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Research on a Positioning Using Chaotic Sequence

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1. Introduction

Chaos is nonperiodic and could generate sequences with infinite length theoretically [1][2]. Although the sequences look like a noise, the values can be predicted exactly if the dynamics and the initial value are provided. On the other hand, positioning is an important subject to pinpoint one's position. In positioning, though the GPS is famous, another system using the m-sequence for ITS (Intelligent Transport Systems) has been also proposed [3].

In this study, a possibility of the chaotic sequence for a kind of positioning is investigated. At first, a chaotic sequence generated by the skew tent map with a given number is stored. When a few number of successive values are obtained from the storage, the position of the head of the successive values is estimated. The least squares method is used for the judgment of the position. As varying the length of the sequence and the signal-to-noise ratio of the data-reading channel, the error rate is evaluated by computer simulations.

2. Chaotic sequence

In this study, the skew tent map is used to generate chaotic sequences. The skew tent map is known as the simplest one-dimensional map generating chaos. The map is shown in Fig. 1. If the top coordinate is given as $(a, 0.5)$, the equation describing the skew tent map is as

$$x_{n+1} = \frac{x_n}{0.5+a} + \left(0.5 - \frac{a}{0.5+a}\right) \qquad (1)$$

$$(-0.5 \leq x_n \leq a)$$

$$x_{n+1} = \left(0.5 + \frac{a}{0.5-a}\right) - \frac{x_n}{0.5-a} \qquad (2)$$

$$(a < x_n \leq 0.5).$$

Because of the sensitive dependence on initial values and parameters, we could generate a number of different chaotic sequences by changing the initial value x_0 or the parameter a.

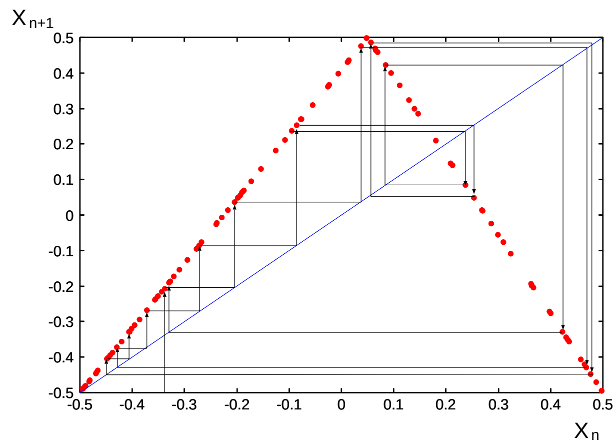

Figure 1: Skew tent map.

3. System model

A chaotic sequence x_0, x_1, x_2, $\cdots$, x_N (N : Chaotic sequence length) is generated by giving an initial value x_0 to the skew tent map with a fixed value of the parameter of a. These values are memorized in a storage keeping the order. We consider to estimate the position of the head of a few number of successive values when the successive values are obtained from the storage. In the case of no noise, only one value is enough to decide the position by virtue of the nonperiodic feature of chaotic sequences. However, under noisy environment, some number of successive values are necessary to decide the position. We consider the case that M successive values (M : Data length) are extracted from the storage. When an additive white Gaussian noise is added to the values, the error rate to decide the position is evaluated. In this study, the least squares method is used for the judgment of the position.

4. Simulation result

In the following computer simulations, the parameter of the skew tent map is fixed as $a = 0.05$. The computer simulations are carried out as

(1) fixing the chaotic sequence length and changing the signal-to-noise ratio,

(2) changing the chaotic sequence length and fixing the data length or the signal-to-noise ratio,

(3) setting a threshold.

4.1. Fixing chaotic sequence length

At first, we carried out computer simulations as fixing the chaotic sequence length as $N = 100$ and changing the signal-to-noise ratio for the data lengths $M = 3, 4, 5, 8$.

The graph of the error rate is shown in Fig. 2. The data in the graph is the average of 100,000 results for different initial values.

We can say that increasing the data length makes the error rate small.

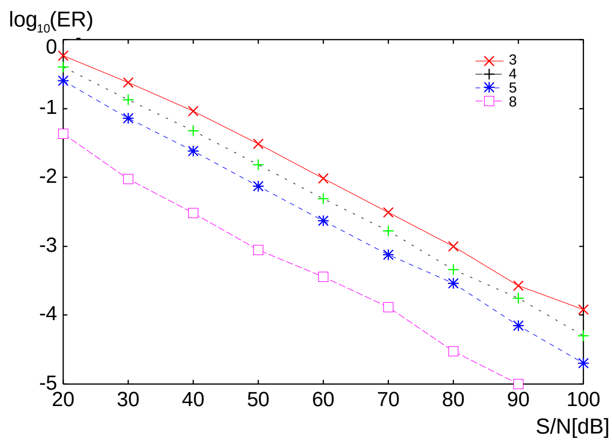

Figure 2: Error rate for the fixed chaotic sequence length ($N = 100$).

4.2. Changing chaotic sequence length

Secondly, we carried out computer simulations as changing the chaotic sequence length from $N = 100$ to $N = 100000$. In the simulations, 100,000 trials are carried out as fixing the initial value of the chaotic sequence as $x_0 = 0.111111$. Figure 3 shows the error rate when the data length is fixed as $M = 3$ and the signal-to-noise ratio is varied as S/N=70dB, 80dB, 90dB, and 100dB. While Fig. 4 shows the error rate when the signal-to-noise ratio is fixed as S/N =100dB and the data length is varied as $M = 3, 4, 5$, and 8.

4.3. Setting threshold

In this study, the read-out data has to be compared with all of the memorized data in the storage in order to decide the position of the read-out data. Thus, if the data length becomes

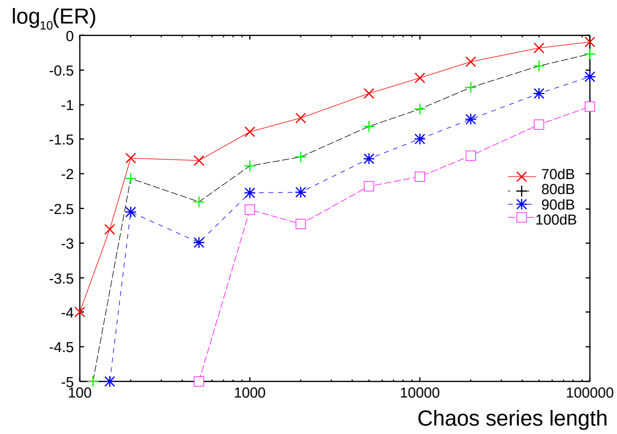

Figure 3: Error rate for the fixed data length ($M = 3$).

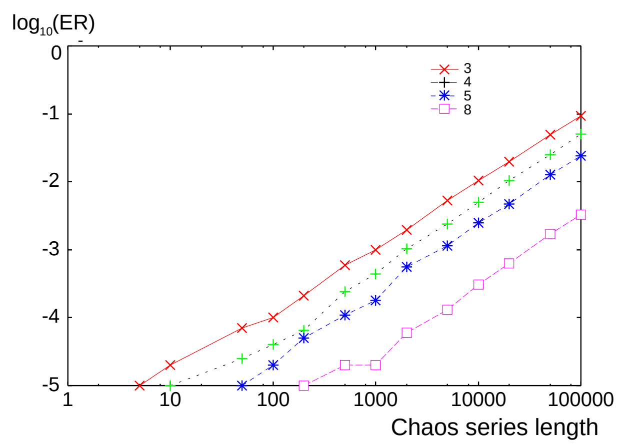

Figure 4: Error rate for the fixed signal-to-noise ratio (S/N=100dB).

large, the computational effort will increase. Hence, we consider to set a threshold in order to reduce that. Namely, if the read-out data is enough to be close to the data extracted from the storage, further extraction is given up and the position is decided as a right one.

In order to estimate an appropriate value of the threshold, the calculated values by the least squares method are shown in Figs. 5 and 6 for the cases of $M = 3$ and 8, respectively. In the figures, black circles show the minimums and grey (red) squares show the second smallest values. Other conditions are set as $N = 1000$ and S/N =100dB.

From the figures, we can say that the position is correctly decided if the threshold is set within $[10^{-9},\ 10^{-10}]$ for $M = 3$. Further, for the case of $M = 8$, the threshold around 10^{-7} would decide a correct position with a high probability.

Figure 7 shows the error rate when the threshold is given to the system. The chaotic sequence length is fixed as $N = 1000$ and the signal-to-noise ratio is fixed as S/N =100dB. The data length is changed as $M = 3, 5$, and 8.

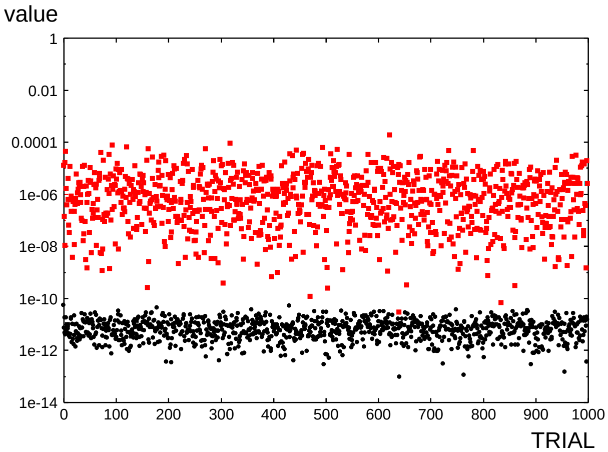

Figure 5: The minimum and the second smallest value by the least squares method ($M = 3$).

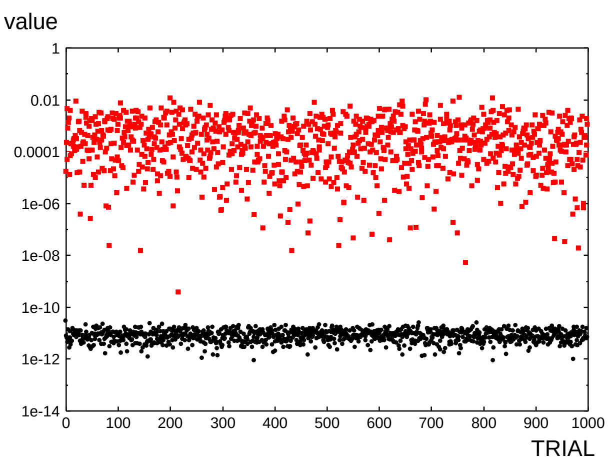

Figure 6: The minimum and the second smallest value by the least squares method ($M = 8$).

At last, we show the graph of the stop position of comparing for the decision in Fig. 8. For the threshold value more than 10^{-10}, the computation time becomes less than the half of the case without the threshold.

5. Conclusions

In this study, a positioning system using chaotic sequence has been proposed and the basic property of the system has been investigated by computer simulations.

Acknowledgement

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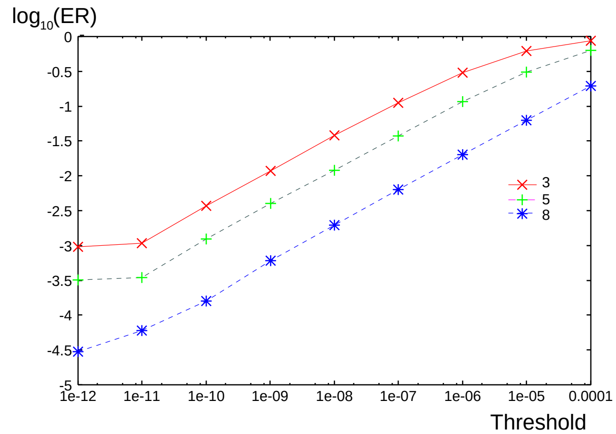

Figure 7: Error rate with the threshold.

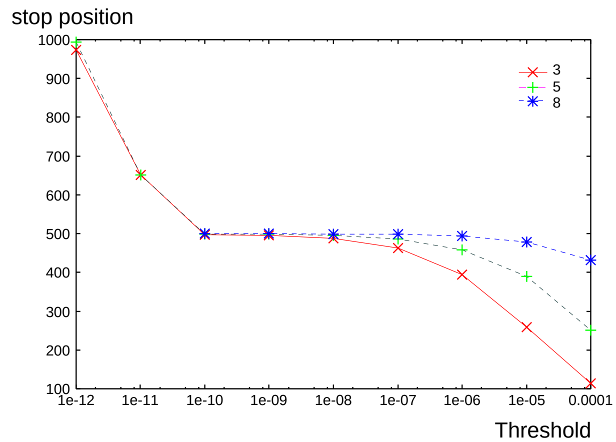

Figure 8: Stop position with the threshold.

Reference

[1] E. Atlee Jackson, *Perspectives of Nonlinear Dynamics 1*, Cambridge University Press, 1991.

[2] Katsufusa Syono, *Chaos Engineering*, Springer-Verlag Tokyo, 2002.

[3] Takaaki Hasegawa, "A Vehicular Driving Assistant System Using Spread Spectrum Techniques," IEICE Trans. Fundamentals, vol. E81-A, no. 11, pp. 2334-2340, 1988.