Classical Wave Propagation Phenomena in Two-Layer Cellular Neural Networks

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1. Introduction

Studies of dynamic phenomena in arrays composed of oscillatory and chaotic elements are very important in such investigations as they provide a universal model for phenomena observed in other domains. From the middle of previous decade, the investigations of the spatio-temporal dynamics in cellular neural networks have been widely carried out, and many papers have been published. The obtained sound results are pattern formations and various forms of autowaves such as excitability waves, concentration waves and so on, which have been observed in many disciplines ranging from biology to physics and chemistry, etc. Up to the present, the CNN arrays, which are composed of Chua's oscillators [2]-[4], second nonlinear circuit obtained by the suitably "reducing" Chua's circuit [5] and etc. [6][7] have been mainly considered in order to generate these phenomena. A common technique in all these cases is almost using CNNs to approximate the various types of nonlinear partial differential equations, especial the well-known reaction-diffusion equations that have shown to generate Turing pattern and propagation phenomena in various continuous media.

The purpose of this paper is to investigate another class of spatio-temporal phenomena occurring in a simpler CNN structure - mutually coupled two-layer CNNs. We found that this type of propagation phenomena has the properties of reflection, permeation, superposition and etc., which are completely different from those nonlinear propagation phenomena. In particular, the conditions for generating the classical wave will be discussed.

2. Two-Layer CNNs

Firstly, let us recall simply the mutually coupled two-layer CNNs [7]-[9], which are described based on the well-known Chua-Yang's CNNs [1]. The system equations are formulated as follows:

$$\left.\begin{array}{rcl}\frac{dx_{1,ij}}{dt} & = & -x_{1,ij}+I_1\\ & + & \sum_{C(k,l)\in N_r(i,j)} A_1(i,j;k,l)y_{1,kl}\\ & + & \sum_{C(k,l)\in N_r(i,j)} B_1(i,j;k,l)u_{1,kl}\\ & + & \sum_{C(k,l)\in N_r(i,j)} C_1(i,j;k,l)y_{2,kl}\\ \frac{dx_{2,ij}}{dt} & = & -x_{2,ij}+I_2\\ & + & \sum_{C(k,l)\in N_r(i,j)} A_2(i,j;k,l)y_{2,kl}\\ & + & \sum_{C(k,l)\in N_r(i,j)} B_2(i,j;k,l)u_{2,kl}\\ & + & \sum_{C(k,l)\in N_r(i,j)} C_2(i,j;k,l)y_{1,kl}\end{array}\right\} \quad (1)$$

$$\left.\begin{array}{l}y_{1,ij}=0.5(|x_{1,ij}+1|-|x_{1,ij}-1|)\\ y_{2,ij}=0.5(|x_{2,ij}+1|-|x_{2,ij}-1|)\end{array}\right\} \quad (2)$$

where (i,j) stands for the position of a cell in the array, for $1 \le i \le M$ and $1 \le j \le N$, and the subscript 1 and 2 stand for first layer and second layer of the two-layer CNN array. The CNNs are efficient for image processing applications such as center point detection, skeletonization and etc., which have been reported in our previous studies [7][8]. In the following paragraph, we will investigate the spatio-temporal phenomena in the two-layer CNNs.

3. Investigation of Classical Wave Propagation

We had shown that the two-layer CNNs could reproduce the pattern formation and active propagation phenomena in reference 9, in which there is a common property that at least one of temporal eigenvalues of two-layer CNNs has positive real part, even if the property is also for image processing applications of the CNNs. However, when all of the temporal eigenvalues have zero or negative real parts, what will be taken place in the CNNs? This work will be discussed in this section.

3.1. Necessary Condition

Let us consider an autonomous two-layer CNN with mutual couple between layers. Its equation is described as the following form:

$$\left.\begin{array}{rcl}\dot{x}_{1,ij} &=& -x_{1,ij}+(a_1+1)y_{1,ij}+c_1y_{2,ij}\\ && +d_1\nabla^2 y_{2,ij}\\ \dot{x}_{2,ij} &=& -x_{2,ij}+(a_2+1)y_{2,ij}+c_2y_{1,ij}\\ && +d_2\nabla^2 y_{1,ij}.\end{array}\right\} \quad (3)$$

Where a_1,a_2,c_1,c_2,d_1 and d_2 are parameters. By expanding the Laplacian operator in discrete form, and comparing with Eq.(1), the two-layer CNN has the following template:

$$A_1 = a_1+1, C_1 = \begin{pmatrix} 0 & d_1 & 0 \\ d_1 & -4d_1+c_1 & d_1 \\ 0 & d_1 & 0 \end{pmatrix},$$

$$A_2 = a_2+1, C_2 = \begin{pmatrix} 0 & d_2 & 0 \\ d_2 & -4d_2+c_2 & d_2 \\ 0 & d_2 & 0 \end{pmatrix},$$

$$B_1 = 0, B_2 = 0, I_1 = 0, I_2 = 0. \quad (4)$$

On the other hand, the Laplacian operator in Eq.(3) acts on the output variables instead of the state variables, and the each output cell is a piecewise-linear nonlinear function of the cell state. If we assume that all initial states of the system at the beginning are always in the linear region, then the Eq.(3) can be written as follows:

$$\left.\begin{array}{l}\dot{x}_{1,ij} = a_1x_{1,ij}+c_1x_{2,ij}+d_1\nabla^2 x_{2,ij}\\ \dot{x}_{2,ij} = a_2x_{2,ij}+c_2x_{1,ij}+d_2\nabla^2 x_{1,ij}.\end{array}\right\} \quad (5)$$

where $x_{1;ij}$, $x_{2;ij}$ are in linear region, and $1 \leq i \leq M$, $1 \leq j \leq N$.

Thus, the above linear differential equation can be solved by decoupling it into MN-decoupled systems of two first-order linear differential equations, and considering that the MN orthonormal space-dependent eigenfunction $\phi_{MN}(m,n;i,j)$ of the discrete Laplacian operator can be assumed as follows for most boundary conditions:

$$\nabla^2\phi_{MN}(m,n;i,j) = -k^2_{MN}\phi_{MN}(m,n;i,j) \quad (6)$$

where M and N are the CNN dimensions, m and n are the summation indexes for the current space variables i and j ($i = 0,1,\cdots,M-1$; $j = 0,1,\cdots,N-1$), and k^2_{mn} are the corresponding spatial eigenvalues. In particular, for the zero-flux boundary condition, the spatial eigenfunction and eigenvalue can be assumed as the following form:

$$\nabla^2\phi_{MN}(m,n;i,j) = \cos\frac{(2i+1)m\pi}{2M}\cos\frac{(2j+1)n\pi}{2N} \quad (7)$$

and

$$k^2_{mn} = 4(\sin^2\frac{m\pi}{2M}+\sin^2\frac{n\pi}{2N}) \quad (8)$$

Then, the expected solution of the Eq.(5) can be expressed as a weighted sum of $M \times N$ orthogonal space dependent eigenfunctions $\phi_{MN}(m,n;i,j)$ in the following form:

$$\left.\begin{array}{l}x_{1,ij}(t) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1}(\alpha_{mn}e^{\lambda_{mn1}t}+\beta_{mn}e^{\lambda_{mn2}t})\phi_{MN}(m,n;i,j)\\ x_{2,ij}(t) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1}(\gamma_{mn}e^{\lambda_{mn1}t}+\delta_{mn}e^{\lambda_{mn2}t})\phi_{MN}(m,n;i,j)\end{array}\right\} \quad (9)$$

where α_{mn}, β_{mn}, γ_{mn}, δ_{mn} are constants depending on the initial conditions. λ_{mn1}, λ_{mn2} are the roots of the following characteristic Eq.(10), which are influenced by spatial eigenvalue k^2_{mn} corresponding to spatial eigenfunction.

$$\det\left|\lambda_{mn}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}a_1 & c_1-k^2_{mn}d_1\\ c_2-k^2_{mn}d_1 & a_2\end{bmatrix}\right| = 0 \quad (10)$$

and

$$\lambda_{mn}[k^2_{mn}] = \frac{1}{2}\Big[(a_1+a_2)\pm \sqrt{(a_1-a_2)^2+4(k^2_{mn}d_2-c_2)(k^2_{mn}d_1-c_1)}\Big] \quad (11)$$

The equation (9) is important because the cell states of the two-layer CNN are given as the time-dependent weighted sums of spatial eigenfunctions. From this equation, it can be derived that, if one of the temporal eigenvalues has positive real part, the two-layer CNN becomes unstable in this linear region, and the cell states do arise and enter into the nonlinear region (i.e., saturation region or partial saturation region). In this situation, the two-layer CNNs can be used for modeling active media for nonlinear phenomena such as pattern formations and autowaves [7], and for image processing applications. However, if all of temporal eigenvalues have zero or negative real part, the all of states of the CNN are in linear region. Thus, the CNNs have the properties of the linear space, such as superposition etc. Therefore, we can use this CNN to model passive media with non-loss or loss in linear space.

3.2. Classical Wave Propagation

In this subsection, the investigations on passive propagation phenomena with/without loss are carried out in two-layer CNNs, which enable us to find out some basic properties of these phenomena. In the following simulation, the CNN array consists of 100×100 cells with zero-flux boundary condition, and the initial state of the first layer CNN is set as Fig.1(a), and the second layer is set as zero initial state. If the parameter set: $a_1 = 0$,

$a_2 = 0$, $d_1 = -0.5$, $d_2 = 0.5$, $c_1 = -5$, and $c_2 = 5$ is selected, from Eq.(11) we have that all of temporal eigenvalues of the CNN are complex numbers with zero real part and the isolated cell without couple behaves as oscillator with limited cycle. In other words, the two-layer CNN is a system without loss. This type of two-layer CNN can be used for modeling passive media without loss. Fig.1 shows the simulation result. Analysis of the simulation result shows that circular waves propagating from the initialized position are generated. These circular waves propagate in all directions through the plane array and their amplitudes decrease with propagating. When the waves collide with each other, they don't disappear and reflect, but permeate through and superpose on each other. When they collide with the boundary, the waves are reflected. Moreover, after the waves are repeatedly reflected and superposed, the initialized positions become invisible. These behaviors behave those the classical wave does. The wave propagation phenomenon keeps up forever duo to zero real parts of temporal eigenvalues. This dynamic process can be observed from the graph of the sum of State-error Square varying with the integration time shown by Fig.8. The sum of State-error Square is defined as:

$$S_{CNN}(t) = \sum_{i=1}^{M}\sum_{j=1}^{N}((x_{1,ij}(t+\Delta t) - x_{1,ij}(t))^2 + (x_{2,ij}(t+\Delta t) - x_{2,ij}(t))^2) \qquad (12)$$

From the graph, we can see that the sum of State-error Square converges on a horizontal line. On the other hand, if we select the parameter set to satisfy all temporal eigenvalues of the CNN having negative real parts, then the CNN is a damped system, each cell state will decay to zero, which can be derived from Eq.(9). This situation is suitable for model passive media with loss. For example, if the parameter set as $a_1 = -0.04$, $a_2 = 0$, $d_1 = -0.5$, $d_2 = 0.5$, $c_1 = -5$, and $c_2 = 5$ is selected, then all of temporal eigenvalues of the CNN are complex numbers with negative real part and the isolated cell without couple behaves damped oscillator. The simulation result is shown by Fig.3. From the analysis of the simulation result, we can see that the propagation phenomenon has the same properties with the above one. However, the network is calm in the finals due to the temporal eigenvalues having negative real part. The dynamic process can be similarly demonstrated from the graph of sum of State-Error Square shown by Fig.4.

4. Conclusions

In this paper, we have investigated the spatio-temporal phenomena in the two-layer CNNs, and found the necessary condition for these phenomena. We can use the two-layer CNN to approximate the passive media as well. Based on our research results, the two-layer CNN can be considered not only for modeling active media, but also for modeling passive media with/without loss. All of these facts show that the two-layer CNNs have a real potential for expansion.

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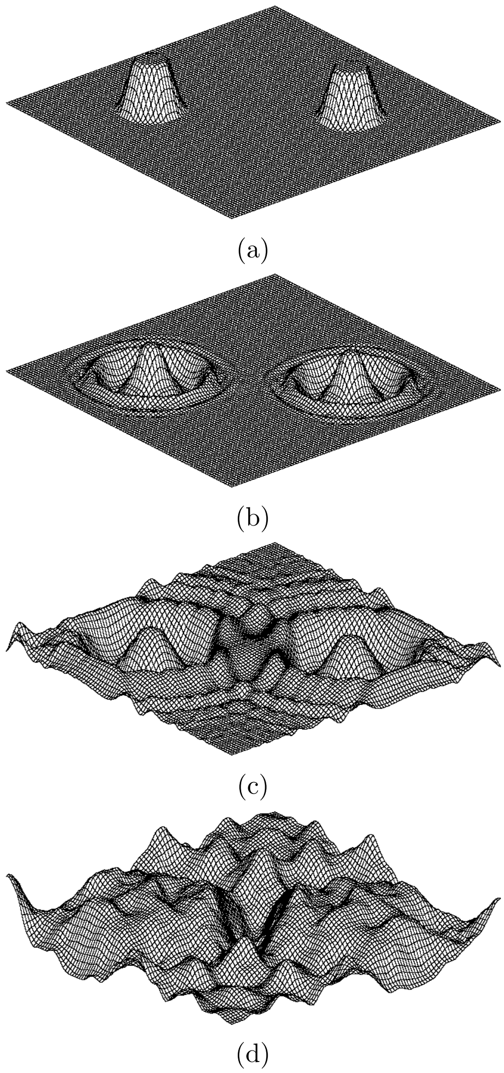

Figure 1: A simulation for classical wave propagation without loss. (a) is initial state, (b)-(d) show three snapshots observed in different iterates.

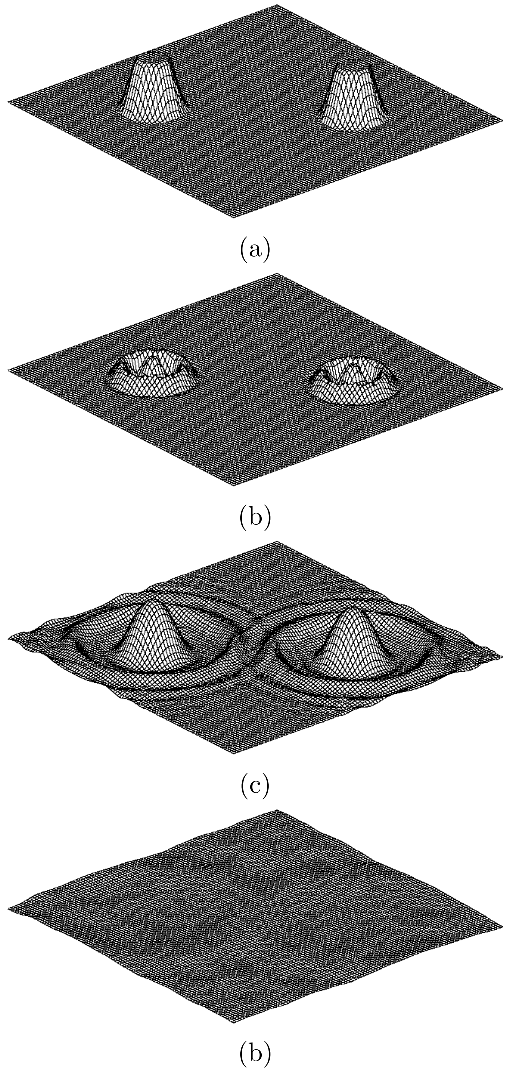

Figure 3: A simulation for classical wave propagation with loss. (a) is initial state, (b) (c) and (d) show four snapshots observed in different iterates.

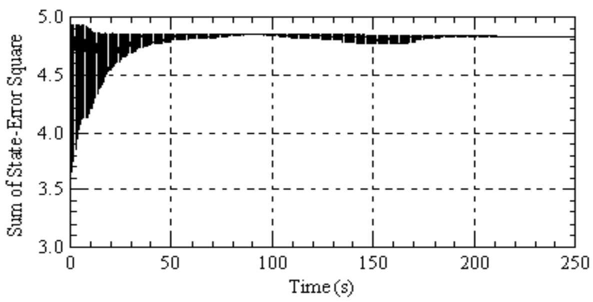

Figure 2: Graph of sum of state-error square via time for classical wave propagation without loss.

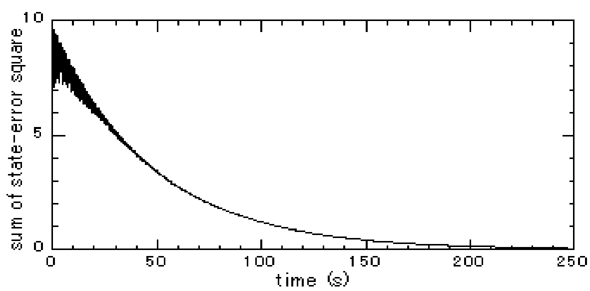

Figure 4: Graph of sum of state-error square via time for classical wave propagation with loss.