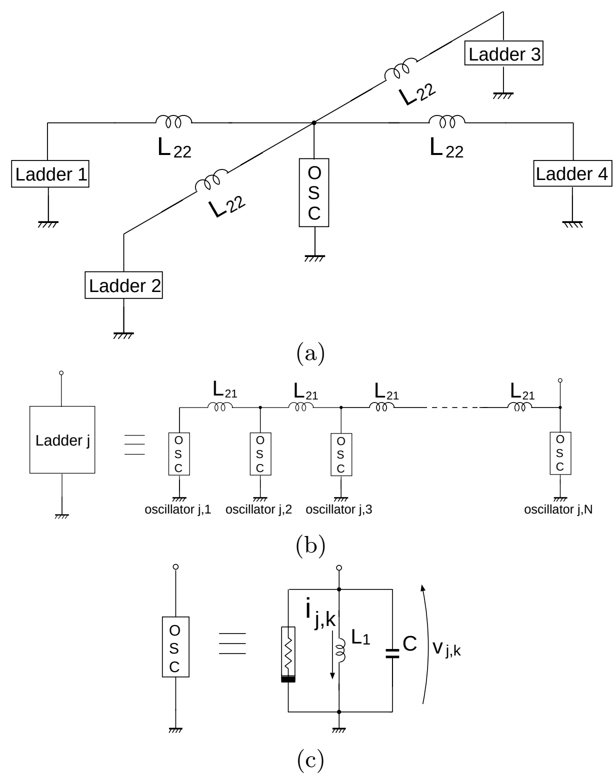


Figure 1: Circuit Model. (a) Coupled ladders system. (b) Coupled oscillators as a ladder. (c) van der Pol oscillator.

[Center Oscillator]

$$\dot{x}_c = y_c \tag{2}$$

$$\dot{y}_c = -x_c + \alpha_1(\sum_{i=1}^{M} x_{i,N} - Mx_c) + \varepsilon(y_c - \frac{1}{3}y_c^3)$$

[Edge Oscillators] (j=1 ∼ 4)

$$\dot{x}_{j,1} = y_{j,1} \tag{3}$$

$$\dot{y}_{j,1} = -x_{j,1} + \alpha_1(x_{j,2} - x_{j,1}) + \varepsilon(y_{j,1} - \frac{1}{3}y_{j,1}^3)$$

[Middle Oscillators] (j=1 ∼ 4, k=2 ∼ N-1)

$$\dot{x}_{j,k} = y_{j,k} \tag{4}$$

$$\dot{y}_{j,k} = -x_{j,k} + \alpha_1(x_{j,k+1} - 2x_{j,k} + x_{j,k-1}) + \varepsilon(y_{j,k} - \frac{1}{3}y_{j,k}^3)$$

[Adjacent Oscillators of Center Oscillator] (j=1 ∼ 4)

$$\dot{x}_{j,N} = y_{j,N} \tag{5}$$

$$\dot{y}_{j,N} = -x_{j,N} + \alpha_1(x_{j,N-1} - x_{j,N}) + \alpha_2(x_c - x_N) + \varepsilon(y_{j,N} - \frac{1}{3}y_{j,N}^3)$$

where

$$t = \sqrt{L_1 C}\tau,\ i_{j,k} = \sqrt{\frac{Cg_1}{3L_1 g_3}}x_{j,k},\ v_{j,k} = \sqrt{\frac{g_1}{3g_3}}y_{j,k},$$

$$\alpha_1 = \frac{L_1}{L_{21}},\ \alpha_2 = \frac{L_1}{L_{22}},\ \varepsilon = g_1\sqrt{\frac{L_1}{C}},\ \frac{d}{d\tau} = \text{“}\cdot\text{”}.$$

It should be noted that α corresponds to the coupling of the oscillators and ε corresponds to the nonlinearity of the oscillators. Throughout the paper, we fix $N = 8$, $\alpha_1 = 0.050$, $\varepsilon = 0.250$ and $\Delta\tau = 0.01$ and calculate (2)-(5) by using the fourth-order Runge-Kutta method.

Next, we explain the mechanism of the generation of the phase-inversion wave by using the change of the instantaneous oscillation frequencies according to the synchronization states and phase difference between adjacent oscillators. It has been already known that oscillation frequency of in-phase synchronization of oscillators coupled by inductors is different from that of anti-phase synchronization. Namely, f_{in}, oscillation frequency of in-phase synchronization, is smaller than f_{anti}, oscillation frequency of anti-phase synchronization. Further, the difference betweens f_{in} and f_{anti} increases as coupling inductance increases [1].

Throughout the paper, we define the phase difference between two adjacent oscillators and the instantaneous frequency of $\mathrm{OSC}_{j,k}$ as follows:

$$\Phi_{j(k,k+1)}(n) = \frac{\tau_k(n) - \tau_{k+1}(n)}{\tau_k(n) - \tau_k(n-1)} \times \pi$$

$$f_{j,k}(n) = \frac{1}{2(\tau_k(n) - \tau_k(n-1))} \tag{6}$$

where $\tau_{j,k}(n)$ is time when the voltage of $\mathrm{OSC}_{j,k}$ crosses 0[V] at n-th time.

All oscillators are in-phase synchronization and phase-inversion waves are generated at the edge of ladder1 and ladder2.

Throughout this paper, center oscillator is shown OSC_c and instantaneous frequency of $\mathrm{OSC}_{j,k}$ is shown $f_{j,k}$ and phase difference between $\mathrm{OSC}_{j,k}$ and $\mathrm{OSC}_{j,k+1}$ is shown $\Phi_{j(k,k+1)}$.

3. A Pair of Phase-Inversion Waves on a Ladder

Figure 2 shows an example of phase-inversion waves on a ladder which is composed by eight oscillators. Vertical axes are sum of two voltages of adjacent oscillators and horizontal axes are time. White regions in the diagram correspond to the states that sum of voltages of the two oscillators are close to zero, namely adjacent two oscillators synchronized at anti-phase. While, black regions correspond to the states that sum of voltages of the two oscillators with large amplitude, namely adjacent two oscillators synchronized at in-phase. In this figure, we can see a pair of phase-inversion waves reflects at the both edges of the array and continuously exists.

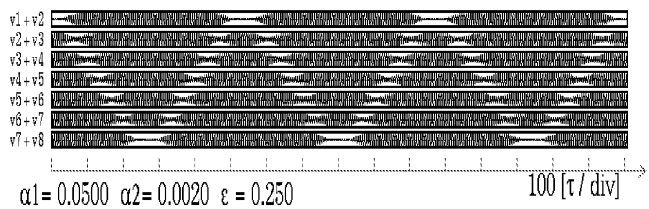
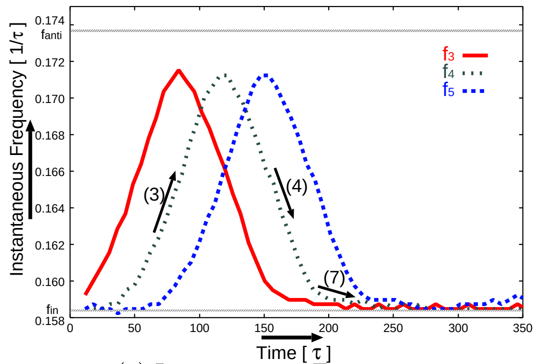


Figure 2: Example of a pair of phase-inversion waves.

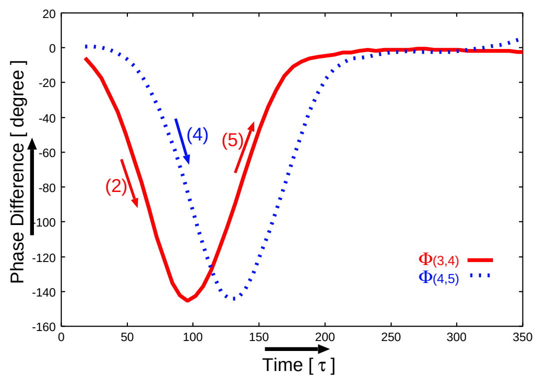
3.1. Mechanism of propagation

Propagation mechanism of a pair of phase-inversion waves are explained according to the phase differences and the instantaneous frequencies. Figures 3 shows phase differences and instantaneous frequencies, where $\Phi_{k,k+1}$ is phase difference between OSC_k and OSC_{k+1} and f_k is instantaneous frequency of OSC_k.

1. Let us assume that phase-inversion waves are going to reach OSC_4 from OSC_1.
2. First phase-inversion wave which changes phase difference from in-phase synchronization to anti-phase synchronization $\Phi_{3,4}$ changes from 0 to π.
3. As $\Phi_{3,4}$ approaches π, f_4 changes from f_{in} to f_{anti}.
4. The change of f_4 causes increase of $\Phi_{4,5}$.
5. Second phase-inversion wave which changes phase difference from anti-phase synchronization to in-phase synchronization reaches OSC_3 before $\Phi_{3,4}$ reaches π. Therefore, $\Phi_{3,4}$ starts to change to 0 again.
6. f_4 starts to change to f_{in} again before f_4 reaches f_{anti}.
7. f_4 reaches f_{in} after $\Phi_{3,4}$ reaches 0 and OSC_1∼ OSC_4 become stable as in-phase synchronization.



(a) Instantaneous Frequency



(b) Phase Difference

Figure 3: Mechanisum of propagation at the middle of array.

4. Two Pairs of Phase-Inversion Waves on a Cross

Figure 4 shows two pair of phase-inversion waves that continuously exist on the cross. These phase-inversion waves continuously exist alterneting ladders.

Two pairs of phase-inversion waves are generated at $OSC_{1,1}$ and $OSC_{2,1}$ and propagate to the oscillator of center. And two pairs of phase-inversion waves do not reflect and penetrate to other two ladders. The penetration mechanism of two pairs of phase-inversion waves are explained according to the phase differences and the instantaneous frequencies too.

4.1. Alternating Propagation Mecanisums

Mechanism of penetration at a cross point are explained according to the phase differences and the instantaneous frequencies. Figures 5 shows phase differences and instantaneous frequencies.

Initial values of $\Phi_{1(8,c)}$, $\Phi_{2(8,c)}$, $\Phi_{3(8,c)}$ and $\Phi_{4(8,c)}$ are 360° instead of 0° in Figure 5(c), because the graphs are better to be understood.

1. Let us assume that first phase-inversion waves on ladder1 and ladder2 are going to reach $OSC_{1,8}$ and $OSC_{2,8}$.
2. $\Phi_{1(8,c)}$ and $\Phi_{2(8,c)}$ begin to change simultaneously. And f_c starts to change quickly from f_{in} to f_{anti} (around τ=188.3).
3. $\Phi_{3(8,c)}$ and $\Phi_{4(8,c)}$ begin to increase (around τ=220).
4. $f_{1,8}$ and $f_{2,8}$ starts to change to f_{in} again by second phase-inversion waves(around τ=248.6).
5. $\Phi_{1(8,c)}$ and $\Phi_{2(8,c)}$ are not large values by the influence which f_c increased quickly (360-308=52[degree] when around τ=242.7).
6. Because, $f_{1,8}$ and $f_{2,8}$ slowly decrease and $\Phi_{1(6,7)}$ and $\Phi_{2(6,7)}$ are values close to 0°, $f_{1,7}$ and $f_{2,7}$ slowly decrease. Threfore, $\Phi_{1(7,8)}$ and $\Phi_{2(7,8)}$ also change toward 0° slowly(around τ=250).
7. Because, $\Phi_{1(7,8)}$ and $\Phi_{2(7,8)}$ are smaller than $-100°$ and f_c is larger than $f_{1,8}$ and $f_{2,8}$, $\Phi_{1(8,c)}$ and $\Phi_{2(8,c)}$ keep increaseing(around τ=268).
8. $f_{1,8}$ and $f_{2,8}$ start to increase again(around τ=268).
9. $\Phi_{1(8,c)}$ and $\Phi_{2(8,c)}$ become mostly in-phase. $\Phi_{3(8,c)}$ and $\Phi_{4(8,c)}$ are large values. Because, $\Phi_{3(8,c)}$ and $\Phi_{4(8,c)}$ are attracted into anti-phase once, f_c quickly increases again(around τ=268).
10. $\Phi_{1(8,c)}$ and $\Phi_{2(8,c)}$ start to change toward anti-phase synchronization(around τ=280).
11. Because phase states between the OSC_c and adjacent OSC_c do not stabilize in anti-phase with these parameters, $f_{3,8}$ and $f_{4,8}$ start to change toward f_{in} again. Therefore, $\Phi_{3(8,c)}$ and $\Phi_{4(8,c)}$ keep increasing. $\Phi_{3(8,c)}$ and $\Phi_{4(8,c)}$ exceed 540° and start to change toward 720°, f_c begins to decrease quickly toward f_{in} again(around τ=280).
12. Because, between $OSC_{1,8}$ and OSC_c and between $OSC_{2,8}$ and OSC_c are attracted to anti-phase synchronization, $f_{1,8}$ and $f_{2,8}$ change to f_{anti} once and start to change toward f_{in} again(around τ=290).
13. Because, $\Phi_{1(8,c)}$ and $\Phi_{2(8,c)}$ reache to 540°, decreasing speed of f_c slow. f_c starts to decrease toward f_{in} again because $\Phi_{1(8,c)}$ and $\Phi_{2(8,c)}$ keep changing toward 720°.

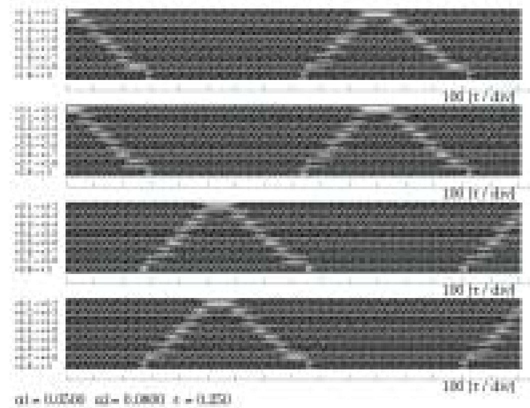


Figure 4: Alternating propagation of phase-inversion waves

14. Phase states between OSC_c and $\mathrm{OSC}_{1,8}$ and between OSC_c and $\mathrm{OSC}_{2,8}$ become in-phase synchronization and stable because $\Phi_{1(8,c)}$ and $\Phi_{2(8,c)}$ reach to 720° and $f_{1,8}$ and $f_{2,8}$ reach to f_{in}(around τ=330).
15. $\Phi_{3(8,c)}$ and $\Phi_{4(8,c)}$ are almost 600°. The phase differences are far from 740°. Therefore, decreasing speed of f_c slows once. And, $f_{3,8}$ and $f_{4,8}$ begin to change toward f_{anti} again (around τ=300)
16. $f_{3,8}$ and $f_{4,8}$ begin to increase. But, f_c is larger than $f_{3,8}$ and $f_{4,8}$ till around 335τ. Therefore, $\Phi_{3(8,c)}$ and $\Phi_{4(8,c)}$ keep to increase till 720° (around τ=310).
17. Phase states between OSC_c and $\mathrm{OSC}_{3,8}$ and OSC_c and $\mathrm{OSC}_{4,8}$ become to in-phase syncronizarion. Phase states between OSC_c and $\mathrm{OSC}_{1,8}$ and OSC_c and $\mathrm{OSC}_{2,8}$ also become to in-phase synchronization. f_c changes to f_{in} quickly.
18. $f_{3,8}$ and $f_{4,8}$ are also attracted to f_{in}.

    As observed above, the phase-inversion wave is penetrated from ladder1,2 to ladder3,4.

5. Conclusions

In this study, four ladders of van der Pol oscillators were coupled by an oscillator and four inductors as a cross. When two pairs of phase-inversion waves were generated at each edges of two ladders, the phase-inversion waves which alternated in two pairs of ladders were observed. By using the relationship between phase states and instantaneous oscillation frequencies, we explained the mechanisms of the propagation and the alternating propagation of wave.

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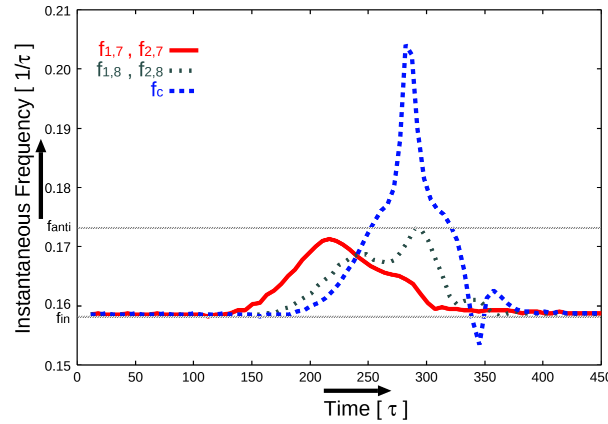
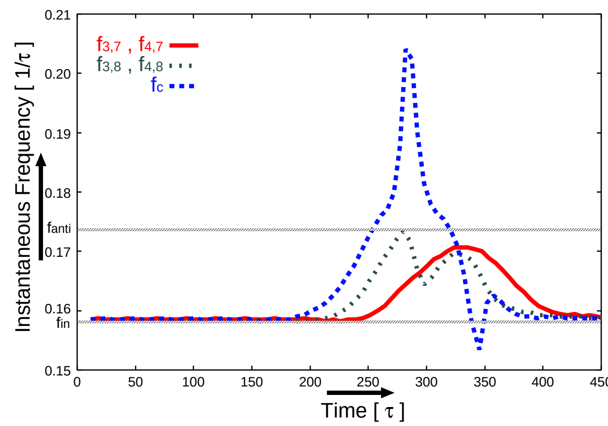
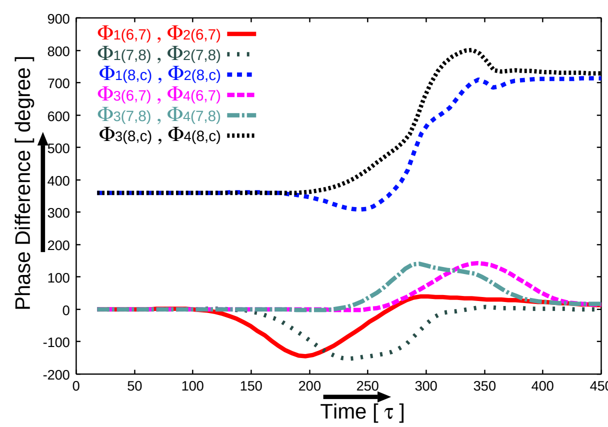


(a) Frequency(ladder1,center)



(b) Frequency(ladder3,center)



(c) Phase Difference

Figure 5: Machanisum of alternating propagation of phase-inversion waves.