2004 RISP International Workshop on Nonlinear
Circuit and Signal Processing (NCSP'04)
Hawaii, USA, Mar. 5-7, 2004.

Response of Coupled Chaotic Circuits to Various Type Signals

Tomoya Hayashi and Yoshifumi Nishio

Tokushima University
2-1 Minami-Josanjima, Tokushima, Japan
Phone:+81-88-656-7470, FAX:+81-88-656-7471
Email: {hayatan or nishio}@ee.tokushima-u.ac.jp

Abstract

In this study, we investigate synchronization phenomena of coupled chaotic circuits to input signals in time domain. Synchronization state of two chaotic signals changes depending on the input signals. By using phase difference of two chaotic signals, it can express the states of quasi-synchronization. We confirmed its behavior to parameter of input signals in time domain.

1. Introduction

Studies on engineering application of chaos, such as chaos communication systems and chaos cryptosystems, attract many researchers attentions. Because chaotic signals process infinite information, it could be possible to create novel engineering systems with great advantages. However, at this moment, there are only a few systems with some kinds of advantages over conventional engineering systems. Hence basic researches on chaotic phenomena with future engineering applications in mind are still quite important.

On the other hand, the authors have researched about many oscillatory circuits connected by a resistor [1]-[3]. Using this connection of circuits, chaotic signals generated from chaotic circuits change the state of synchronization to minimize the total of currents flowing in coupled resistor. Moreover, we confirm the phenomena of the coupled chaotic circuits when sinusoidal input is added [4]-[5]. The coupled system changes the state of synchronization against the input signal with very limited range of angular frequency. Therefore this coupled system is useful for signal processing of input signals.

In this study, we investigate synchronization phenomena of coupled chaotic circuits to input signals in time domain. Synchronization state of two chaotic signals changes depending on the input signals. By using phase difference of two chaotic signals, it can express the states of quasi-synchronization. We confirmed its behavior to parameter of input signals in time domain.

2. Circuit model

Figure 1 shows the circuit model. The circuit in Fig. 1(a) is a three-dimensional autonomous circuit generating chaos. and was proposed by Inaba and Mori [4]. In the circuit in Fig. 1(b), two Inaba's circuits are coupled by one coupling resistor R and an input signal is added to the coupling resistor as a current I_3.

The circuit equations governing the circuit in Fig. 1(b) are given as Eq. (1). We assume the $i - v$ characteristics of the diodes in the circuit by the two-segment piecewise linear function as Eq. (2).

$$\begin{aligned} L_1\frac{dI_k}{dt} &= ri_k + rI_k - v_k - R\sum_{j=1}^{3} I_j \\ L_2\frac{di_k}{dt} &= ri_k + rI_k - v_k - v_d(i_k) \\ C\frac{dv_k}{dt} &= i_k + I_k \\ &(k = 1, 2) \end{aligned} \tag{1}$$

$$v_d(i_k) = 0.5(r_d i_k + E - |r_d i_k - E|) \tag{2}$$

By using the variables and the parameters in Eq. (3), the circuit equations in Eq. (1) are normalized as Eq. (4).

$$\begin{aligned} &I_k = \sqrt{\frac{C}{L_1}}Ex_k, \quad i_k = \sqrt{\frac{C}{L_1}}Ey_k, \\ &v_k = Ez_k, \quad t = \sqrt{L_1C}\tau, \quad \alpha = \frac{L_1}{L_2}, \\ &\beta = r\sqrt{\frac{C}{L_1}}, \quad \gamma = R\sqrt{\frac{C}{L_1}}, \quad \delta = r_d\sqrt{\frac{C}{L_1}} \end{aligned} \tag{3}$$

$$\begin{aligned} \frac{dx_k}{d\tau} &= \beta(x_k + y_k) - z_k - \gamma\sum_{j=1}^{3} x_j \\ \frac{dy_k}{d\tau} &= \alpha\left\{\beta(x_k + y_k) - z_k - f_d(y_k)\right\} \\ \frac{dz_k}{d\tau} &= x_k + y_k \\ &(k = 1, 2) \end{aligned} \tag{4}$$

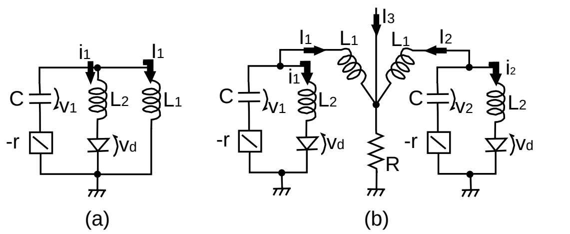

Figure 1: Chaotic circuit and the coupled chaotic circuits. $\alpha = 7.0$, $\beta = 0.14$ and $\delta = 100$.

3. Input signals

It is possible to consider various kinds of signals as input signal x_3 in future, for example, simple signals represented by basic mathematical functions, chaotic signals obtained from other chaotic circuits, audio/visual signals in real engineering systems and so on.

In this study, as the first step toward the total understanding of the response of the coupled circuits to various input signals, we consider the case that a simple sinusoidal, rectangular and chaotic signal is input.

Namely, we use the following sinusoidal functions with the normalized amplitude A_m and the normalized angular frequency ω as the input x_3.

$$x_3(\tau) = A_m \sin \omega\tau \tag{5}$$

Function of rectangular input signal is defined as follows. It is defined with Eq. 5, using the parameter angular frequency ω and A_m.

$$x_3 = \begin{cases} -A_m & \sin(\omega\tau) < 0 \\ A_m & \sin(\omega\tau) > 0 \end{cases} \tag{6}$$

And for chaotic input signals, we use the chaotic input signal which is generated from same chaotic signal used coupled chaotic circuits. The differential equations of chaotic circuits generating chaotic input signal are shown as Eq. (7). We wrote it subscript in Eq. (7) for convenience.

ε is the parameter for the frequency of oscillation of chaotic circuits. A_{mc} controls amplitude of input chaotic signal. Chaotic signal x_4 is amplified by A_{mc} and inputted to coupled chaotic circuits as chaotic input signal x_3.

$$\begin{aligned} \frac{dx_4}{d\tau} &= \varepsilon_4\beta(x_4 + y_4) - \varepsilon_4^2 z_4 - \varepsilon_4\gamma x_4 \\ \frac{dy_4}{d\tau} &= \alpha\left\{\varepsilon_4\beta(x_4 + y_4) - \varepsilon_4 z_4 - f_d(y_4)\right\} \\ \frac{dz_4}{d\tau} &= x_4 + y_4 \end{aligned} \tag{7}$$

$$x_4(\tau) = A_{mc} \times x_3(\tau) \tag{8}$$

4. Quasi-synchronization of chaotic signals

In this study, we use the quasi-synchronization state of two chaotic signals. In this state, a chaotic signal $S_i(t)$ from a sub circuit of coupled system and the other chaotic signal from the other sub circuit $S_j(t)$ have the relation such as $|S_i(t) \pm S_j(t - T)| < \varepsilon$ where a constant T represents phase shift between two signals and a constant ε is much smaller than the average amplitude of each chaotic signal. This is not mathematical definition and is used only for qualitative explanation of the observed phenomena.

In previous research[4], we have investigated the synchronization of coupled chaotic circuits to input signal which time passed enough. In order to investigate the synchronization, we define the phase difference of the two chaotic signals. By using the time when the chaotic signals take extrema as shown in Fig. 2, the phase difference θ[deg] is defined as follows.

$$\theta = \frac{\tau_{20} - \tau_{10}}{\tau_{11} - \tau_{10}} \times 360. \tag{9}$$

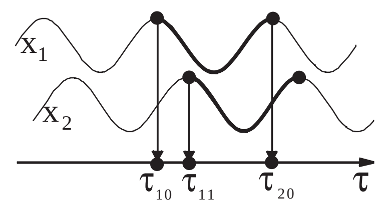

Figure 2: Definition of the phase difference θ.

5. Response to input signal in time domain

In this section, response of coupled chaotic circuits is investigated, to various type signals in time domain. Calculated phase difference θ which shows the synchronization state of chaotic signals in coupled chaotic circuits, are shown in Fig. 3- 10.

θ is calculated on the basis of one chaotic signal of two chaotic signals. From this reason, with the signal made in to a standard, phase difference has two displays in one state. Then in order to display simply, we make the state where there is no input signal as a standard, θ is shown by $180[\mathrm{deg}]$ using $|\theta - 180|$. And in every results in Fig. 3- 10, input of signal is begin after 50,000 iterations(τ).

In there results, phase difference θ of two chaotic signals varied depending on the input signals. Obviously, θ changes only to input signal with limited frequency. (Fig. 3, 4, 6, 8, 10) Then value of θ is denepnding on the amplitude of input signals. (Fig. 5, 7, 9) And between two region, without a response of coupled chaotic circuits (θ is round $0[\mathrm{deg}]$) and with a response (θ is around $120[\mathrm{deg}]$), θ is changes according to the value of parameter which a input signal have. θ varies not suddenly but gradually and changes like a sinusoidal signal.

6. Conclusions

We investigated that the quasi-synchronization phenomena in coupled chaotic circuits to various type signals. By using phase difference θ of chaotic signals, states of quasi-synchronization of chaotic signals are shown in time domain to varied parameters of input signals. It is confirmed that phase difference θ varied like a sinusoidal signal between region with coupled chaotic circuits response to input signal and not response.

References

[1] Yoshifumi Nishio and Akio Ushida, "Spatio-Temporal Chaos in Simple Coupled Chaotic Circuits," IEEE Transactions on Circuits and Systems I, vol. 42, no. 10, pp. 678-686, Oct. 1995 .

[2] Yoshifumi Nishio, Martin Hasler and Akio Ushida, "Markov Chain Modeling of Chaotic Wandering in Simple Coupled Chaotic Circuits," Proc. of ECCTD'01, vol. 2, pp. 105-108, Aug. 2001.

[3] Yoshifumi Nishio and Akio Ushida, "Chaotic Wandering and Its Analysys in Simple Coupled Chaotic Circuits," IEICE Transactions on Fundamentals, vol. E85-A no. 1, pp. 248-255, Jan. 2002.

[4] Tomoya Hayashi, Yoshifumi Nishio, Martin Hasler and Akio Ushida, "Response of Coupled Chaotic Circuits to Sinusoidal Input Signal," Proc. of ISCAS'02, vol. 3, pp. 461-464, May 2002.

[5] Tomoya Hayashi, Yoshifumi Nishio, Martin Hasler and Akio Ushida, "Information Transmission Using Signal Processing by Coupled Chaotic Circuits," Proc. of NOLTA'02, vol. 2, pp. 821-824, Oct. 2002.

[6] Naohiko Inaba and Shinsaku Mori, "Chaotic Phenomena in Circuits with a Linear Negative Resistance and an Ideal Diode," Proc. of MWSCAS'88, pp. 211-214, Aug. 1988.

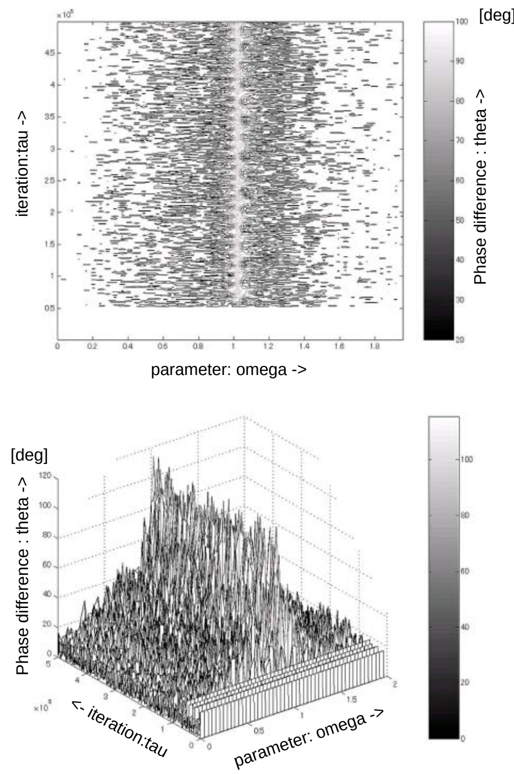

Figure 3: Sinusoidal signal is input: Phase difference θ to τ(iteration) and ω. $\alpha = 7.0$, $\beta = 0.14$, $\gamma = 0.03$, $\delta = 100$, $A_m = 1.17$.

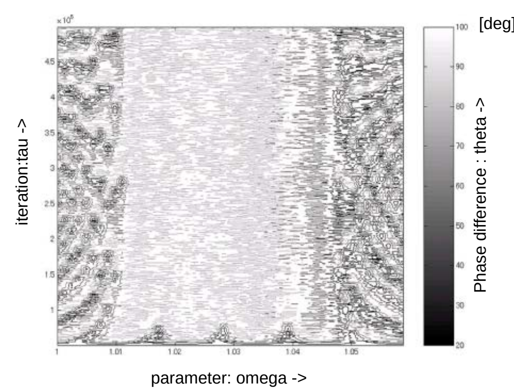

Figure 4: Sinusoidal signal is input: θ to τ(iteration) and ω. $\alpha = 7.0$, $\beta = 0.14$, $\gamma = 0.03$, $\delta = 100$, $A_m = 1.17$.

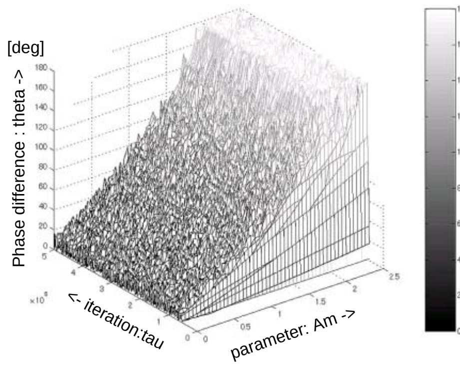

Figure 5: Sinusoidal signal is input: θ to τ(iteration) and A_m. $\alpha = 7.0$, $\beta = 0.14$, $\gamma = 0.03$, $\delta = 100$, $\omega = 1.037$.

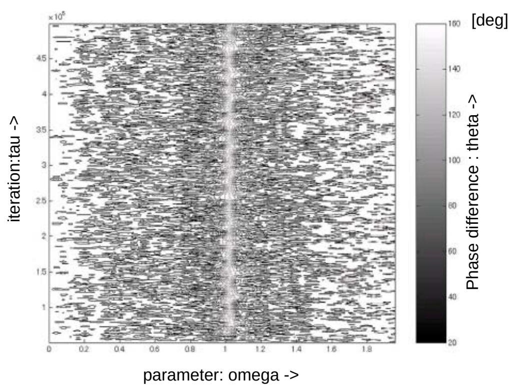

Figure 6: Rectangular signal is input: θ to τ(iteration) and ω. $\alpha = 7.0$, $\beta = 0.14$, $\gamma = 0.03$, $\delta = 100$, $A_m = 1.17$.

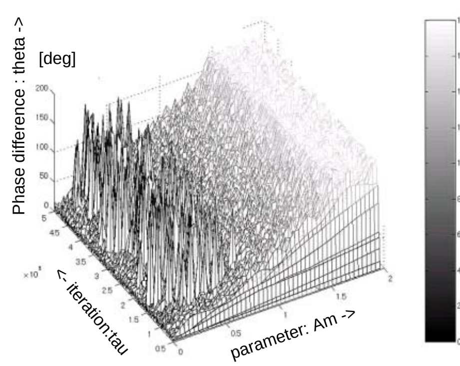

Figure 7: Rectangular signal is input: θ to τ(iteration) and A_m. $\alpha = 7.0$, $\beta = 0.14$, $\gamma = 0.03$, $\delta = 100$, $\omega = 1.037$.

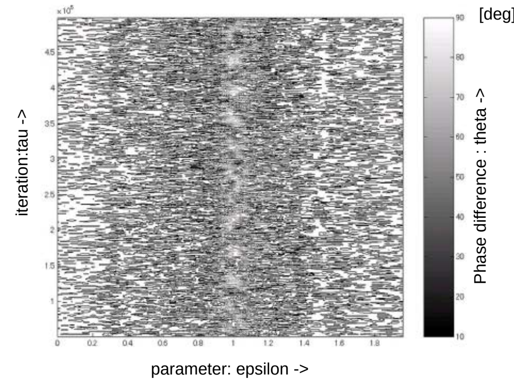

Figure 8: Chaotic signal is input : θ to τ(iteration) and ε. $\alpha = 7.0$, $\beta = 0.14$, $\gamma = 0.03$, $\delta = 100$, $A_{mc} = 1.0$, β(input signal)$= 1.0$.

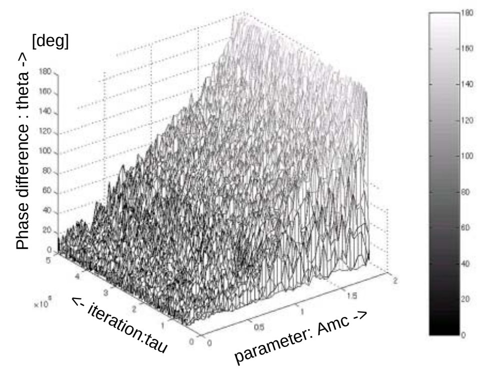

Figure 9: Chaotic signal is input : θ to τ(iteration) and A_{mc}. $\alpha = 7.0$, $\beta = 0.14$, $\gamma = 0.03$, $\delta = 100$, $\varepsilon = 1.037$, β(input signal)$= 1.0$.

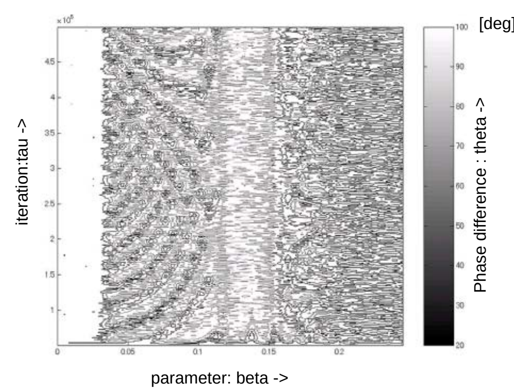

Figure 10: Chaotic signal is input : θ to τ(iteration) and β. $\alpha = 7.0$, $\beta = 0.14$, $\gamma = 0.03$, $\delta = 100$, $A_{mc} = 1.0$, $\varepsilon = 1.0$.