2004 RISP International Workshop on Nonlinear
Circuit and Signal Processing (NCSP'04)
Hawaii, USA, Mar. 5-7, 2004.

Solving Ability of Hopfiend NN with Chaos Noise for Maximum Independent Set Problems

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1. Introduction

In a certain graph, it is very usuful to find a maximum indepandent set because we can apply the maximum independent set to various engineering problems. However, Maximum Independent Set Problems is so-called NP-difficult problems that we can't solve the problems in time polynomial in nodes. In consequense, we need to find some kind of solving method that can solve the problems effectively. Today,it has confirmed that GA contrast favorably with a heuristics method in DIMACS Bench Mark graph. In this study, we use Hopfield Neural Network (Hopfield NN) with chaos noise in Maximum Independent Set Problems and confirm the validity, because Hopfield NN with chaos noise achieved concrete results in areas like combinational optimization problems sucu as TSP and QAP. By computer simulations, we confirm that Hopfield NN with chaos noise can find some good solutions better than GA.

2. Maximum Independent Set Problems

Maximum Independent Set Problems is one of the combinational optimization problems. The Maximum Independent Set Problems are finding the largest subset of vertices of a graph such that none of these vertices are connected by an edge (i.e., all vertices are independent of each other). If we define what Maximum Independent Set Problems is in graph theory, we can say as follows.

First, universal set $G = (V, E)$ is defined as undirected graph that is formed N-nodes. Where, set V is set of nodes and E is set of Edges. In subset U of V, when there is no-Edges in between two nodes that belong to U. Set U is independent set of G. When set U is independent set and U is not contained by another independent sets in G, U is local maximum independent set. If the local maximum independent set has maximum nodes, it is maximum independent set.

In following figure (Fig 1), $U_1 = \{1, 3, 4\}$, $U_2 = \{2, 6\}$ and $U_3 = \{1, 3, 4, 6\}$ are independent sets. U_2 and U_3 are local maximum independent set and U_3 is maximum independent set.

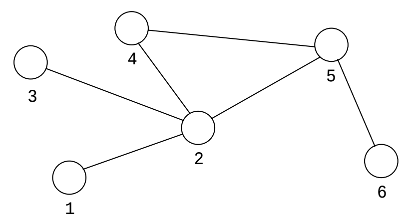

Figure 1: example graph .

3. Method of Maximum Independet Set Problems by Hopfield NN

In this method, N neurons are needed in the graph that has N nodes because we related neurons to each nodes and we made these work. And we decided limiting condition and objective functions for Hopfield NN with chaos noise. The limiting condition is Eq.(1) because fire neurons don't have Edges. Objective functions are Eq.(2) and Eq.(3) because sum of firing neurons is to the maximization and the neuron has few Edges fire. there is those equations follow. ($d_{i,j}$ is incidence matrix that is expressed if there is a Edge between i node and j node 1, if not 0.)

$$\frac{A}{2}\sum_{i,j}^{N} d_{i,j}\, x_i\, x_j \Rightarrow 0 \tag{1}$$

$$B\sum_{i}^{N}(1 - x_i)^2 \Rightarrow 0 \tag{2}$$

$$C\sum_{i}^{N} degree[i]\, x_i \Rightarrow min \tag{3}$$

By adove equations, the energy function is Eq.(4) in Hopfield NN with chaos noise.

$$E = -\frac{A}{2}\sum_{i,j}^{N} d_{i,j}\, x_i\, x_j - B\sum_{i}^{N} x_i^2 + \sum_{i}^{N}\Big(-2B + C\cdot degree[i]\Big)x_i \tag{4}$$

And the energy function in Hopfield NN is following Eq.(5).

$$E = -\frac{1}{2}\sum_{i,j}^{N} \omega_{ij}\, x_i\, x_j - \sum_{i}^{N} \theta_i\, x_i \tag{5}$$

Compare Eq.(4) and Eq.(5),coupling coefficient ω_{ij} and threshold value θ_i are discribed such as follow equation.

$$\omega_{ij} = -Ad_{i,j}(1-\delta_{ij}) - B \tag{6}$$

$$\theta_i = 2B - C\cdot degree[i] \tag{7}$$

And, internal state of neurons update asynchronously by Eq.(8). In this equation, f is a sigmoid function (Eq.(9)), and z_i is chaos noise.

$$x_i(t+1) = f\left[\sum_{i}^{N} \omega_{ij}\, x_j(t) + \theta_i + \beta z_i\right] \tag{8}$$

$$f(x) = \frac{1}{1+\exp(-x/\epsilon)} \tag{9}$$

4. Chaos Noise

In these years, it is studied that chaos is used effective. Recently, there are the study using Hopfield NN with chaos noise for combinational optimization problems as TSP and QAP. And those given good results. Reason, we use the Hopfield NN with chaos noise for Maximum Independent Set Problems. We show time series of chaos noise the following figure (Fig 2). Here we adopt the logistick map as an example of chaos noise, which is described as follows:

$$z_i(t+1) = \alpha z_i(t)(1-z_i(t)) \tag{10}$$

where α is a bifurcation parameter.

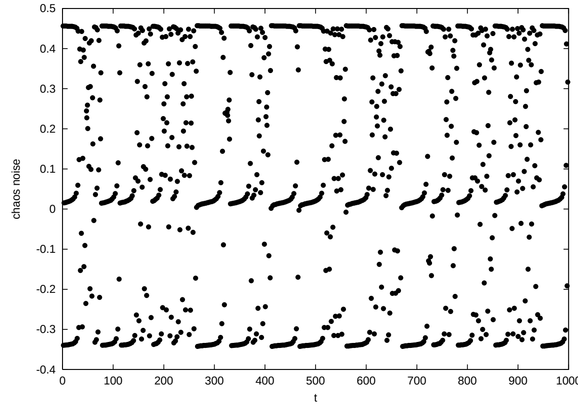

Figure 2: chaos noise.

5. Graph Normalization

In this study, we propose graph normalization to fix the parameters of Hopfield NN and we tyr to improve the performance of Hopfield NN with chaos noise. We consider that the graph characteristics must be an important factor and we normalized the graphs for fix the parameters. The more each node's Edges abounding, the factors both primary and secondary are added. Moreover, for the parameters independent the graphs, we divide the factors both primary and secondary by maximum order of the graph.

6. Analysis Result

In this study, the problems used here was chosen from the site DIMACS which collects the bench mark problems. We carried out computer simulation for brock800_1, brock800_2, brock800_3 and brock800_4. The parameters of Hopfield NN are fixed as A=2, B=1, C=1 and ε=0.81. The bifurcation parameter of chaos noise is fixed as α=3.828, and the amplitude of the injected chaos noise is fixed as β=6.12. We had confirme that these graphs are made for hard to find maximum independent set in the graph, and those maximum independent sets have a much lower rate of nodes than aggregate number of the graph's nodes. We show analysis results in Table 1. (Edges is sum of Edges in the graph, Opt. is the graph's maximum independent set, Best is best solution in 10 loops, Avg. is 10 loops average, [1] is thebibliography[1]'s solution and Our is Hopfield NN with chaos noise's solutions.) We can see that the case of the Hopfield NN with caoth noise has much better performance than the case of GA both Best and Avg.. embrace that sequence better than another methods both best and 10 loops average. In this occasion, we decided the bifurcation parameter α=3.828.

Table 1 Analysis Result for DIMACS benchmark graphs

Data	Edges	Opt.	[1]		our	
			Best	Avg.	Best	Avg.
brock800-1	207505	23	19	18.7	21	21.0
brock800-2	208166	24	20	18.8	21	21.0
brock800-3	207333	25	19	18.7	22	20.7
brock800-4	207643	26	19	18.5	21	20.7

7. Conclusions

In this study, we solved Maximum Independent Set Problems by Hopfield NN with chaos noise. And we confirmed that conclusion in brock800_1, brock800_2, brock800_3 and brock800_4 (DIMACS benchmark graph) are better. As tasks for the future, we make consideration Graph Normalization graph independent and more effective objective function.

Acknowledgement

This work was party supported by Kayamori Foundation of Informational Science Advancement's Resarch Grant.

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