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Markov Chain Modeling of Intermittency Chaos and its Application to Hopfield NN for QAP

Yoko Uwate * Yoshifumi Nishio * Akio Ushida *

Abstract — In this study, a modeling method of the intermittency chaos using the Markov chain is proposed. The performances of the intermittency chaos and the Markov chain model are investigated when they are injected to the Hopfield Neural Network for a quadratic assignment problem. Computer simulated results show that the proposed modeling is good enough to gain similar performance of the intermittency chaos.

1 INTRODUCTION

Intermittency chaos [1] is deeply related to *the edge of chaos* [2] and many people suggest that such a behavior between order and chaos gains better performance for various kinds of information processing than fully developed chaos. One good example of this is an application of chaos to the Hopfield Neural Networks (Hopfield NN) [3] solving combinatorial optimization problems to avoid trappings of the solutions into a local minimum. Hayakawa *et al.* pointed out the chaos near the three-periodic window of the logistic map gains the best performance for solving traveling salesman problems (TSP) [4]. However, the reason why the intermittency chaos exhibits such a good performance has not been clarified. Therefore, it is very important to make simpler models of the good characteristics of the intermittency chaos and to investigate their detailed properties.

In this study, we propose a modeling method of the intermittency chaos obtained from the logistic map by using the Markov chain. Various people have already proposed the Markov chain modelings of chaotic systems [5]-[7]. The modelings have successfully applied to the chaos-based spread spectrum communication systems for the purposes of the noise cleaning of chaotic sequences [5] and the analytical estimation of the performance [6]. Further, the modeling has been extended to generate more complex nonlinear phenomena such as self-similarity [7]. These modelings are effective in the sense that the models could generate almost all the phenomena observed from the original chaotic system. However, it is not appropriate to reveal the reasons of the good performance of the intermittency chaos. Therefore, in this study, we pay our attentions only on the distribution of the lengths of the laminar parts and the burst parts, which seems to be the most distinguished feature of the intermittency chaos. The proposed modeling using the Markov chain is completely different from those in the references on the point that each state in the Markov chain is not the quantized value (or the interval) of the variable but the behavior of the successive orbits. As a result, the model becomes very simple and enhances the feature of the intermittency chaos. In order to confirm that the proposed model has the good property of the intermittency chaos, we investigate the performances when the intermittency chaos and the Markov chain model are injected to the Hopfield NN for quadratic assignment problems (QAP), which is said to be much more difficult to be solved than TSP. Computer simulated results show that the proposed modeling is good enough to gain similar performance of the intermittency chaos.

*Department of Electrical Engineering, Tokushima University, 2-1 Minami-Josanjima, Tokushima 770-8506, Japan. Email: {uwate,nishio,ushida}@ee.tokushima-u.ac.jp, Tel: +81-88-656-7470, Fax: +81-88-656-7471.

2 INTERMITTENCY CHAOS

We consider the logistic map to generate chaotic time series;

$$\hat{z}(t+1) = \alpha \hat{z}(t)(1 - \hat{z}(t)). \tag{1}$$

Varying parameter α, Eq. (1) behaves chaotically via a period-doubling cascade. Further, it is well known that the map produces intermittent bursts just before periodic-windows appear. Figure 1 shows an example of the intermittency chaos near the three-periodic window. As we can see from the figure, the chaotic time series could be divided into two phases; laminar part of periodic behavior with period 3 and burst part of spread points over the invariant interval. As increasing α, the ratio of the laminar parts becomes larger and finally the three-periodic window appears.

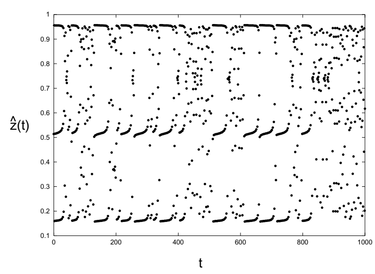

Figure 1: Intermittency chaos for α=3.827940.

3 MARKOV CHAIN MODELING

In this section, we model the intermittency chaos by using the Markov chain.

At first, we distinguish the laminar part and the burst part of the intermittency chaos. Because we treat only the intermittency chaos near the three-periodic window, we regard three successive sequences starting from a point whose value is 0.9444 or more as one-period of the laminar part. Other points are regarded as the burst part.

In order to make the Markov chain model precisely, we counted the length of the laminar parts. The frequency of each period of the laminar part during 100,000 iterations of the logistic map is shown in Fig. 2. We can see that the graph does not obey any simple scaling rules. Namely, the length of the laminar part is bounded and the maximum value of the length takes a peak. Though we omit the explanation of this reason, this property can be easily derived from the mechanism of the intermittency chaos. We consider that this is the most distinguished feature of the intermittency chaos.

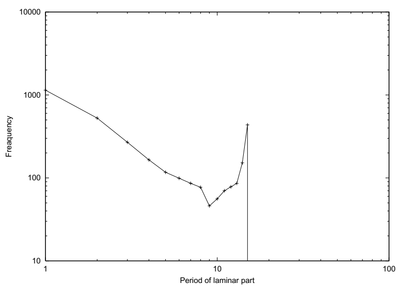

Figure 2: Distribution of period of laminar part. (Intermittency chaos for α=3.827940.)

In order to model the above-mentioned feature of the intermittency chaos, we propose the Markov chain as shown in Fig. 3 where $P(S_k|S_l)$ means the transition probability from the state S_l to the state S_k, and

$$P(S_{k+1}|S_k)+P(S_0|S_k)=1 \qquad (k=0\sim L-1) \quad (2)$$

must be satisfied. In this Markov chain, the state S_0 corresponds to the burst part. The states $S_1 \sim S_L$ correspond to the laminar parts {0.956,0.160,0.514} and the subscript k of S_k indicates the length of the continuing laminar part at that time.

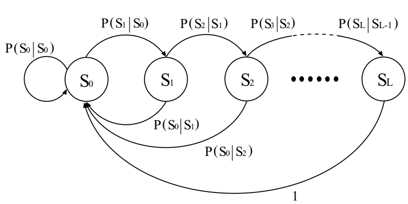

Figure 3: Markov chain.

If we denote the stationary probability for the state S_k as $Q(S_k)$, the transition probabilities satisfy the following equations.

$$Q(S_k)=\begin{cases} \sum_{l=0}^{L-1} P(S_0|S_l)Q(S_l)+Q(S_L) & (k=0) \\ P(S_k|S_{k-1})Q(S_{k-1}) & (0<k\leq L) \end{cases} \quad (3)$$

$$\sum_{k=0}^{L} Q(S_k)=1. \quad (4)$$

We derive the stationary probabilities of the Markov chain from the simulated data of the intermittency chaos by counting the number of the corresponding state. Further, the transition probabilities are calculated from the stationary probabilities by using Eq. (3).

Figure 4 shows the time series of the obtained Markov chain model for $L=15$. In order to check the statistical property of the obtained time series, we counted the length of the laminar part. The result is shown in Fig. 5. We can say that the result is very close to that in Fig. 2.

Further, we produce the Markov chain models for various sizes of the maximum length L of the laminar part. The comparison of the properties of the original intermittency chaos and the Markov chain model is summarized in Table 1. We can confirm that the properties of the Markov chain models are similar to those of the original intermittency chaos.

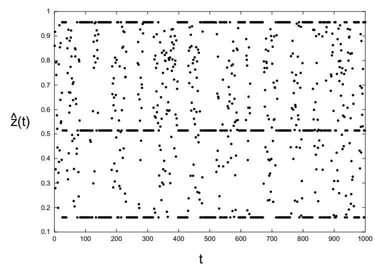

Figure 4: Time series of the Markov chain model for L=15.

4 Application to Hopfield NN for QAP

As an example of applications of the intermittency chaos and the Markov chain model, we investigate their performances when they are injected to the Hopfield NN for QAP in order to avoid the local minimum trapping problems.

QAP is expressed as follow: given two matrices, distance matrix C and flow matrix D, and find the permutation $\mathbf{p}$ which corresponds to the minimum

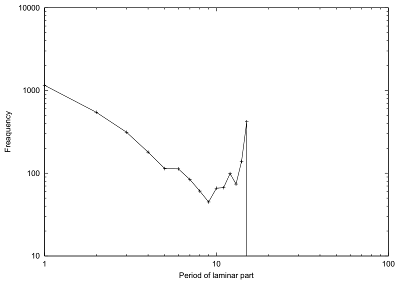

Figure 5: Distribution of period of laminar part. (Markov chain model for L=15.)

Table 1: Properties of intermittency chaos and Markov chain model.

	Ratio of laminar part		Average length of laminar part	
L	Chaos	Markov	Chaos	Markov
7	0.4215	0.4156	3.1495	3.0789
9	0.4714	0.4580	3.8271	3.7139
11	0.5049	0.5049	4.3486	4.2729
13	0.5348	0.5289	4.9732	4.9216
15	0.5558	0.5524	5.4389	5.2984
17	0.5798	0.5700	5.9185	5.7266
19	0.5905	0.6083	6.5688	6.5358
21	0.6251	0.6313	7.0888	7.1835
23	0.6422	0.6424	7.4430	7.3444
25	0.6445	0.6427	7.4853	7.4764
30	0.6878	0.6804	9.2968	8.8704
40	0.7261	0.7196	11.5463	11.1663
50	0.7655	0.7580	13.6257	12.8656
70	0.8240	0.8167	19.7186	19.2385
100	0.8563	0.8532	24.8647	24.4863

value of the objective function $f(\mathbf{p})$ in Eq. (5).

$$f(\mathbf{p}) = \sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij} D_{p(i)p(j)}, \qquad (5)$$

where C_{ij} and D_{ij} are the (i, j)-th elements of C and D, respectively, $p(i)$ is the i-th element of vector $\mathbf{p}$, and N is the size of the problem. There are many real applications which are formulated by Eq. (5).

Because QAP is very difficult, it is almost impossible to solve the optimum solutions in larger problems. The largest problem which is solved by deterministic methods may be only 20 in recent study. Further, computation time is very long to obtain the exact optimum solutions. Therefore, it is usual to develop heuristic methods which search nearly optimal solutions in reasonable time. For solving N-element QAP by the Hopfield NN, $N\times N$ neurons are required and the following energy function is defined to fire (i, j)-th neuron at the optimal position:

$$E = \sum_{i,m=1}^{N}\sum_{j,n=1}^{N} w_{im;jn} x_{im} x_{jn} + \sum_{i,m=1}^{N} \theta_{im} x_{im}. \qquad (6)$$

The neurons are coupled each other with the synaptic connection weight. Suppose that the weight between (i, m)-th neuron and (j, n)-th neuron and the threshold of the (i, m)-th neuron are described by:

$$\begin{aligned} w_{im;jn} &= -2\Big\{A(1-\delta_{mn})\delta_{ij} \\ &\quad + B\delta_{mn}(1-\delta_{ij}) + \frac{C_{ij}D_{mn}}{q}\Big\} \qquad (7) \\ \theta_{im} &= A + B \end{aligned}$$

where A and B are positive constants, and δ_{ij} is Kronecker's delta. The states of $N\times N$ neurons are unsynchronously updated due to the following difference equation:

$$x_{im}(t+1) = g\left(\sum_{j,n=1}^{N} w_{im;jn} x_{jn}(t) + \theta_{im} + \beta z_{im}(t)\right) \qquad (8)$$

where g is sigmoidal function, $z_{im}(t)$ is the intermittency chaos or the Markov chain model, and β limits amplitude of the injected time series. Note that we normalize $\hat{z}_{im}$ by Eq. (9) before the injection.

$$z_{im}(t+1) = \frac{\hat{z}_{im}(t) - \bar{z}}{\sigma_z} \qquad (9)$$

where $\bar{z}$ is the average of $\hat{z}(t)$, and σ_z is the standard deviation of $\hat{z}(t)$. Further, we use the method suggested by Sato *et al.* (1.1 in [8]) to decide firing of neurons.

5 Simulated Results

The problem used here is chosen from the site QAPLIB which collects the bench mark problems [9]. We carried out computer simulations for the problems with various sizes. The results for "Nug12" are shown in this section. The global minimum of this problem is known as 578. The parameters of the Hopfield NN are fixed as $A = 1.0$, $B = 1.0$, $q = 100$ and $\varepsilon = 0.02$ and the amplitude of the injected time series is fixed as $\beta = 0.6$. The number of updating the network N_{it} is $10,000$.

In order to evaluate the performance precisely, we neglect the once-appeared-solutions and use the two functions proposed in our previous study [10].

5.1 Depth_1

The first function *Depth_1* is defined as

$$Depth\_1 = \sum_{k=0}^{n} \{f(\mathbf{p}_k) - D_\infty\}^2 \qquad (10)$$

where D_∞ is a constant which is large enough to include the energies of all solutions, n is the number of the accepted solutions and the energy $f(\mathbf{p}_k)$ is calculated by Eq. (5) using the permutation $\mathbf{p}_k$ corresponding to the k-th solution.

The calculated result of *Depth_1* is shown in Fig. 6. We confirm that both the intermittency chaos and the Markov chain model exhibit similar tendency such that *Depth_1* decreases as the maximum length of the laminar part L increases. However, the *Depth_1* for the Markov chain model is not as large as those for the intermittency chaos.

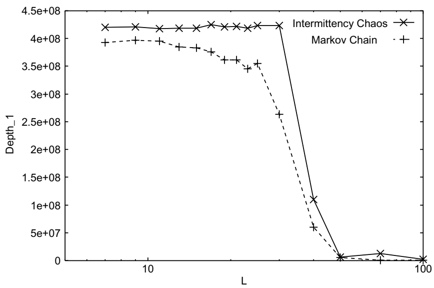

Figure 6: Result of *Depth_1* ($D_\infty = 1000$).

5.2 Depth_2

Next, we calculate *Depth_2* proposed to enhance the performance for finding a lot of good solutions [10]. The function *Depth_2* is defined as follows:

$$Depth\_2 = \sum_{k\in\mathbf{k}_g}^{n} \{f(\mathbf{p}_k) - D_{th}\}^2 - \sum_{k\notin\mathbf{k}_g}^{n} \{f(\mathbf{p}_k) - D_{th}\}^2 \quad (11)$$

$$\text{where} \quad \mathbf{k}_g = \{k \mid f(\mathbf{p}_k) \le D_{th}\}.$$

The calculated result of *Depth_2* is shown in Fig. 7. The Markov chain model gains similar performance to the intermittency chaos. This result shows that the Markov chain model has the characteristic to find a lot of nearly optimal solutions which is the important characteristic of the intermittency chaos.

6 Conclusions

In this study, a modeling method of the intermittency chaos using the Markov chain has been proposed. The performances of the intermittency chaos and the Markov chain model were investigated when they were injected to the Hopfield Neural Network for QAP. Computer simulated results showed that the proposed modeling was good enough to gain similar performance of the intermittency chaos.

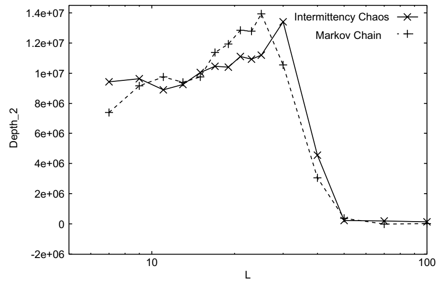

Figure 7: Result of *Depth_2* ($D_{th} = 812$).

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