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Search of Many Good Solutions of QAP by Connected Hopfield NNs with Chaos Noise

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1. Introduction

Combinatorial optimization problems have an ability to describe various actual problems mathematically. However, there is a large disparity between to describe problems mathematically and to solve them. In order to solve a very difficult problem, it takes a long time (e.g. more than the cosmos age) by current computer system. That makes no sense actually. The method using the Hopfield Neural Network (NN) [1] has been proposed for combinatorial optimization problems as an approximate mean based on search. In this method, if we choose connection weights between neurons appropriately according to given problems, we can obtain a good solution by the energy minimization principle. However, the solutions are often trapped into a local minimum and do not reach the global minimum. In order to avoid this critical problem, several people proposed the method adding some kinds of noise to the Hopfield NN. Many researchers suggested that intermittency chaos near the three-periodic window of the logistic map gains the best performance [2]. In order to make clear the reason why intermittency chaos is better than fully developed chaos, we have investigated the performance with the burst noise generated by the Gilbert model for traveling salesman problems [3, 4] and quadratic assignment problems(QAP) said to be one of most difficult to solve in the combinatorial optimization problems [5]. In those results, we have confirmed that the Hopfield NN with noise can find a lot of various solutions. It is very important to find a lot of various solutions when the networks do not know the optimal solution. However, the Hopfield NN with noise has some problems: "The solution of the network keeps staying at the same state during a certain period." "The solution of the network comes into the states found before a number of times." By avoiding these problems, the network could find a lot of nearly optimal solutions.

In this study, we connect some Hopfield NNs with chaos noise like hierarchical networks in order to find a lot of nearly optimal solutions. The weights of neurons are configured as the same and each network operates by itself. We propose a method connecting some Hopfield NNs with chaos noise and evaluate this method by solving QAP. In the simulated results, we confirm that the connected Hopfield NNs with chaos noise can search a broad range of energy function and find many nearly optimal solutions.

2. Solving QAP with Hopfield NN

Various methods are proposed for solving QAP which is one of the NP-hard combinatorial optimization problems. QAP is expressed as follow: given two matrices, distance matrix $\mathbf{C}$ and flow matrix $\mathbf{D}$, and find the permutation $\mathbf{p}$ which corresponds to the minimum value of the objective function $f(\mathbf{p})$ in Eq. (1).

$$f(\mathbf{p}) = \sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij} D_{p(i)p(j)}, \tag{1}$$

where C_{ij} and D_{ij} are the (i, j)-th elements of $\mathbf{C}$ and $\mathbf{D}$, respectively, $p(i)$ is the i-th element of the vector $\mathbf{p}$, and N is the size of the problem. There are many real applications which are formulated by Eq. (1). One example of QAP is to find an arrangement of factories to make a cost the minimum. The cost is given by the distance between the cities and the flow of the products between the factories.

Other examples are the placement of logical modules in an IC chip, the distribution of medical services in a large hospital, and so on.

Because the QAP is very difficult, it is almost impossible to solve the optimum solutions in larger problems. The largest problem whose optimal solution can be obtained may be only 36 in recent study [6]. Further, computation time is very long to obtain the exact optimum solutions. Therefore, it is usual to develop heuristic methods which search nearly optimal solutions in reasonable time.

For solving an N-element QAP by the Hopfield NN, $N \times N$ neurons are required and the following energy function is defined:

$$E = \sum_{i,m=1}^{N} \sum_{j,n=1}^{N} w_{im;jn} x_{jn} + \sum_{i,m=1}^{N} \theta_{im} x_{im}. \tag{2}$$

The neurons are coupled each other with the synaptic connection weight. Suppose that the weight between (i, m)-th neuron and (j, n)-th neuron and the threshold of the (i, m)-th neuron are described by:

$$\begin{aligned} w_{im;jn} &= -2\Big\{A(1-\delta_{mn})\delta_{ij} \\ &\quad +B\delta_{mn}(1-\delta_{ij}) + \frac{C_{ij}D_{mn}}{q}\Big\} \\ \theta_{im} &= A + B \end{aligned} \tag{3}$$

where A and B are positive constants, q is a normalization parameter to correspond given problems, and δ_{ij} is the Kronecker's delta. The states of $N \times N$ neurons are asynchronously updated due to the following difference equation:

$$x_{im}(t+1) = g\left(\sum_{j,n=1}^{N} w_{im;jn} x_{jn}(t) - \theta_{im} + \beta z_{im}(t)\right) \tag{4}$$

where g is a sigmoidal function defined as follows:

$$g(x) = \frac{1}{1 + \exp\left(-\frac{x}{\varepsilon}\right)} \tag{5}$$

z_{im} is an additional noise, and β limits the amplitude of the noise.

Also, we use the method suggested by Sato *et al.* (1.1 in [7]) to decide firing of neurons.

3. Chaos Noises

In this section, we describe chaos noise injected to the Hopfield NN. The logistic map is used to generate chaos noise:

$$\hat{z}_{im}(t+1) = \alpha \hat{z}_{im}(t)(1 - \hat{z}_{im}(t)). \tag{6}$$

Varying parameter α, Eq. (6) behaves chaotically via a period-doubling cascade. When we inject chaos noise to the Hopfield NN, we normalize $\hat{z}_{im}$ by Eq. (7).

$$z_{im}(t+1) = \frac{\hat{z}_{im}(t) - \bar{z}}{\sigma_z} \tag{7}$$

Where $\bar{z}$ is the average of $\hat{z}(t)$, and σ_z is the standard deviation of $\hat{z}(t)$. Figure 1 shows an example of the time series of the chaos noise.

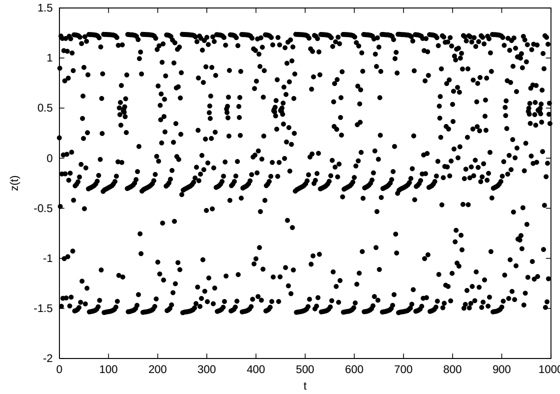

Figure 1: Chaos noise. α=3.8274.

4. Connected Hopfield NNs

In this study, we connect some Hopfield NNs with chaos noise like hierarchical networks in order to find a lot of nearly optimal solutions. We consider that the connected Hopfield NNs with chaos noise can search a broad range of energy function.

The connected Hopfield NNs with chaos noise is shown in Fig. 2. In this figure, ● show the firing neuron, $\otimes$ show the connection neuron. The weights of neurons are configured as the same and each network operates by itself. We consider that reflecting the firing pattern of one network to the firing pattern of the next network by connecting with the neurons between the two networks is important. The K-th network selects one neuron from the firing neurons of the (K-1)-th network at random, and we connect the selected neuron with the neuron at the same position as the selected neuron in the K-th network. The output of the connected neuron in the K-th network is set to zero.

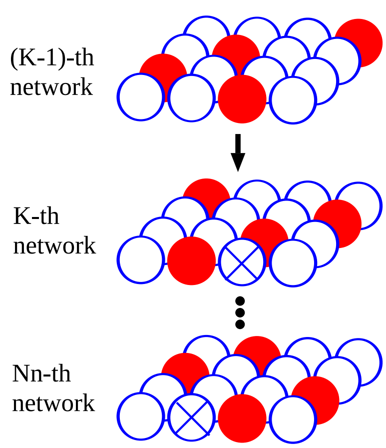

Figure 2: Connected Hopfield NNs.

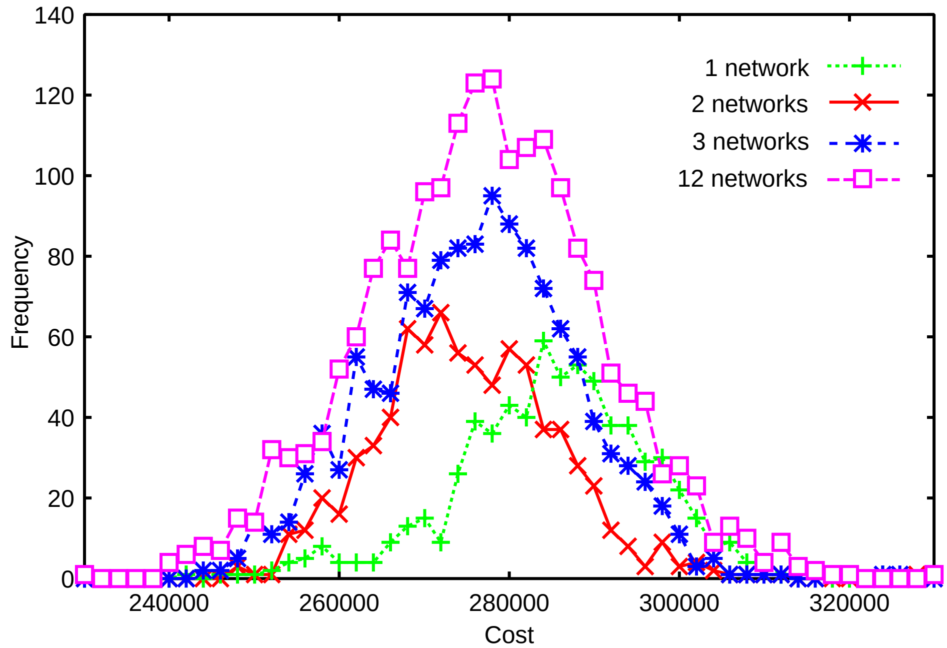

Figure 3: Frequency distribution of solutions.

5. Simulated Results

The problem used here was chosen from the site QAPLIB which collects the bench mark problems. We carried out computer simulation for "Tai12a" using connected 2∼20 Hopfield NNs with chaos noise. The global minimum of this problem is known as 224416. The parameters of the Hopfield NN are fixed as $A = 0.94$, $B = 0.94$, $q = 30000$ and $\varepsilon = 0.02$ and the amplitude of the injected chaos noise is fixed as $\beta = 0.5$. The total number of updating the network is 12000. For example, the case of connecting 4 networks, the number of updating the per 1 network is 3000, and the total number of updating the 4 networks is 12000.

Next, we explain how to accept solutions. The connected Hopfield NNs with the chaos noise searches various solutions. However, the state of the Hopfield NN sometimes stays around a group of several solutions. We consider that such a behavior is not useful to find the optimal or nearly optimal solutions. So, we take the only-different-solutions method. Namely, we take into account the solutions which have not found ever.

5.1. Frequency distribution

The results of the frequency distribution when the some Hopfield NNs with chaos noise are connected is shown in Fig. 3. For comparison, the result of the only 1-Hopfield NN with chaos noise is shown in this figure. The horizontal axis is cost calculated by Eq. (1) and vertical axis is frequency. The frequency means the number of the accepted solutions with the corresponding costs found during 12000 iterations. We can see that a lot of the nearly optimal solutions are found for the case of connected some Hopfield NNs with chaos noise. When the number of connection Hopfield NNs with chaos noise is large, the networks can find many good solutions. On the other hand, only a small number of the nearly optimal solutions are found for the only 1-Hopfield NN with chaos noise.

5.2. $Depth\_1$

Until now, we have proposed two methods [8] to appreciate finding a lot of nearly optimal solutions.

The first evaluation method $Depth\_1$ is defined as

$$Depth\_1 = \sum_{k=0}^{n} \{f(\mathbf{p}_k) - D_\infty\}^2 \tag{8}$$

where D_∞ is a constant which is large enough to include the costs of all solutions, n is the number of the accepted solutions and the cost $f(\mathbf{p}_k)$ is calculated by Eq. (1) using the permutation $\mathbf{p}_k$ corresponding to the k-th accepted solution.

The calculated result of $Depth\_1$ is shown in Fig. 4. The horizontal axis is number of connected networks and vertical axis is $Depth\_1$. In this figure, $Depth\_1$ becomes larger as the number of the connected networks increases. We consider that the connected Hopfield NNs with chaos noise can obtain good performance to find a lot of solutions.

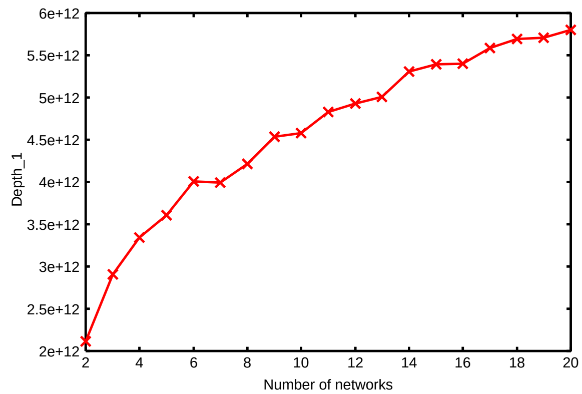

Figure 4: $Depth\_1$ for Tai12a ($D_\infty = 340000$).

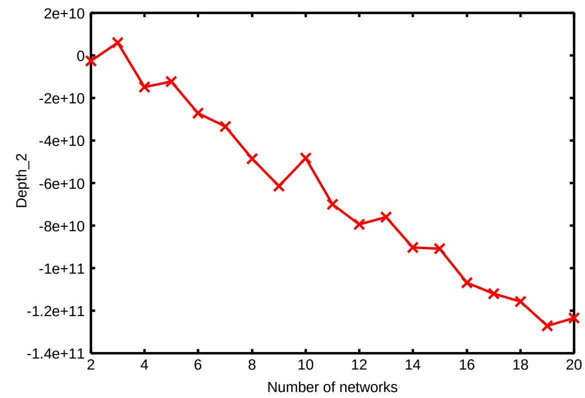

Figure 5: Depth_2 for Tai12a ($D_{th} = 285000$).

5.3. Depth_2

The second evaluation method $Depth\_2$ is defined as

$$Depth\_2 = \sum_{k \in \mathbf{k}_g}^{n} \{f(\mathbf{p}_k) - D_{th}\}^2 - \sum_{k \notin \mathbf{k}_g}^{n} \{f(\mathbf{p}_k) - D_{th}\}^2 \quad (9)$$

$$\text{where} \quad \mathbf{k}_g = \{k \mid f(\mathbf{p}_k) \leq D_{th}\}.$$

This evaluation has an advantage such that we can set the threshold D_{th} according to the requirement. We consider that finding a lot of bad solutions makes the performance of the network worse. However, the value of $Depth\_1$ increases even if the obtained solution is very bad. Hence, in this evaluation, we not only set up a threshold but give a penalty according to the cost. Namely, if the network finds a solution with the cost more than a given threshold value, the value of $Depth\_2$ is reduced.

The calculated result of $Depth\_2$ is shown in Fig. 5. The horizontal axis is number of connected networks and vertical axis is $Depth\_2$. In this figure, $Depth\_2$ becomes smaller as the number of the connected networks increases. This result shows that the networks find a lot of nearly optimal solutions as well as far optimal solutions. From this result, we can see that there is an appropriate number of connecting networks to find only good solutions.

6. Conclusions

In this study, we connected some Hopfield NNs with chaos noise like hierarchical networks in order to find a lot of nearly optimal solutions. The weights of neurons are configured as the same and each network operates by itself. We proposed method connecting some Hopfield NNs with chaos noise and evaluated this method by simulating QAP. In the simulated results, the connection method got good performance to find a lot of nearly optimal solutions of QAP. We confirmed that the connected Hopfield NNs with chaos noise can search a broad range of energy function and find many nearly optimal solutions.

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