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Analysis of Voltage Distribution of Interconnects

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1. Introduction

The analysis and design of high speed LSI chips are becoming more and more important, because PCBs connecting LSI chips may cause the signal delays and crosstalks, which sometimes happen to faulty switching operations. In the last decade, many papers have been published on the transient analysis of lossy interconnects [1]-[3],[7]. The recursive convolution methods using moment-matching technique [2],[3] can be efficiently applied to the lossy interconnects terminated by nonlinear elements. However, one of serious problems in the moment-matching method is that the poles calculated by the Maclaurin expansion and Padè approximation may happen to cause the large errors when the poles are located far from the origin. To overcome the problem, Nakhla et al. propose CFH (complex frequency hopping) [4] which can calculate the exact poles by properly changing the origins of the Taylor expansions on imaginary axis. On the other hand, an equivalent circuit technique is shown in [5], where the interconnects are replaced by the discrete π-type and/or T-type models. It can be efficiently applied to the relatively short interconnect.

In this paper, we propose an asymptotic equivalent circuit technique based on a complex frequency domain. Firstly, we calculate the exact poles and the residues of the admittance matrix, and after then , the admittance matrix is described by the partial fractions. Secondly, the equivalent circuit is synthesized with the essential poles and residues of the partial fractions. Lastly, replacing the interconnect by the equivalent circuit, we calculate the transient response with Spice. In section 2, we show how to calculate the exact poles and residues. Our asymptotic equivalent circuit shown in section 3.

Although most papers have only discussed the transient responses at the near and far ends of interconnects, knowing the voltage distribution on the interconnect is also important for designing ICs because it may give the large crosstalks to the other equipments. In this paper, two methods are shown for calculating the distribution which are based on the numerical Laplace transformation and the asymptotic equivalent circuit. We show the algorithms in section 4, and some interesting illustrative examples in section 5.

2. Calculation of the poles and the residues of interconnect

Now, consider a uniformly coupled N conductors' interconnect. The Telegraph equation is described by

$$\left.\begin{aligned} \frac{d\mathbf{V}(x,s)}{dx} &= -(\mathbf{R}+s\mathbf{L})\mathbf{I}(x,s) \\ \frac{d\mathbf{I}(x,s)}{dx} &= -(\mathbf{G}+s\mathbf{C})\mathbf{V}(x,s) \end{aligned}\right\} \tag{1}$$

Let us introduce the transfer matrix $\mathbf{P}_v(s)$ [6], and calculate the eigenvalues as follows:

$$\mathrm{diag}[\gamma_j(s)^2] = \mathbf{P}_v(s)^{-1}(\mathbf{R}+s\mathbf{L})(\mathbf{G}+s\mathbf{C})\mathbf{P}_v(s) \tag{2}$$

We further introduce the transfer matrix $\mathbf{P}_c(s)$:

$$\mathbf{P}_c(s) = (\mathbf{R}+s\mathbf{L})^{-1}\mathbf{P}_v(s)\mathbf{\Gamma}(s), \quad \mathbf{\Gamma}(s) = \mathrm{diag}[\gamma_j(s)] \tag{3}$$

Then, the input and output relation is described by the admittance matrix as follows:

$$\begin{bmatrix} \mathbf{I}(0,s) \\ -\mathbf{I}(d,s) \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{11}(d,s) & \mathbf{Y}_{12}(d,s) \\ \mathbf{Y}_{21}(d,s) & \mathbf{Y}_{22}(d,s) \end{bmatrix} \begin{bmatrix} \mathbf{V}(0,s) \\ \mathbf{V}(d,s) \end{bmatrix} \tag{4}$$

where

$$\begin{aligned} \mathbf{Y}_{11}(d,s) = \mathbf{Y}_{22}(d,s) &= \mathbf{P}_c(s)\mathrm{diag}[\coth\gamma_j(s)d]\mathbf{P}_v(s)^{-1} \\ \mathbf{Y}_{12}(d,s) = \mathbf{Y}_{21}(d,s) &= -\mathbf{P}_c(s)\mathrm{diag}[\sinh\gamma_j(s)d]^{-1}\mathbf{P}_v(s)^{-1} \end{aligned} \tag{5}$$

we have the following theorem for calculations of the poles [7].

Theorem 1: *The locations of poles satisfying relations (5) are found by solving the following equation:*

$$\left|(\mathbf{R}+s\mathbf{L})(\mathbf{G}+s\mathbf{C}) + \left(\frac{i\pi}{d}\right)^2\mathbf{I}\right| = 0, \quad i = 0,1,2,\ldots \tag{6}$$

On the other hand, the poles for the zero eigenvalue are obtained by the following relation:

$$\lim_{\gamma_i(s)\to 0}\mathbf{P}_c(s)\mathrm{diag}[\sinh\gamma_i(\mathrm{s})\mathrm{d}]^{-1}\mathbf{P}_c(s)^T = \frac{1}{d}(\mathbf{R}+s\mathbf{L})^{-1} \tag{7}$$

Thus, the poles for zero eigenvalues are calculated by $|\mathbf{R}+s\mathbf{L}| = 0.$ Thus, all the poles for $n=0$ are located on negative real axis. Since the relations (6) and (7) are given by the algebraic equation of s, the poles are numerically calculated by the use of the algorithm such as Bairstow. Now, let us calculate the residues of admittance matrices (4).

Theorem 2: The residues of $\mathbf{Y}_{12}(d,s)$ and $\mathbf{Y}_{21}(d,s)$ for the pole

p_i are given by

$$\mathbf{k}_{12,i} = \mathbf{k}_{21,i}$$
$$= \mathbf{P}_c(s)\mathrm{diag}\left[0\cdots\overset{\cdots\quad i\quad\cdots}{\left(\frac{1}{\cosh(\gamma_i(s)d)\dfrac{\partial\gamma_i(s)}{\partial s}d}\right)}\cdots 0\right]\mathbf{P}_v(s)^{-1}\Bigg|_{s=p_i} \tag{8}$$

Corollary: The residue of $\mathbf{Y}_{11}(d,s)$ and $\mathbf{Y}_{11}(d,s)$ are given by

$$\mathbf{k}_{11,i} = \mathbf{k}_{22,i}$$
$$= \mathbf{P}_c(s)\mathrm{diag}\left[0\cdots\overset{\cdots\quad i\quad\cdots}{\left(\frac{1}{\dfrac{\partial\gamma_i(s)}{\partial s}d}\right)}\cdots 0\right]\mathbf{P}_v(s)^{-1}\Bigg|_{s=p_i} \tag{9}$$

Observe that the residues given by (8) and (9) are same except for the sings, because the denominator of (8) has the coefficient $\cosh(\gamma(s)d)$ taking "1" or "-1" depended on $\cosh(\gamma(s)d)|_{s=p_i} = jn\pi, i = 1,2,\cdots,N, n = 1,2,\cdots$.

Using these poles and the residues, the elements of admittance matrix (4) are described by the partial fractions as follows:

$$Y_{11,ij}(s) = Y_{22,ij}(s) = \frac{k_{0,ij}}{s-p_0} + \frac{k_{1,ij}}{s-p_1} + \frac{k_{2,ij}}{s-p_2} + \frac{k_{3,ij}}{s-p_3} + \cdots \tag{10.1}$$

$$Y_{12,ij}(s) = Y_{21,ij}(s) = \frac{k_{0,ij}}{s-p_0} - \frac{k_{1,ij}}{s-p_1} + \frac{k_{2,ij}}{s-p_2} - \frac{k_{3,ij}}{s-p_3} + \cdots \tag{10.2}$$

Thus, the input-output relations described by (4) is realized with the equivalent asymptotic circuit as shown in the next section.

3. Asymptotic equivalent circuit

Note that since the terms corresponding to poles far from the origin in s-plane don't give large effect in the transient analysis, the lossy interconnect can be well-modeled with relatively few terms. In this section, we consider the modeling the admittance matrix (4) described by the partial fractions (10). For simplicity, we consider an uncoupled uniform interconnect. The admittance equation is given by

$$\begin{bmatrix} I(0,s) \\ -I(d,s) \end{bmatrix} = \frac{1}{Z_0(s)}\begin{bmatrix} \coth\gamma(s)d & -[\sinh\gamma(s)d]^{-1} \\ -[\sinh\gamma(s)d]^{-1} & \coth\gamma(s)d \end{bmatrix} \times \begin{bmatrix} V(0,s) \\ V(d,s) \end{bmatrix} \tag{11}$$

where The poles $(i \neq 0)$ satisfy the following relation:

$$\sqrt{(R+sL)(G+sC)} = j\pi i/d, \quad i = 1,2,\ldots \tag{12}$$

Thus, we have

$$p_0 = -u_0, \quad p_i = -u_i \pm jv_i, \quad i = 1,2,\ldots \tag{13}$$

where $u_0 = R/L$, and

$$u_i = \frac{LG+RC}{2LC}, \ v_i = \frac{\sqrt{4LC(RG+(\frac{i\pi}{d})^2)-(LG+RC)^2}}{2LC}$$

Then, the corresponding residues are given by

$$k_{0,11} = \frac{2}{Ld}, \ k_{i,11} = \frac{1}{Z_0(s)D\{\lambda(s)\}d}\Bigg|_{s=p_i} \quad i = 1,2,\ldots \tag{14.1}$$

$$k_{0,12} = -\frac{2}{Ld}, \ k_{i,12} = \frac{(-1)^{i-1}}{Z_0(s)D\{\lambda(s)\}d}\Bigg|_{s=p_i} \quad i = 1,2,\ldots \tag{14.2}$$

where $k_{0,22} = k_{0,11}, \ k_{i,22} = k_{i,11}, \ k_{0,21} = k_{0,12}, \ k_{i,21} = k_{i,12}$

$$D\{\lambda(s)\} = \frac{\partial\lambda(s)}{\partial s} = \frac{L}{2}\sqrt{\frac{G+sC}{R+sL}} + \frac{C}{2}\sqrt{\frac{R+sL}{G+sC}} \tag{15}$$

Since the poles and residues are composed of the complex conjugates, the admittance matrices can be described as follows:

$$\begin{aligned} &Y_{11}(d,s) = Y_{22}(d,s) \\ &\simeq \frac{k_0}{s+p_0} + \sum_{i=1}^{2M}\frac{2\Re\{k_i\}s + 2\Re\{k_ip_i\}}{(s+u_i)^2+v_i^2} \end{aligned} \tag{16.1}$$

$$\begin{aligned} &Y_{12}(d,s) = Y_{21}(d,s) \\ &\simeq -\frac{k_0}{s+p_0} + \sum_{i=1}^{2M}\frac{2\times(-1)^{i-1}(\Re\{k_i\}s + \Re\{k_ip_i\})}{(s+u_i)^2+v_i^2} \end{aligned} \tag{16.2}$$

For simplicity, we set here $k_0 = k_{0,11} = -k_{0,12}$, $k_i = k_{i,11} = |k_{i,12}|$, and $2M+1$ means the order of approximation. Set each term of (16) as follows:

$$Y_0(s) = \frac{b_0}{s+a_0}, \ Y_i(s) = \frac{b_{i1}s+b_{i0}}{s^2+a_{i1}s+a_{i0}}, \ i = 1,2,\ldots,2M \tag{17}$$

Each term of (17) is realized by the circuit by RLC circuit shown by Fig.1 whose parameters are given as follows:

$$\left.\begin{aligned} L_0 = \frac{1}{b_0}, \ R_0 = \frac{a_0}{b_0} \ L_i = \frac{1}{b_{i1}}, \ R_i = \frac{a_{i1}b_{i1}-b_{i0}}{b_{i1}^2} \\ C_i = \frac{b_{i1}^3}{a_{0i}b_{i1}^2+(b_{i0}-a_{i1}b_{i1})b_{i0}} \ G_i = \frac{b_{i1}^2b_{i0}}{a_{0i}b_{i1}^2+(b_{i0}-a_{i1}b_{i1})b_{i0}} \end{aligned}\right\} \tag{18}$$

Observe that the admittance of $(Y_{11}(s), Y_{22}(s))$ and $(Y_{12}(s), Y_{21}(s))$ are composed of the same terms except of their signs. Now, put the admittance having the same sign to $Y_1(s)$, and that having the opposite sign to $Y_2(s)$ as follows:

$$Y_1(s) = \sum_{i=1}^{N}\frac{2\Re\{k_{2i-1}\}s + 2\Re\{k_{2i-1}p_{2i-1}\}}{(s+u_{2i-1})^2+v_{2i-1}^2} \tag{19.1}$$

$$Y_2(s) = \frac{k_0}{s+p_0} + \sum_{i=1}^{N}\frac{2\Re\{k_{2i}\}s + 2\Re\{k_{2i}p_{2i}\}}{(s+u_{2i})^2+v_{2i}^2} \tag{19.2}$$

Then, we can realize the interconnect by the equivalent circuit as shown by Fig.1. Note that the current controlled current sources $I_r(d,s)$ in the figure shows the reflection at the far end, and $I_r(0,s)$ at the near end, respectively.

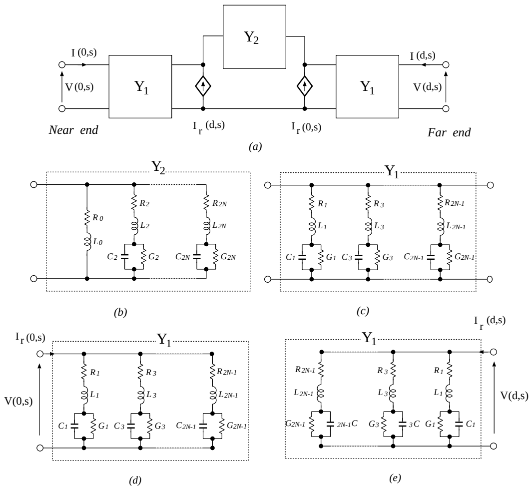


Fig.1: Equivalent circuit of an interconnect.

4. Calculations of the voltage distribution on interconnect

It is important to know the voltage distribution on the interconnect, because the potential may give large crosstalk to the other lines and transistors located close to the lines. These values are estimated by using the voltage waveforms at the near and far ends of the interconnect as follows:

$$\mathbf{V}(x,s) = \mathbf{P}_v(s)\mathrm{diag}\left(\frac{\sinh\gamma_i(s)(d-x)}{\sinh\gamma_i(s)d}\right)\mathbf{P}_v^{-1}(s)\mathbf{V}(0,s)$$

$$+\mathbf{P}_v(s)\mathrm{diag}\left(\frac{\sinh\gamma_i(s)x}{\sinh\gamma_i(s)d}\right)\mathbf{P}_v^{-1}(s)\mathbf{V}(d,s) \qquad (20)$$

In this section, we show two methods depending on the numerical Laplace transformation and the asymptotic equivalent circuit.

4.1 Numerical Laplace transformation method

The transient responses can be obtained by the use of the inverse Laplace transformations [8]. The formula for the Laplace function $V(x,s)$ is given by

$$v(t_n) = \frac{e^{at_n}}{T}\left[\Re\left\{\sum_{k=0}^{K-1} V(x, a+\frac{jk\pi}{T})e^{j2kn\frac{\pi}{K}}\right\} - \frac{1}{2}V(x,a)\right] \qquad (21)$$

for $t_n = n(2T/K), N = 0,1,2,\ldots,K-1$. The formula comes from the inverse Fourier transformation [8], so that it gives the good results in the region $0 \le t \le T$. It is known from many numerical experiments that "a" should be chosen around $aT = 3 \sim 5$ and $K = 2^m$. In this case, the waveforms $v(0,t)$ and $v(d,t)$ at the near and far ends are not given by the analytical forms, because the interconnects are usually terminated by the nonlinear sub-circuits. Therefore, we also need to apply the numerical Laplace transformation to get $V(0,s)$ and $V(d,s)$ as follows:

$$V(x, a+\frac{jk\pi}{T}) = \int_0^\infty e^{-\left(a+\frac{jk\pi}{T}\right)t} v(0,t)dt$$
$$\cong h\left(v(0,0) + v(0,T_m)e^{-\left(a+\frac{jk\pi}{T}\right)T_m}\right)$$
$$+\frac{h}{2}\sum_{k=1}^{N-1} v(0,kh)^{-\left(a+\frac{jk\pi}{T}\right)kh} \qquad (22)$$

where we applied the trapezoid numerical integration formula at $s = a + \frac{jk\pi}{T}$. The step size is chosen by $h = T_m/N$ for the interval $[0, T_m]$. To carry out the algorithm, we first calculate the transient responses with Spice given in section 3. Using these data, the transient waveform of the voltage distribution is calculated by Fortran and/or C-program.

4.2 Asymptotic equivalent circuit technique

The next one uses the asymptotic equivalent circuit satisfying the relation (20). To understand the method, we consider the case of single line. The voltage at "x" from the near end is given by

$$V(x,s) = \frac{1}{\sinh\gamma(s)d}\{V(0,s)\sinh\gamma(s)(d-x) + V(d,s)\sinh\gamma(s)x\} \qquad (23)$$

The poles $i \ge 1$ are the same as those of (13), and $p_{01} = -R/L$, $p_{02} = -G/C$ for $i = 0$.

However, the corresponding residues for p_{01} and p_{02} are zeros, because (23) is nonsingular at the poles. On the other hand, the residues corresponding to $p_i = u_i \pm jv_i$ are given by

$$k_i = V(0,s)\lim_{s\to p_i}(s-p_i)\frac{\sinh\gamma(s)(d-x)}{\sinh\gamma(s)d}$$
$$= V(0,s)\left.\frac{\sinh\gamma(s)(d-x)}{\frac{\partial\gamma(s)}{\partial s}d\cosh\gamma(s)d}\right|_{s=p_i} \qquad (24)$$

The residues for the first term of (23) is given by

$$jk_{i1} = j\frac{2i\pi V(0,s)\sin(i\pi\frac{(d-x)}{d})\times(-1)^i}{d^2\sqrt{4LC\left(RG+\left(\frac{i\pi}{d}\right)^2\right)-(LG+RC)^2}} \qquad (25.1)$$

The residues for the second term of (23) has the same form as follow:

$$jk_{i2} = j\frac{2i\pi V(d,s)\sin(i\pi\frac{(x)}{d})\times(-1)^i}{d^2\sqrt{4LC\left(RG+\left(\frac{i\pi}{d}\right)^2\right)-(LG+RC)^2}} \qquad (25.2)$$

Thus, the first term of (23) is described by the asymptotic partial fractions as follows:

$$V_1(x,s) = \sum_{i=1}^{M}\frac{V(0,s)/s}{s/K_{i1} + (-2u_i)/K_{i1} + (u_i^2+v_i^2)/sK_{i1}} \qquad (26.1)$$

where $K_{i1} = -2k_{i1}$. In the same way, the second term of (23) is given by

$$V_2(x,s) = \sum_{i=1}^{M}\frac{V(d,s)/s}{s/K_{i2} + (-2u_i)/K_{i2} + (u_i^2+v_i^2)/sK_{i2}} \qquad (26.2)$$

where $K_{i2} = -2k_{i2}$

Thus, the asymptotic equivalent circuit realizing (24) is shown by Fig.2, where the parameters are given by $L_{ik} = |1/K_{ik}|$, $R_{ik} = |-2u_i/Kik|$, $C_{ik} = |K_{ik}/(u_i^2+v_i^2)|$,$\{i = 1,\ldots,M,\ k = 1,2\}$. The voltage controlled-current sources $j(0,t) = v(0,t)$ and $j(d,t) = v(d,t)$ are obtained by the transient responses at the near and far ends in Fig.2. Since k_{1i} and k_{i2} have term $\sin(i\pi\frac{(d-x)}{d})\times(-1)^i$, the signs depend on "$i$" and "$x$". The parameters denoted by """in the figure correspond to the terms with the negative signs. Thus, the transient distributed voltage waveform can be given by the voltage difference at $1(\omega)$ resistor. Since the netlists will be automatically obtained by Spice, the method will be also useful.

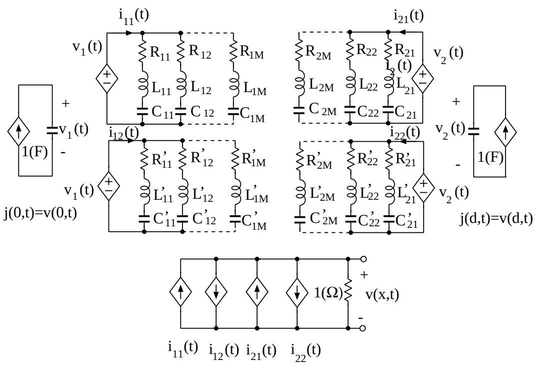
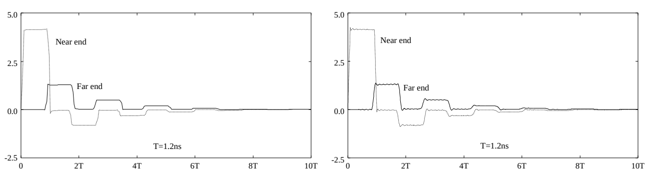


Fig.2: Asymptotic equivalent circuit for getting $v(x,t)$.

5. Illustrative examples

5.1. Interconnect terminated by linear resistors

To investigate the accuracy of our asymptotic method, consider an interconnect terminated by $10[\Omega]$ at near and far ends, respectively.



(a) Solution by the numerical inverse Laplace transformation.

(b) Solution by our asymptotic method with our 15th order.

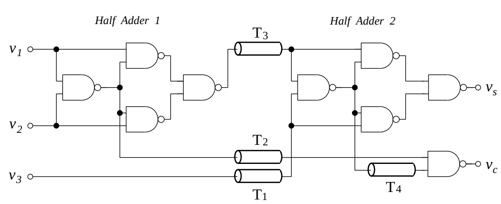
Fig. 3: Comparison our asymptotic method to the numerical inverse Laplace transformation.

$$L = 1[nH/mm], \quad R = 0.5[\Omega/mm], \quad C = 4[pF/mm],$$
$$G = 0.5[mS/mm], \quad d = 5[mm]$$

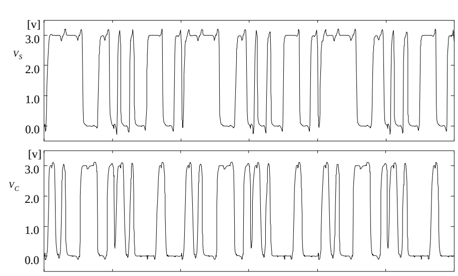
From these results, we found that our method can get the good results with 15th order of the approximation. Furthermore, the time delay is exactly estimated even with the low order model.

5.2. A LSI circuit coupled with interconnects

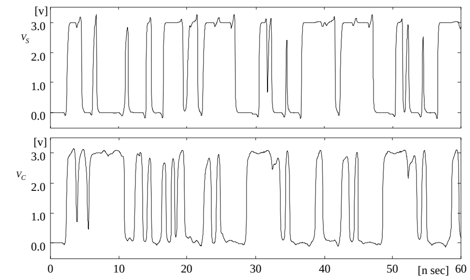
Consider a fulladder circuit coupled with interconnects.



(a) Fulladder circuit consisted of nand circuits.



(b) Transient response of the fulladder without interconnects.



(b) Transient response of the fulladder coupled with interconnects with 37th order.

Fig.4: $r = 500[\Omega/mm],\ L = 10[\mu/mm],\ C = 0.01[pF/mm],$
$G = 0.5[\mu S/mm],\ d = 5[mm]$

In this example, we replaced the interconnects by the asymptotic equivalent circuits, and get the results with transient analysis of Spice. We found from these results that it has large time delay. Our simulator is easily applied to any kind of circuits.

6. Conclusions and remarks

In this paper, we have proposed an asymptotic equivalent circuit for interconnects, and two methods for getting the response of distribution voltage on the interconnect. At first step, we calculate the exact poles and the residues of admittance matrix, and describe it in the form of partial fractions. Secondly, we realize the admittance matrix by the asymptotic equivalent circuit. We found that we can get good result even with our low order approximation. Although we have not yet shown the example for the voltage distribution in this paper, we want show some examples in the conference.

As the future problem, we need to extend our algorithm to multi-coupled interconnects.

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