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Phase-Inversion Waves in Oscillator Ladders Coupled by Inductors

Yusaku ACHI†, Miki AKAZAWA†, Masayuki YAMAUCHI†† and Yoshifumi NISHIO†

† Tokushima University
2-1 Minami-Josanjima, Tokushima, Japan
Phone:+81-88-656-7470, Fax:+81-88-656-7471
Email: yusaku@ee.tokushima-u.ac.jp

††Sophia University

1. Introduction

In world today, many kinds of circuits for various applications have been designed and developed. Especially, digital circuits use signals generated by oscillators. Therefore, oscillators always keep an important position, and various oscillators still continued to be designed and put to practical use. Recently, a lot of companies develop systems of high speed. In these systems, high frequency oscillators are needed. For instance, central processing units and communication instruments are needed oscillators which generate giga heartz. Especially, in communication, not only oscillators generating high frequency but also high frequency oscillators which hop at high speed are needed. A lot of researchers emphasize to develop these oscillators. Moreover, some researches that coupled oscillators apply to algorithm of walking of the robot are done [1]-[4].

Recently, we discovered continuously existing wave of changing phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase in coupled van der Pol oscillators by inductors as a ladder [5]-[7]. This phenomenon is observed in steady state. We call this phenomenon as “phase-inversion wave”. And, the mechanisms of “propagation,” “disappearance,” “reflection in the middle of the array” and “reflection at an edge of the array,” which were the basic characters of the phase-inversion waves were clarified [5].

In this study, some ladders which are composed of van der Pol oscillators coupled by inductors are coupled by some inductors. We investigate various phenomena of the phase-inversion waves by changing initial values and parameters. The observed phenomena are classified and the relation between the phenomena and the parameters are clarified.

2. Circuit Model

The circuit model used in this study is shown in Fig. 1. N van der Pol oscillators are coupled by coupling inductors L_0. We carried out computer calculations for the cases of $N = 8$. In the computer calculations, we assume the $v - i$ characteristics of the nonlinear negative resistors in each circuit as the following function.

$$i_r(v_{j,k}) = -g_1 v_{j,k} + g_3 v_{j,k}^3 \quad (g_1, g_3 > 0) \tag{1}$$

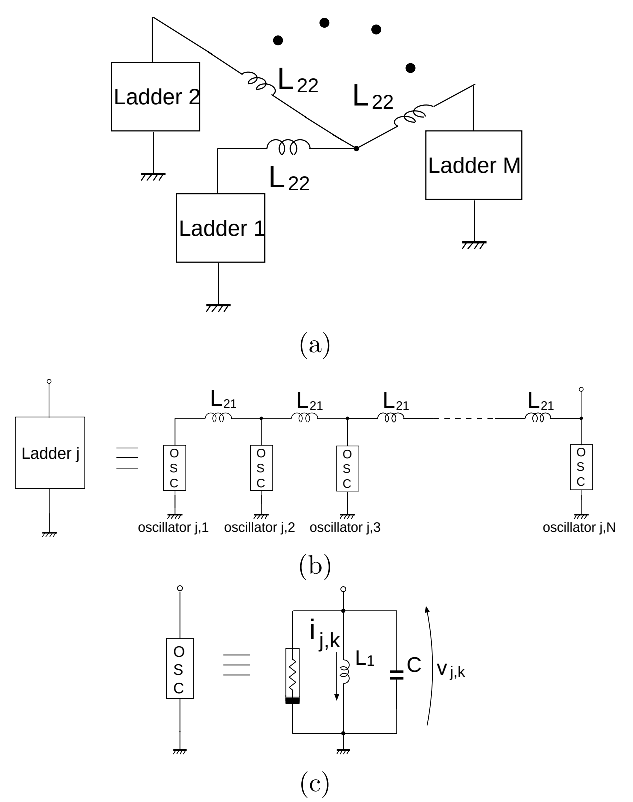


Figure 1: Circuit Model. (a) Coupled ladders system. (b) Coupled oscillators as a ladder. (c) van der Pol oscillator.

The circuit equations governing the circuit in Fig. 1 are written as:
[Edge Oscillators]

$$\dot{x}_{j,1} = y_{j,1} \tag{2}$$

$$\dot{y}_{j,1} = -x_{j,1} + \alpha_1(x_{j,2} - x_{j,1}) + \varepsilon(y_{j,1} - \frac{1}{3}y_{j,1}^3)$$

[Middle Oscillators]

$$\dot{x}_{j,k} = y_{j,k} \tag{3}$$

$$\dot{y}_{j,k} = -x_{j,k} + \alpha_1(x_{j,k+1} - 2x_{j,k} + x_{j,k-1}) + \varepsilon(y_{j,k} - \frac{1}{3}y_{j,k}^3) \quad (k = 2, 3, \dots, N-1)$$

[Adjacent Oscillators of Jointed Point]

$$\dot{x}_{j,N} = y_{j,N} \tag{4}$$

$$\dot{y}_{j,N} = -x_{j,N} + \alpha_1(x_{j,N-1} - x_{j,N}) + \frac{\alpha_2}{M}(x_c - x_N) + \varepsilon(y_{j,N} - \frac{1}{3}y_{j,N}^3)$$

where

$$t = \sqrt{L_1 C}\tau,\ i_{j,k} = \sqrt{\frac{Cg_1}{3L_1 g_3}}x_{j,k},\ v_{j,k} = \sqrt{\frac{g_1}{3g_3}}y_{j,k},$$

$$\alpha_1 = \frac{L_1}{L_{21}},\ \alpha_2 = \frac{L_1}{L_{22}},\ \varepsilon = g_1\sqrt{\frac{L_1}{C}},\ \frac{d}{d\tau} = \text{“}\cdot\text{”}.$$

It should be noted that α corresponds to the coupling of the oscillators and ε corresponds to the nonlinearity of the oscillators. Throughout the paper, we fix $N = 8$, $\alpha_1 = 0.050$, $\varepsilon = 0.250$ and $\Delta\tau = 0.01$ and calculate (2)-(4) by using the fourth-order Runge-Kutta method.

3. Phase-Inversion Waves

Value of α_2 and oscillator which are generated phase-inversion waves are changed. And we observe phase-inversion waves in this circuit. Further, we fix that a pair of phase-inversion waves is generated at edge oscillators.

Complex phenomena are observed in domain of α_2 which is not written in tables. These phenomena can not been classified.

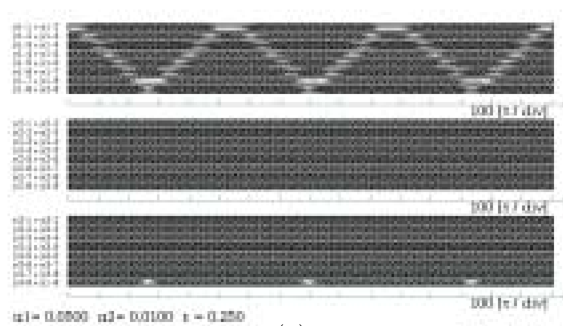
3.1. Coupled Three Ladders

Observed phenomena are classified into 3 patterns by changing α_2. Each number of generated at edge of ladder was summarized in Tab. 1. Figure 2 shows an example of phase-inversion waves on the circuit.

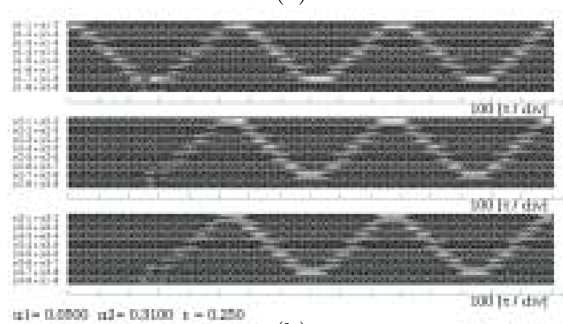
1−(a) The phase-inversion waves propagate to the joint of ladder and reflect. The phase-inversion waves do not penetrate to other ladders and continuously exist.

1−(b) The phase-inversion waves propagate to the joint of ladder. And the phase-inversion waves reflect at the joint of ladder and penetrate to other two ladders. The phase-inversion waves are continuously existing on all ladders.

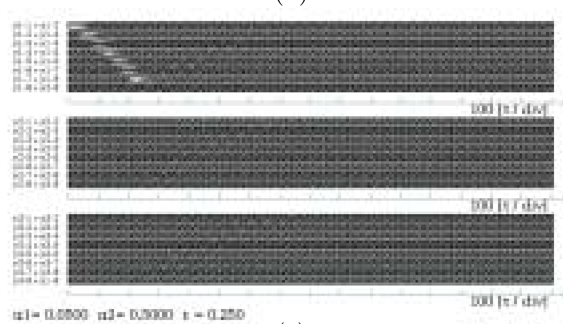
1−(c) The phase-inversion waves propagate to the joint of ladder and disappear.



(a)



(b)



(c)

Figure 2: $M = 3$. Waves generated from $\mathrm{OSC}_{1,1}$.

3.2. Coupled Four Ladders

Observed phenomena are classified into 4 patterns by changing α_2. Each number of generated at edge of ladder

Table 1: $M = 3$.

(a) Waves are generated from $OSC_{1,1}$.

α_2	Classification	Figure
~0.011	1−(a)	2(a)
0.300~0.376	1−(b)	2(b)
0.044~0.049 0.408~	1−(c)	2(c)

(b) Waves are generated from $OSC_{1,1}$ and $OSC_{2,1}$.

α_2	Classification
~ 0.019	1-(a)
0.035 ~ 0.038 0.220 ~	1-(b)
0.044 ~ 0.046	1-(c)

(c) Waves are generated from all edges of ladders.

α_2	Classification
In all almost the areas	1-(a)

was summarized in Tab. 2. Figure 3 shows an example of phase-inversion waves on the circuit.

2−(a) The phase-inversion waves propagate to the joint of ladder and reflect. The phase-inversion waves do not penetrate to other ladders and continuously exist.

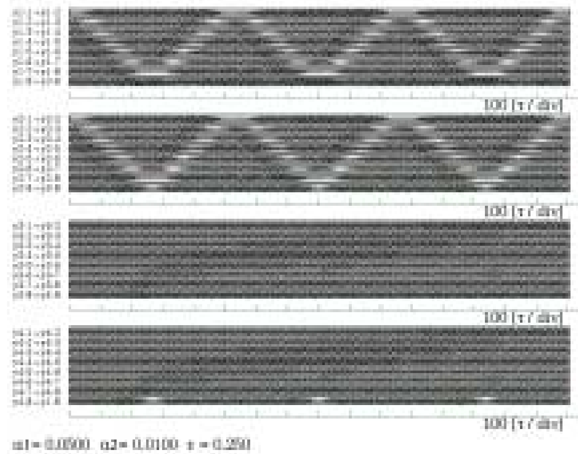
2−(b) The phase-inversion waves propagate to the joint of ladder. And the phase-inversion waves do not reflect and penetrate to other two ladders. The phase-inversion waves continuously exist on two ladders.

2−(c) The phase-inversion waves propagate to the joint of ladder. And the phase-inversion waves reflect at the joint of ladder and penetrate to other two ladders.

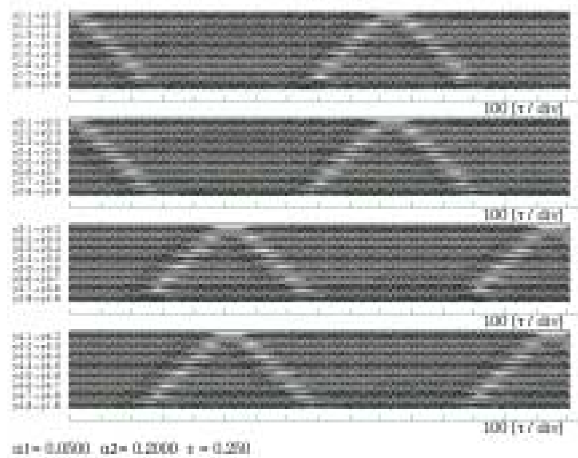
2−(d) The phase-inversion waves become extinction while the phase-inversion waves are propagating.

4. In-and-Anti-Phase Synchronization

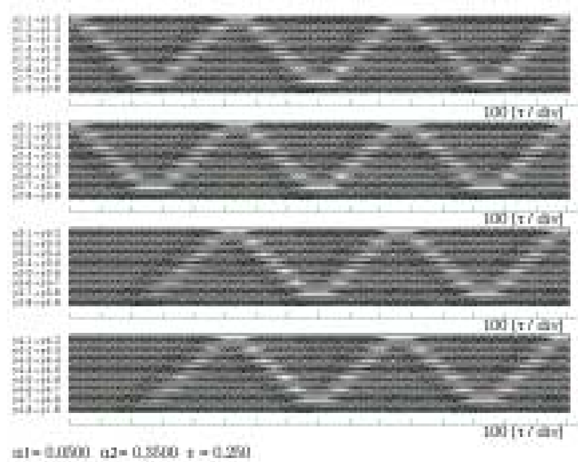
In this section, we observes the propagation of the phase-inversion wave in the another type of synchronization. Another type of synchronization is "in-and-anti-phase synchronization" which in-phase and anti-phase are existing alternately. In this section, three ladders were coupled, and put it into the state of "in-and-anti-phase synchronization". When a pair of phase-inversion waves are generated from $OSC_{1,1}$ and $OSC_{2,1}$, the phase-inversion waves do not reflect and penetrate to the other ladder. The range of this phenomena which observed in this circuit is shown in Tab. 3.



(a)



(b)



(c)

Figure 3: $M = 4$. Waves are generated from $OSC_{1,1}$ and $OSC_{1,2}$.

Table 2: $M = 4$.

(a) Waves are generated from $OSC_{1,1}$.

α_2	Classification
∼0.010	2−(a)
0.085∼0.124 0.188∼0.299	2−(c)
0.043∼0.072 0.288∼	2−(d)

(b) Waves are generated from $OSC_{1,1}$ and $OSC_{2,1}$.

α_2	Classification	Figure
∼ 0.032	2-(a)	3(a)
0.057 ∼ 0.275	2-(b)	3(b)
0.295 ∼	2-(c)	3(c)

(c) Waves are generated from $OSC_{1,1}$, $OSC_{2,1}$ and $OSC_{3,1}$.

α_2	Classification
∼ 0.032	2-(a)
0.116 ∼	2-(c)

(d) Waves are generated from all edges of ladders.

α_2	Classification
In all almost the areas	2-(a)

Table 3: $M = 3$.
Waves are generated from $OSC_{1,1}$ and $OSC_{2,1}$.

α_2	Figure
0.083 ∼ 0.094	Fig.4

5. Conclusions

In this study, some ladders which are composed of van der Pol oscillators coupled by inductors are coupled by some inductors as a cross. We investigated various phenomena of the phase-inversion waves such as "reflection," "penetration," and "disappearance" at a joint of ladders, by changing initial values and parameters. When two pairs of phese-inversion waves were generated at each edge of two ladder of coupled four ladders, the phase-inversion waves were do not reflect and penetrate to other two ladders. Further, the phase-inversion waves which existing alternately were observed, when all oscillators are state of in-and-anti-phase synchronization.

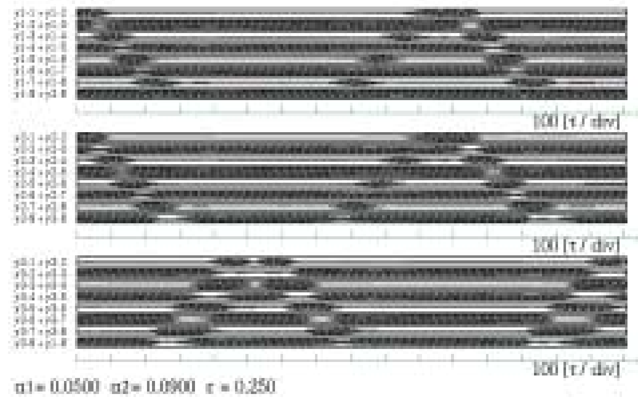


Figure 4: $M = 3$. Waves are generated from $OSC_{1,1}$ and $OSC_{2,1}$.

References

[1] T. Suezaki and S. Mori, "Mutual Synchronization of Two Oscillators," *Trans of IECE*, vol. 48, no. 9, pp. 1551–1557, Sep. 1965.

[2] T. Endo and S. Mori, "Mode Analysis of a Multimode Ladder Oscillator," *IEEE Trans. Circuits and Syst.*, vol. 23, pp. 100-113, Feb. 1976.

[3] S. Ito, H. Yuasa and K. Ito, "Oscillator-Mechanical Model of the Pattern Transition on Quadrupedal Locomotion Based on Energy Expenditure," *Trans. of SICE*, Vol. 32, no. 11, pp. 1535–1543, Nov. 1996 (in Japanese).

[4] A. Fujii, A. Ishiguro and P. E. Hotz, "Generation of Bipedal Locomotion Using a CPG Circuit with Neuromodulation," *Trans. of SICE*, Vol. 38, no. 11, pp. 981–989, Nov. 2002 (in Japanese).

[5] M. Yamauchi, M. Wada, Y. Nishio and A. Ushida, "Wave Propagation Phenomena of Phase States in Oscillators Coupled by Inductors as a Ladder," *IEICE Trans. Fundamentals*, vol.E82-A, no.11, pp.2592-2598, Nov. 1999.

[6] M. Yamauchi, M. Wada, Y.Nishio and A. Ushida, "Collisions between Two Phase-Inversion-Waves in an Array of Oscillators," *Proc. ISCAS'00*, vol.1, pp.679-682, May 2000.

[7] M. Yamauchi, Y. Nishio and A. Ushida, "Reflection and Transmission of Phase-Inversion-Waves in Oscillators Coupled by Two Kinds Inductors," *Proc. ISCAS'01*, vol.3, pp.767-770, May 2001.