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SPICE-Oriented Oscillators Analysis Using Harmonic Balance Method

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Abstract — **In this paper, we consider reactance oscillators having a negative resistances. It is known that the oscillators will have multiple oscillations near at the anti-resonant frequencies of the reactance sub-circuits. We propose here the harmonic balance method for solving the multiple oscillations, whose determining equations are replaced by the coupled equivalent sine and cosine circuits. The circuits can be solved with STC(solution curve tracing circuit) based on the Newton homotopy method. Thus, the stable and unstable oscillations can be easily found with the SPICE simulation.**

1. INTRODUCTION

Analysis of oscillators are very important for designing communication circuits because we need to know the exact oscillator frequency for mixers and modulators. There are many types of oscillators such as Colpitts, Hartley, crystal oscillators which have a unique oscillating frequency. On the other hand, there are many coupled oscillators with the same oscillator frequency which may happen interesting phenomena such as multi-mode oscillations [1]-[2]. These phenomena can be analytically explained by the analysis of the weakly nonlinear coupled oscillators [1]. Furthermore, it is known that the coupled oscillators sometimes have quasi-periodic oscillations and/or chaos phenomena [3]. In this paper, we consider the steady-state analysis of oscillator circuits having multiple oscillations. It is very difficult task to find out all the stable and unstable oscillations. There have been published many papers concerning to the steady-state analysis of oscillator circuits having a unique oscillation [4]-[9]. There are two types of the techniques such as time-domain and frequency-domain analyses. The references [5]-[7] belong to the former technique, and [8],[10] to the latter. Note that in the harmonic balance method, if we assume the DC and the K higher harmonic components for N-variables, the determining equation is described by $N(2K+1)$ algebraic equations, because the fundamental oscillator frequency must be considered as the additional variable in the analysis.

In the illustrative example, in section 4, we consider a reactance oscillator consisted of Cauer circuit and a negative resistor. We can easily synthesize the reactance Cauer circuit by specifying the resonant and anti-resonant frequencies. Thus, we can realize an oscillator having multiple oscillations whose oscillations will be arisen around the anti-resonant frequency points [11]-[13]. We propose here a SPICE-oriented Newton homotopy algorithm based on the harmonic balance method for the analysis of the multiple oscillations, which we needs not to derive the troublesome circuit equation, determining equation and the Jacobian matrix. The analysis can be easily carried out with the transient analysis of SPICE.

2. NEWTON HOMOTOPY METHOD TO THE DETERMINING EQUATION

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In order to show the ideas of our algorithm, let us consider a reactance oscillator with a negative resistance as shown by Fig. 1.

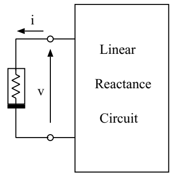


Fig. 1 A reactance oscillator.

Assume that the nonlinear characteristic of the negative resistance is discribed by the power series as follows:

$$i = C \ + C\, v + C\, v \ + C\, v \ + \cdots. \qquad (1)$$

We also assume the waveform in the following form;

$$v = V \ + V \ \cos\omega t + \sum_{k}^{K}[V_{\ k-} \ \cos k\omega t + V_{\ k-} \ \sin k\omega t]. \qquad (2)$$

Observe that the waveform does not contain $\sin\omega t$-component, because we can arbitrarily choose the time origin in the analysis of the autonomous system. Applying the *harmonic balance method* to the circuit equation shown by Fig. 1, we have the *determining equation* as follows:

$$\left.\begin{array}{l} F\ (V\ ,V\ ,\cdots,V_{\ K-}\ ,\omega)=0\ \cdots\ \mathrm{DC}\\ F\ (V\ ,V\ ,\cdots,V_{\ K-}\ ,\omega)=0\ \cdots\ \cos\omega t\\ F\ (V\ ,V\ ,\cdots,V_{\ K-}\ ,\omega)=0\ \cdots\ \sin\omega t\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ F_{\ K-}\ (V\ ,V\ ,\cdots,V_{\ K-}\ ,\omega)=0\ \cdots\ \cos K\omega t\\ F_{\ K}(V\ ,V\ ,\cdots,V_{\ K-}\ ,\omega)=0\ \cdots\ \sin K\omega t \end{array}\right\}, \qquad (3)$$

where the variabale ω denotes the fundamental frequency component to be determined, and K is the highest frequency component. Thus, the determining equation is described by a set of algebraic equations consisted of $(2K+1)$-variables and the same number of equations. Note that it is not easy to solve the nonlinear equation because it may have many solutions for the multiple oscillator circuit.

For simplicity, we set (3) to

$$\mathbf{f}(\mathbf{v}) = \mathbf{0}, \quad \mathbf{v} \in \mathbf{R}^{\ K} \ , \quad \mathbf{f}(.) : \mathbf{R}^{\ K} \ \mapsto \mathbf{R}^{\ K} \ , \qquad (4)$$

where $\mathbf{v} = [V\ ,V\ ,\cdots,V_{\ K-}\ ,\omega]^T$. Applying the *Newton homotopy method* [14] to solve (4), we have the following relation:

$$\mathbf{F}(\mathbf{v},\rho) = \mathbf{f}(\mathbf{v}) + (\rho - 1)\mathbf{f}(\mathbf{v}\ ) = \mathbf{0}, \qquad (5)$$

where ρ is an additional variable, and $\mathbf{v}$ is an initial guess. Thus, the solution curve starts at $(\mathbf{v}\ ,\rho = 0)$, and the solutions satisfying (4) is obtained at $\rho = 1$ in

Of course, the method can be applied to oscillators containing the multiple nonlinear resistors.

2K+2 dimensional space. The curve can be traced by the application of the *arc-length method* as follows:

$$\left.\begin{array}{l}\mathbf{F}(\mathbf{v},\rho)=\mathbf{0}\\ \sum_{i}^{K}\left(\frac{dv_i}{ds}\right)+\left(\frac{d\rho}{ds}\right)=1\end{array}\right\}, \quad (6)$$

where s is the arc-length from the starting point on the solution curve. These algebraic-differential equation can be transformed into a set of nonlinear equations by *backward-difference method* [16]. The algorithm is exactly equal to the transient analysis of SPICE, so that we can use SPICE for the curve tracing algorithm [17]. The circuit diagram is shown in Figs. 2(a) and (b).

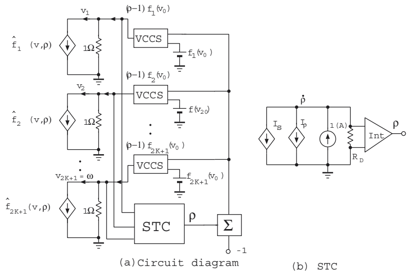


Fig. 2 Circuit diagram of the Newton homotopy method

In the above figure, "VCCS" denotes a voltage-controlled current source, and the current sources $\hat{f}_i(v,\rho)$ and I_s, I_ρ of STC are given by

$$\left.\begin{array}{l}\hat{f}_i(\mathbf{v},\rho)=F_i(\mathbf{v},\rho)-v_i,\quad i=1,2,\ldots,2K+1\\ I_s=\sum_{i}^{K}\left(\frac{dv_i}{ds}\right),\quad I_\rho=\left(\frac{d\rho}{ds}\right).\end{array}\right\}. \quad (7)$$

Note that when we assume the voltage difference at 1[Ω] resistance to be equal to $-v_i$, then we have

$$-v_i=F_i(\mathbf{v},\rho)-v_i,\quad i=1,2,\ldots,2K+1, \quad (8)$$

which satisfy the relation $\mathbf{F}(\mathbf{v},\rho)=\mathbf{0}$. On the other hand, STC(solution curve tracing circuit) realizes the second term of (6) by setting arc-length s with time t, where R_D is a sufficiently large dummy resistance to avoid a L-J cut set. Thus, we can easily trace any homotopy path from an arbitrarily chosen initial guess $\mathbf{v}$ and find the multiple oscillations at $\rho=1$ using the transient analysis of SPICE.

3. EQUIVALENT CIRCUIT MODEL OF THE DETERMINING EQUATION

In this section, we consider the equivalent sine and cosine circuits corresponding to the determining equation (4) for a weakly nonlinear circuit [15]. The equivalent circuits consist of the resistive sub-circuits, each of which has the same topology as the original one except that the reactance elements are replaced by the voltage-controlled current sources and/or the current-controlled voltage sources.

3.1 Nonlinear inductor

Assume the inductor flux is described by the current-controlled characteristic as follows:

$$\hat{\phi}(i_L)=L\ i_L+L\ i_L+L\ i_L+\cdots. \quad (9)$$

Nonlinear characteristics must be described by the power series functions.

Let us consider the terms until the Kth order higher harmonic components of ω as follows:

$$i_L=I_{L,}\ +\sum_{k}^{K}[I_{L,\ k-}\ \cos k\omega t+I_{L,\ k}\sin k\omega t]. \quad (10)$$

Substituting (10) into (9), we have the Fourier expansion as follows

$$\hat{\phi}(i_L)\cong\Phi_{L,}\ +\sum_{k}^{K}[\Phi_{L,\ k-}\ \cos k\omega t+\Phi_{L,\ k}\sin k\omega t], \quad (11)$$

where $\Phi_{L,}\ ,\Phi_{L,}\ ,\cdots,\Phi_{L,\ K}$ are analytical functions of $I_{L,}\ ,I_{L,}\ ,\cdots,I_{L,\ K}$. Thus, the inductor voltage is given by

$$\begin{array}{l}\hat{v}_L(i_L)=\frac{d\hat{\phi}(i_L)}{dt}\\ =\sum_{k}^{K}[-k\omega\Phi_{L,\ k-}\ \sin k\omega t+k\omega\Phi_{L,\ k}\cos k\omega t].\end{array} \quad (12)$$

Thus, the sine and cosine components for the kth order higher harmonic component are respectively given by

$$\begin{array}{l}V_{L,\ k}=-k\omega\Phi_{L,\ k-}\ ,\ V_{L,\ k-}\ =k\omega\Phi_{L,\ k},\\ k=1,2,\ldots,K,\end{array} \quad (13)$$

where $V_{L,\ k-}\ ,V_{L,\ k}$ are the amplitudes of the voltages for the kth frequency components. Thus, the inductor is replaced by a pair of the coupled current-controlled voltage sources as shown in Fig. 3.

3.2 Nonlinear capacitor

Assume that the capacitor charge is described by the voltage-controlled characteristic as follows:

$$\hat{q}(v_C)=C\ v_C+C\ v_C+C\ v_C+\cdots. \quad (14)$$

Set the voltage waveform v_C as follows:

$$v_C=V_C\ +\sum_{k}^{K}[V_{C,\ k-}\ \cos k\omega t+V_{C,\ k}\sin k\omega t]. \quad (15)$$

Substituting (15) into (14), we have

$$\hat{q}(v_C)\cong Q_C\ +\sum_{k}^{K}[Q_{C,\ k-}\ \cos k\omega t+Q_{C,\ k}\sin k\omega t], \quad (16)$$

where $Q_C\ ,Q_C\ ,\ldots,Q_{C,\ K}$ are analytical functions of $V_C\ ,V_C\ ,\ldots,V_{C,\ K}$. Then, the voltage-current characteristic of capacitor is given by

$$\begin{array}{l}\hat{i}_C(v_C)=\frac{d\hat{q}(v_C)}{dt}\\ =\sum_{k}^{K}[-k\omega Q_{C,\ k-}\ \sin k\omega t+k\omega Q_{C,\ k}\cos k\omega t].\end{array} \quad (17)$$

Thus, the sine and cosine components for the kth order harmonic are respectively given by

$$\begin{array}{l}I_{C,\ k}=-k\omega Q_{C,\ k-}\ ,\ I_{C,\ k-}\ =k\omega Q_{C,\ k},\\ k=1,2,\ldots,K,\end{array} \quad (18)$$

where $-k\omega Q_{C,\ k-}\ ,k\omega Q_{C,\ k}$ are the currents of the kth frequency components of the nonlinear capacitors. Thus, the capacitor is replaced by a pair of the voltage-controlled current sources as shown in Fig. 3.

3.3 Nonlinear resistor

There are two types of the voltage-controlled and current-controlled nonlinear resistors. We consider here the voltage-controlled resistor as follows:

$$\hat{i}_G(v_G) = H_G \;\; + H_G \;\; v_G + H_G \;\; v_G + H_G \;\; v_G + \cdots . \quad (19)$$

Assume the voltage waveform v_G as follows:

$$v_G = V_G \;\; + \sum_{k}^{K} [V_{G,\; k-} \;\; \cos k\omega t + V_{G,\; k} \sin k\omega t]. \quad (20)$$

Substituting (20) into (19), we have

$$\hat{i}_G(v_G) \cong \hat{I}_G \;\; + \sum_{k}^{K} [\hat{I}_{G,\; k-} \;\; \cos k\omega t + \hat{I}_{G,\; k} \sin k\omega t]. \quad (21)$$

Thus, the kth order higher harmonics of the sine and cosine components are respectively given by

$$I_{G,\; k} = \hat{I}_{G,\; k},\; I_{G,\; k-} \;\; = \hat{I}_{G,\; k-} \;\; , \quad k = 1, 2, \ldots, K. \quad (22)$$

Note that the voltage-controlled nonlinear resistors are described by the voltage-controlled resistors as shown in Fig. 3.

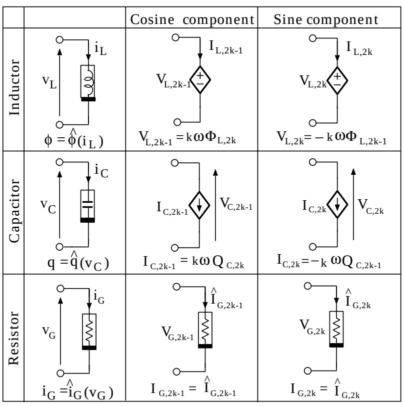


Fig. 3 Equivalent cosine and sine circuits

4. AN ILLUSTRATIVE EXAMPLE

4.1 Steady-state analysis

Consider a Cauer oscillator with a negative resistor as shown in Fig. 4. Let the nonlinear characteristic be given by

$$i_G = -C \;\; v_G + C \;\; v_G, \quad C \;\; = 1. \;\; C \;\; = 1. \quad (23)$$

At first, we design the reactance Cauer sub-circuit such that it has the following resonant and ant-resonant frequencies:

$$\left. \begin{array}{l} \text{Anti}-\text{resonant frequencies}: \omega \;\; = 1,\; \omega \;\; = 4,\; \omega \;\; = 6 \\ \text{Resonant frequencies}: \;\; \omega \;\; = 2,\; \omega \;\; = 5 \end{array} \right\}$$

Then, we have the following circuit parameters shown in Fig. 4. We also introduce small resistances for all the inductors.

This characteristic can be easily realized by the analog behavior model in SPICE.

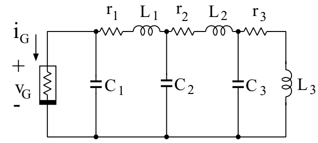


Fig. 4 Cauer Oscillator

$$C \;\; = 0.1, \quad C \;\; = 0.343, \quad C \;\; = 0.439, \quad L \;\; = 0.417$$
$$L \;\; = 0.262, \quad L \;\; = 1.058, \quad r \;\; = \cdots = r \;\; = 0.01$$

Note that since the nonlinear voltage-current characteristic of the resistor is symmetrical with respect to the origin, we can only consider the odd order higher harmonic components. Hence, we can assume the waveform v_G as follows:

$$\left. \begin{array}{l} v_G(t) = V \;\; _C \cos \omega t + \sum_{k}^{K} [V_{k} \;\; ,_C \cos(2k+1)\omega t \\ \qquad + V_{k} \;\; ,_S \sin(2K+1)\omega t] \end{array} \right\}. \quad (24)$$

Observe that we neglect the term of $\sin \omega t$ in (24) because of the autonomous system. Assume that the inductor and capacitor characteristics are linear. Then, the inductor and capacitor are replaced by the simple linear current-controlled voltage sources and voltage-controlled current sources in the sine and cosine curcuits as shown in Fig. 5. In the simulation, although we considered until the 5th order higher harmonic component ($K = 2$), the equivalent circuit for the fundamental oscillator frequency combined by the Newton homotopy method is shown in Fig. 5 for simplicity.

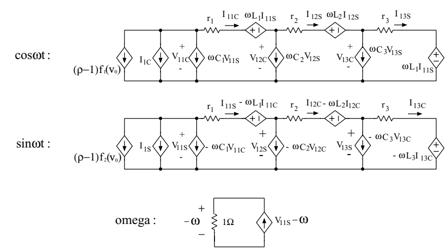
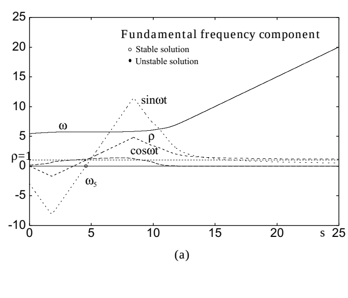


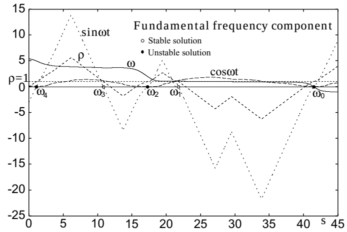
Fig. 5 Equivalent circuit describing the determining equation for the fundamental frequency component

Observe that each sub-circuit has the same structure as the original one as shown in Fig. 4. The results of solution curves are shown in Fig. 6(a) for $s < 0$ and (b)$s > 0$, where the solid line corresponds to the frequency ω. We have 6 solutions at $\rho = 1$ as follows:

$$\omega \;\; = 0,\; \omega \;\; = 1.0878,\; \omega \;\; = 2.0001,$$
$$\omega \;\; = 3.6770,\; \omega \;\; = 4.999,\; \omega \;\; = 5.722.$$



(a)



(b)

Fig. 6(a) Homotopy curves, $s < 0$, (b) $s > 0$

4.2 Verification of the solutions

Now we need to check the stabilities of the 6 solutions in Fig. 6 by the variational technique, where the variational equation at the solution $(\hat{V}, \hat{I})$ is directly obtained by setting $v = \hat{V} + \Delta v, i = \hat{I} + \Delta i$ in the circuit equation, and have

$$\left[\begin{array}{c} \Delta \dot{v} \\ \Delta \dot{i} \end{array} \right] = \mathbf{A} \left[\begin{array}{c} \Delta v \\ \Delta i \end{array} \right]. \tag{25}$$

The stabilities are checked by calculating the eigenvalues of **A**. If it has the eigenvalues containing at least one positive real part, the solution will be unstable and otherwise it will be stable. Thus, we have the following results:

Stable solutions: at ω , ω , and ω
Unstable solutions: at ω , ω , and ω

Observe that the solutions only at the anti-resonant frequencies ω , ω , and ω are stable. These results agree with the properties from the reference [11] and are described in our method by showing the unstable solutions with zero-amplitude ocsillations.

The transient waveforms starting from the initial conditions estimated by our harmonic balance method are shown in Figs. 7(a), (b) and (c). The frequency spectrums with FFT are shown in Figs. 8(a), (b) and (c). The waveforms at $\omega = 1.0556$ is the most distorted, and contains many higher harmonics. All of them have the small errors because the waveforms still have the higher harmonics larger than 5th higher harmonic component.

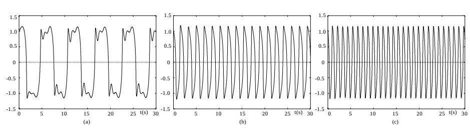


Fig. 7 The results of transient analysis whose initial conditions are estimated from our harmonic balance method.
(a) For ω , (b) For ω ,(c) For ω

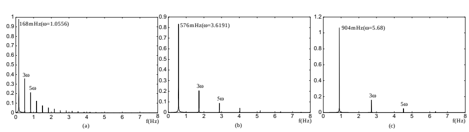


Fig. 8 Frequency spectrum for Fig.8 .
(a) $\omega = 1.0556$, (b) $\omega = 3.6191$, (c) $\omega = 5.68$

5. CONCLUSIONS AND REMARKS

In this paper, we proposed a simple algorithm using SPICE simulator for calculating multiple oscillations of a reactance oscillator, where we need not to derive any troublesome circuit equations, and to solve the determining equations. We only need to derive the Fourier coefficients for nonlinear elements in the functional forms, which can be done by the use of software such as Mathematica [18].

The Cauer oscillator with multiple oscillations sometimes happen to have a quasi-periodic oscillations. It seems that two stable oscillations are arisen in the same time, independently. For the future problem, we want to extend the algorithm to the practical circuits such as crystal oscillators.

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