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Phase-Wave Propagation Phenomena in One-Dimensional Two-Layer CNNs

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1. Introduction

Dynamic property of networks of oscillatory and chaotic elements is one of the very lively studied topics. Many papers of international conferences and journals are devoted uniquely to studies of spatially extended systems, active media, and coupled lattices showing important areas where studies of dynamic phenomena in coupled oscillators find potential applications. Since Cellular Neural Networks (CNNs) were invented in 1988 [1], the CNN paradigm provides a flexible framework (or universal model) to describe spatio-temporal dynamics in discrete space and - perhaps more importantly from a practical point of view - allows for efficient VLSI implementation of analogue, array-computing structure. Many nonlinear phenomena such as pattern formation, autowaves and etc, which come from many disciplines ranging from biology to physics and chemistry, have been reproduced in the CNNs, and the related literatures have been reported [8]-[11].

On the other hand, a lot of studies on phase relationship of mutually coupled oscillators or chaotic circuits have been widely carried out. Endo and Mori [2]-[4] have studied van der Pol oscillators as a ladder, ring or two-dimensional array, and confirmed that several modes of synchronization. We have also discovered very interesting wave propagation phenomena of phase-difference in van der Pol oscillators coupled by inductors as a ladder [5]. Unfortunately, the basic circuits of the above models contain inductive and high-order nonlinearly resistive devices resulting in circuit sophistication, so that it is not suitable to develop the implementation of the large-scale arrays of these circuits. In this paper, we will investigate these dynamic behaviors encountered in a simpler circuit structure. This structure is one-dimensional array of two-layer CNN cells, in which there exists resistive couple between cells and each cell has a well-known piecewise-linear nonlinear output function at the output stage. Depending on the application of couple coefficients between cells, the initial conditions of cells and the boundary conditions imposed on the array, we observed that many kinds of very interesting propagation phenomena of phase difference, such as synchronization, phase-wave and phase-inversion-wave, can be regenerated in the one-dimensional two-layer CNN array.

2. One-Dimensional Two-Layer CNN Array

In this section, we describe the circuit array used for investigating the propagation phenomena of phase differences between adjacent cells. This array is a one-dimensional two-layer CNN with constant template, with circuit topology simpler than those reported in literatures [2]-[5], which is developed from the original Chua-Yang CNNs. In other words, the array consists of the cells of two-layer CNN with resistive couple. The state equation of each cell is defined as follows:

$$\left.\begin{aligned} \dot{x}_{1;i} &= -x_{1;i} + A_1 * y_{1;i} + B_1 * u_{1;i} + C_1 * y_{2;i} + I_1 \\ \dot{x}_{2;i} &= -x_{2;i} + A_2 * y_{2;i} + B_2 * u_{2;i} + C_2 * y_{1;i} + I_2 \end{aligned}\right\} \quad (1)$$

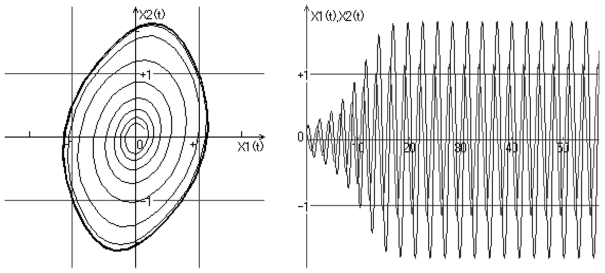
with output equation

$$\left.\begin{aligned} y_{1;i} &= 0.5(|x_{1;i}+1| - |x_{1;i}-1|) \\ y_{2;i} &= 0.5(|x_{2;i}+1| - |x_{2;i}-1|) \end{aligned}\right\} \quad (2)$$

$$i = 1, 2, ..., N.$$

where A, B, C, and I are respectively the feedback template, control template, coupled template, and bias current, and the subscript 1 and 2 stand for first layer and second layer of the two-layer CNN array. In this paper, we restrict our discussions to the following form of template.

$$\begin{aligned} A_1 &= \left[\ d_1 \quad a_1 \quad d_1\ \right],\ C_1 = \left[\ 0 \quad c_1 \quad 0\ \right], \\ A_2 &= \left[\ d_2 \quad a_2 \quad d_2\ \right],\ C_2 = \left[\ 0 \quad c_2 \quad 0\ \right], \\ & B_1 = B_2 = 0,\ I_1 = I_2 = 0. \end{aligned} \quad (3)$$



(a) (b)

Figure 1: (a) Phase portrait in the $x_{1;i}$–$x_{2;i}$ plane of the limit cycle of the isolated two-layer CNN Cell. (b) Time evolution of $x_{1;i}$ and $x_{2;i}$.

It has been proved [6][7] that for a particular choice its parameters a_1, a_2, c_1, and c_2, this nonlinear second-order cell in absence of couple to its neighbor cells oscillates with a limit cycle centered to the origin and symmetric with respect to the phase plane axis, which is shown in Fig. 1 by adopting $a_1 = 1.6$, $a_2 = 0.7$, $c_1 = -2$, and $c_2 = 4$.

3. Phase-Wave Propagation Phenomena

3.1. Phase-Waves

The phase-waves were firstly introduced in literature [5], which are observed from the changes of phase difference between adjacent cells in a ladder composed of van der Pol oscillators coupled by inductors. The phase difference is defined as follows:

$$\phi_{i,i+1}(n) = 2\pi \times \frac{t_i(n) - t_{i+1}(n)}{t_i(n) - t_i(n-1)} \tag{4}$$

where $t_i(n)$ is time when the state $x_{1;i}$ crosses 0[V] at n-th period. In all of the following Figs.4, 5 and 6, the vertical axes are the sum of the first layer outputs of adjacent cell, and the horizontal axes are time. Hence, the diagrams show qualitatively how phase differences between adjacent cells change as time goes, where white regions correspond to the state that two adjacent cells/oscillators are anti-phase synchronization, and black regions to the in-phase synchronization. Figure 4(a) shows wave propagation phenomenon of phase differences, which is observed from one-dimensional array, composed of 14 two-layer CNN cells with zero-flux condition. The initial condition of the first layer of this array is set as Fig.3, where the black squares denote cells having value 1 and the white squares value -1. The initial condition of the second layer is zero. The parameter set: $a_1 = 1.2$, $a_2 = 1.1$, $c_1 = -2$, $c_2 = 4$, $d_1 = 0.1$,

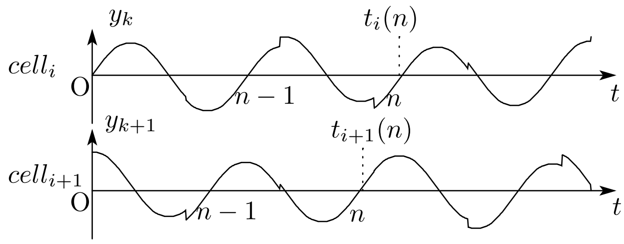


Figure 2: Definition of phase difference.

and $d_2 = -0.1$ is adopted. Analysis of this simulation indicates that the anti-phase between adjacent cells is transferred gradually from one end to the other end of the CNNs array, and reflected at the end of array. This transfer process of phase continuously exists in the CNN array, so called "phase-inversion-wave". In the following sections, we investigate the influence of the array boundary conditions, the cell initial states and the coupling parameters on the phenomena.

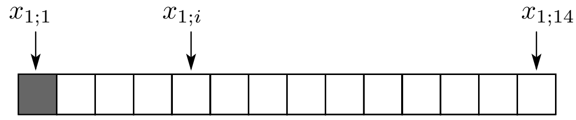


Figure 3: A initial condition for one-dimensional array of 14 cells.

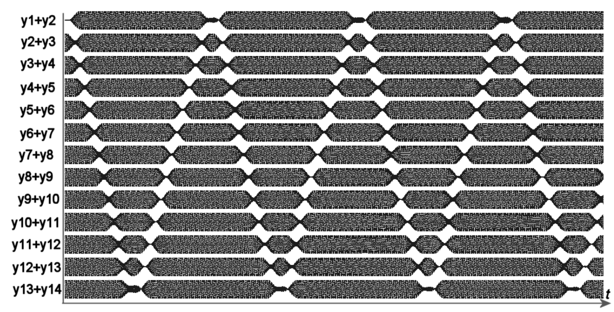


Figure 4: Phase-inversion-wave in a one-dimensional array of 14 cells.

3.2. Influence of Coupling Parameters

In this section, the influence of coupling parameters between adjacent cells on the propagation of phase-waves is investigated in a one-dimensional array composed nine two-layer CNN cells with zero-flux boundary condition. The initial state of the first layer of array is shown in

Fig.5, and the second layer initial state is zero. The parameter set is the same with the last simulation except parameters d_1 and d_2. We respectively carry out three simulations by adopting parameters $d_1 = 0.1$ and $d_2 = -0.1$, $d_1 = 0.15$ and $d_2 = -0.15$, and $d_1 = 0.2$ and $d_2 = -0.2$. The simulation results are respectively shown in Figs.6 (a), (b) and (c). As we have seen, the propagation speed of phase-wave is directly proportional to the coupling parameters (coupling conductance) d_1 and d_2 between adjacent cells.

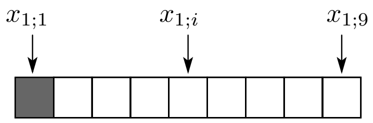


Figure 5: A initial condition for one-dimensional array of 9 cells.

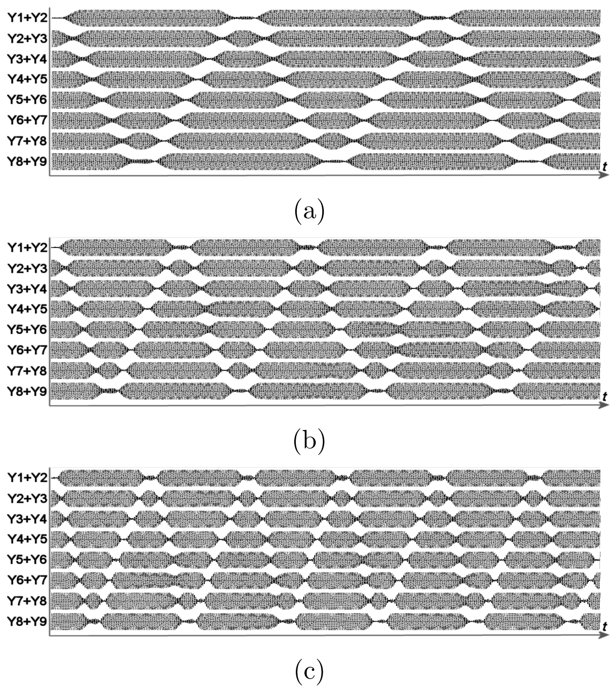


Figure 6: Influence of couple parameters on phase-inversion-wave

3.3. Influence of Initial States

We investigate the influence of different initial states on the phase-wave propagation in a one-dimensional array composed 18 two-layer CNN cells with zero-flux boundary condition. The parameter set: $a_1 = 1.1$, $a_2 = 1.1$, $c_1 = -3$, $c_2 = 5$, $d_1 = -0.03$, and $d_2 = 0.03$ is adopted. Figure 7 shows three different initial conditions. We obtain three different patterns of wave propagation phenomena shown in Fig.8. The analysis of the simulation indicates that the wave reflections take place at both ends of array or the positions that two phase-waves collide with each other.

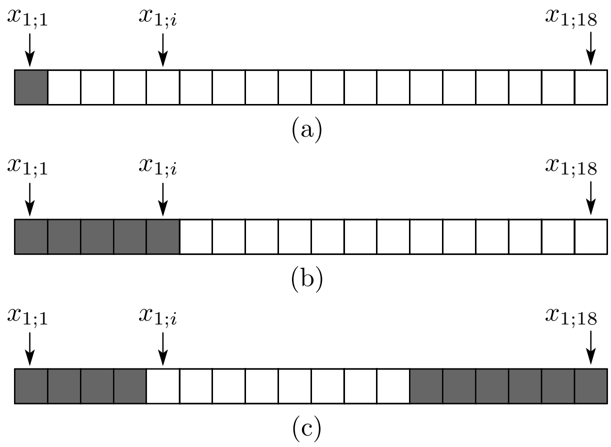


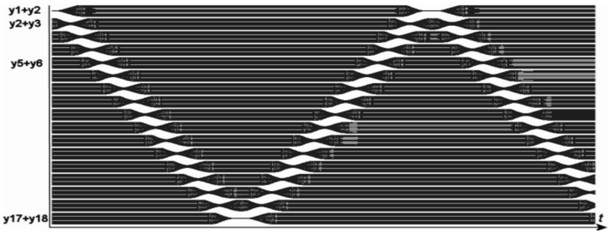
Figure 7: Three initial states for one-dimensional array of 18 cells.

3.4. Influence of Boundary Conditions

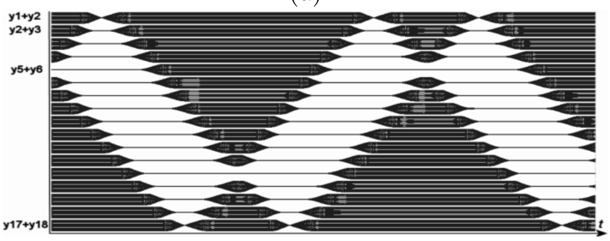
In this section, two simulations are carried our to observe what phenomena arise with adopting two different boundary conditions – zero-flux and ring in the same circuit array composed of 14 cells. The parameter set is the same with the last simulation. Two different phase-inversion-waves are shown in Fig.9, where (a) is for zero-flux boundary and (b) is for ring boundary. We found that the phase-wave don't be reflected at the both ends of array, but pass across the boundary and appear from the opposite side again in the case of ring boundary condition.

4. Conclusions

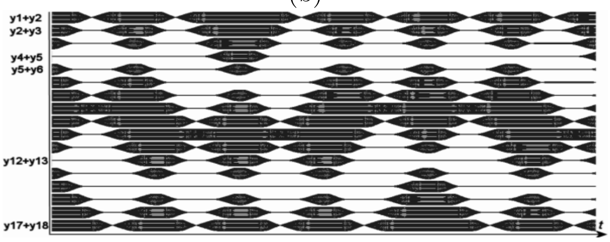
In this study, we have proposed a simpler circuit structure – one-dimensional arrays of two-layer CNN cells for investigating the phase-wave propagation phenomena. We found that the propagation phenomena of phase-wave, especially phase-inversion-wave, can be reproduced in the CNN array composed of almost numbers of cells. Moreover, we have investigated qualitatively the influences of the coupling parameters, the initial states, and the boundary conditions on these propagation phenomena. In future works, we will try to find out the patterns of these dynamics and clarify their mechanism.



(a)

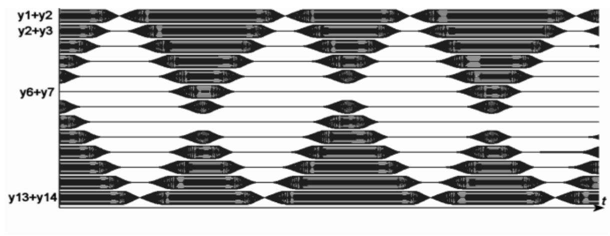


(b)

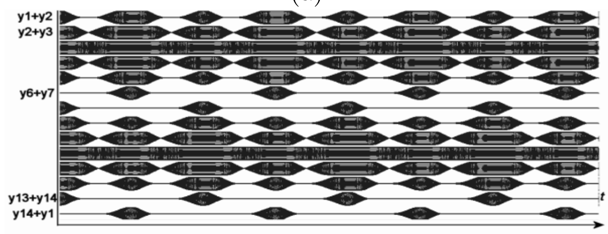


(c)

Figure 8: Influence of three kinds of initial conditions on phase-inversion-wave



(a)



(b)

Figure 9: Influence of boundary conditions on phase-inversion-wave

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